Es 1

1) $\int x^3 e^{-x} dx$ = per parti $\quad h(x) = e^{-x} \quad H(x) = -e^{-x}$

$f(x) = x^3 \quad f'(x) = 3x^2$

$= -e^{-x} \cdot x^3 - \int (-e^{-x}) \cdot 3x^2 dx = -e^{-x} x^3 + 3 \int e^{-x} x^2 dx =$

= per parti $\quad h(x) = e^{-x} \quad H(x) = -e^{-x} \quad =$

$f(x) = x^2 \quad f'(x) = 2x$

$= -e^{-x} x^3 + 3 \left[-e^{-x} x^2 - \int (-e^{-x}) \cdot 2x \, dx \right] =$

$= -e^{-x} x^3 - 3e^{-x} x^2 + 6 \int e^{-x} x \, dx =$ per parti

$f(x) = x \to f'(x) = 1$

$h(x) = e^{-x} \to H(x) = -e^{-x}$

$= -e^{-x} x^3 - 3e^{-x} x^2 + 6 \left[-e^{-x} \cdot x - \int (-e^{-x}) \cdot 1 \, dx \right] =$

$= -e^{-x} x^3 - 3e^{-x} x^2 - 6xe^{-x} - 6e^{-x} + c$

2) $\int x^2 \sin x \, dx$ = per parti = $h(x) = \sin x \quad H(x) = -\cos x$

$f(x) = x^2 \quad f'(x) = 2x$

$= -\cos x \cdot x^2 - \int (-\cos x) \cdot 2x \, dx = -\cos x \, x^2 + 2 \int \cos x \cdot x \, dx$

= per parti $h(x) = \cos x \quad H(x) = \sin x$ $= -\cos x \cdot x^2 +$

$f(x) = x \quad f'(x) = 1$

$+ 2\left[\sin x \cdot x - \int \sin x \cdot 1 \, dx\right] = -\cos x \cdot x^2 + 2 \sin x \cdot x + 2\cos x + c$

3) $\int \arcsin x \, dx$ = per parti $h(x) = 1 \quad H(x) = x$ $=$

$f(x) = \arcsin x \quad f' = \frac{1}{\sqrt{1-x^2}}$

$= x \cdot \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} \, dx = x \arcsin x + \sqrt{1-x^2} + c$

$\int \frac{x}{\sqrt{1-x^2}} dx =$ cambio variabile $= \int \frac{1}{\sqrt{y}} \left(-\frac{1}{2}\right) dy =$

$y = 1 - x^2 \quad dy = -2x \, dx$

$-\frac{1}{2} dy = x \, dx$

$= -\frac{1}{2} \int y^{-1/2} dy = -\frac{1}{2} \frac{1}{-\frac{1}{2}+1} y^{-\frac{1}{2}+1} + c = -y^{\frac{1}{2}} + c = -\sqrt{1-x^2} + c$

4) $\int x(\lg x)^2\,dx =$ per parti $=$ $h(x)=x \quad H(x)=x^2/2$

$f(x)=(\lg x)^2 \quad f'(x)=2\lg x\cdot\frac{1}{x}$

$= \frac{x^2}{2}(\lg x)^2 - \int \frac{\cancel{x^2}}{\cancel{2}}\cdot\cancel{2}\lg x\cdot\frac{1}{\cancel{x}}\,dx =$ per parti $\quad h(x)=x \to H=\frac{x^2}{2}$

$f=\lg x \quad f'=\frac{1}{x}$

$= \frac{x^2}{2}(\lg x)^2 - \left[\frac{x^2}{2}\lg x - \int \frac{\cancel{x^2}}{2}\cdot\frac{1}{\cancel{x}}\,dx\right] = \frac{x^2}{2}(\lg x)^2 - \frac{x^2}{2}\lg x +$

$+ \frac{1}{2}\int x\,dx = \frac{x^2}{2}(\lg x)^2 - \frac{x^2}{2}\lg x + \frac{1}{4}x^2 + c.$

5) $\int x \operatorname{arctg} x\,dx =$ per parti $\quad h(x)=x \to H(x)=x^2/2$

$f(x)=\operatorname{arctg} x \quad f'(x)=\frac{1}{1+x^2}$

$= \frac{x^2}{2}\operatorname{arctg} x - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int 1\,dx +$

$\frac{1}{2}\frac{x^2}{(1+x^2)} = \frac{1}{2}\frac{x^2+1}{x^2+1} = \frac{1}{2} + \frac{1}{2(x^2+1)}$

$+\frac{1}{2}\int \frac{1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}x + \frac{1}{2}\operatorname{arctg} x + c$

6) $\int \frac{\cos x}{1+(\sin x)^2} dx = \begin{array}{l} y=\sin x \\ dy = \cos x\, dx \end{array} = \int \frac{dy}{1+y^2} =$

$= \operatorname{arctg} y + c = \operatorname{arctg}(\sin x) + c$

7) $\int \frac{1}{x^3} e^{\frac{1}{x}} dx = \quad y=\frac{1}{x} \quad dy = -\frac{1}{x^2} dx \qquad \frac{1}{x^3} = \frac{1}{x} \cdot \left(\frac{1}{x^2}\right)$

$= -\int y e^y dy =$ per parti: $\quad \begin{array}{l} h(y) = e^y \rightarrow H(y) = e^y \\ f(y) = y \rightarrow f'(y) = 1 \end{array} =$

$= -\left[y e^y - \int 1 \cdot e^y dy \right] = -y e^y + e^y + c = -\frac{1}{x} e^{\frac{1}{x}} + e^{\frac{1}{x}} + c$

8) $\int \frac{1}{x^2+6x+9} dx = \int \frac{1}{(x-3)^2} dx = \frac{1}{-2+1} (x-3)^{-2+1} + c = -\frac{1}{(x-3)} + c$

$x^2+6x+9=0 \qquad x_{1,2} = 3 \Rightarrow x^2+6x+9 = (x-3)^2$

9) $\int \frac{1+x}{x^2+2}dx = \int \frac{1}{x^2+2}dx + \int \frac{x}{x^2+2}dx =$ $x^2+2=0$ non ha radici ($\Delta < 0$)

$= \frac{1}{\sqrt{2}} \operatorname{arctg}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{2}\int \frac{2x}{x^2+2}dx = \frac{1}{\sqrt{2}} \operatorname{arctg}\frac{x}{\sqrt{2}} + \frac{1}{2}\lg(x^2+2)$
$+c$

10) $\int \frac{x}{x^2-4}dx$ $\quad x^2-4=(x-2)(x+2)$

fratti semplici $\quad \frac{x}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{(A+B)x + 2A-2B}{(x-2)(x+2)}$

$\Rightarrow \begin{cases} A+B=1 \\ 2A-2B=0 \end{cases} \quad \begin{cases} A=B=\frac{1}{2} \\ A=B \end{cases}$

$\int \frac{x}{x^2-4}dx = \int \frac{1}{2}\frac{1}{x-2}dx + \int \frac{1}{2}\frac{1}{x+2}dx = \frac{1}{2}\lg|x-2| + \frac{1}{2}\lg|x+2|$
$+c$

$= \frac{1}{2}\lg|x-2||x+2| + c = \frac{1}{2}\lg|x^2-4| + c$

altro modo per risolvere:

$$\int \frac{x}{x^2-4}\,dx = \frac{1}{2}\int \frac{2x}{x^2-4}\,dx = \frac{1}{2}\lg|x^2-4| + c.$$

$y = x^2-4 \quad dy = 2x\,dx$

11) $$\int \frac{e^x}{e^{2x}-3e^x+2}\,dx = \begin{matrix} y = e^x \\ dy = e^x dx \end{matrix} = \int \frac{1}{y^2-3y+2}\,dy$$

$y^2-3y+2=0 \Rightarrow y = \frac{3\pm 1}{2} \begin{matrix} \nearrow +2 \\ \searrow +1 \end{matrix}$

$(y^2-3y+2) = (y-2)(y-1)$

fratti semplici: $\frac{A}{y-2} + \frac{B}{y-1} = \frac{1}{y^2+3y+2} = \frac{(A+B)y - A - 2B}{(y-2)(y-1)}$

$$\begin{cases} A+B=0 \\ -A-2B=1 \end{cases} \quad \begin{cases} A=-B \\ -A+2A=1 \end{cases} \quad \begin{cases} B=-1 \\ A=1 \end{cases}$$

$$\int \frac{1}{y^2+3y+2}\,dy = \int \frac{1}{y-2}\,dy + \int \frac{-1}{y-1}\,dy = \lg|y-2| - \lg|y-1| + c =$$

$$= \lg \frac{|y-2|}{|y-1|} + c = \lg \frac{|e^x - 2|}{|e^x - 1|} + c.$$

12) $\displaystyle\int \frac{1}{x(\lg x + 3)} dx$ $\quad y = \lg x \quad dy = \frac{1}{x} dx \quad =$

$$= \int \frac{1}{y^2+3} dy = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{y}{\sqrt{3}} + c = \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{\lg x}{\sqrt{3}}\right) + c$$

Es 2.

1) $\displaystyle\int_{-1}^{1} x e^{-2x} dx =$ per parti $h(x) = e^{-2x} \Rightarrow H(x) = -\frac{1}{2} e^{-2x} =$

$f(x) = x \Rightarrow f'(x) = 1$

$$= \left[-\frac{1}{2} e^{-2x} x\right]_{-1}^{1} - \int_{-1}^{1} \left(-\frac{1}{2} e^{-2x}\right) \cdot 1 \, dx = \left[-\frac{1}{2} e^{-2x} \cdot x\right]_{-1}^{1} + \frac{1}{2} \left[-\frac{1}{2} e^{-2x}\right]_{-1}^{1} =$$

$$= -\frac{1}{2} e^{-2} + \frac{1}{2} e^{2}(-1) - \frac{1}{4} e^{-2} + \frac{1}{4} e^{2} = -\frac{1}{4} e^{2} - \frac{3}{4} e^{-2}$$

2) $\int_0^{\pi^2} \sin\sqrt{x}\, dx =$ $\quad y=\sqrt{x} \quad x=y^2 \quad dx=2y\,dy$

$x=0 \to y=0$

$x=\pi^2 \to y=\pi$

$= \int_0^{\pi} \sin y \cdot 2y\, dy = 2\int_0^{\pi} \sin y \cdot y\, dy =$ per parti: $h(y)=\sin y$ $=$

$H(y)=-\cos y$

$f(y)=y \quad f'(y)=1$

$= 2\left[-\cos y \cdot y\right]_0^{\pi} - 2\int_0^{\pi}(-\cos y)\, dy =$

$= 2\left[-\cos y \cdot y\right]_0^{\pi} + \left[\sin y\right]_0^{\pi} = -2\overbrace{\cos\pi}^{=-1}\cdot\pi + 2\underbrace{\cos 0}_{=1}\cdot 0 +$

$+ \underbrace{\sin\pi}_{=0} - \underbrace{\sin 0}_{=0} = 2\pi$

3) $\int_1^{e} (\lg x)^2\, dx =$ per parti $\quad h(x)=1 \quad H(x)=x$

$f(x)=(\lg x)^2 \quad f'=2\lg x\cdot\frac{1}{x}$

$= \left[x\cdot(\lg x)^2\right]_1^{e} - \int_1^{e} \cancel{x}\cdot 2\lg x\cdot\frac{1}{\cancel{x}}\, dx$ $=$ per parti $h(x)=1 \Rightarrow H(x)=x$

$f(x)=\lg x \Rightarrow f'(x)=1$

$$= \left[x(\lg x)^2\right]_1^e - 2\left\{\left[x\lg x\right]_1^e - \int_1^e x\cdot\frac{1}{x}\,dx\right\} =$$

$$= \left[x(\lg x)^2\right]_1^e - 2\left[x\lg x\right]_1^e + 2\left[x\right]_1^e = e\cdot(\underbrace{\lg e}_{=1})^2 - 1(\underbrace{\lg 1}_{=0})^2 +$$

$$-2e\underbrace{\lg e}_{=1} + 2\cdot 1\cdot\underbrace{\lg 1}_{=0} + 2e - 2\cdot 1 = e - 2e + 2e - 2 = e - 2.$$

4) $\displaystyle\int_0^{1/2} \operatorname{arctg}(2x)\,dx$ per parti $g'(x)=1$ $G(x)=x$

$f(x)=\operatorname{arctg}2x$ $\quad f' = 2\cdot\dfrac{1}{1+4x^2}$

$$= \left[x\operatorname{arctg}2x\right]_0^{1/2} - \int_0^{1/2} x\cdot\frac{2}{1+4x^2}\,dx =$$

$$\int\frac{2x}{1+4x^2}\,dx = \quad y=1+4x^2,\ dy=8x\,dx \quad = \frac{1}{4}\int\frac{1}{y}\,dy = \frac{1}{4}\lg|y| + c = \frac{1}{4}\lg(1+4x^2)+c$$

$$= \left[x \operatorname{arctg} 2x\right]_0^{1/2} - \left[\frac{1}{4} \lg(1+4x^2)\right]_0^{1/2} =$$

$$= \frac{1}{2} \underbrace{\operatorname{arctg} 1}_{= \pi/4} - 0 \cdot \operatorname{arctg} 0 - \frac{1}{4} \lg(1+1) + \frac{1}{4} \overbrace{\lg(1)}^{= 0} =$$

$$= \frac{\pi}{8} - \frac{1}{4} \lg 2 .$$

5) $$\int_{-1}^{0} \frac{1-2x}{x^2+2x-3}\, dx$$

fratti semplici

$$x^2+2x-3 = (x-1)(x+3)$$

$$\frac{1-2x}{x^2+2x-3} = \frac{A}{x-1} + \frac{B}{x+3} = \frac{(A+B)x + 3A - B}{(x-1)(x+3)}$$

$$\begin{cases} A+B = -2 \\ 3A-B = 1 \end{cases} \quad \begin{cases} 4A = -1 \\ B = 3A-1 \end{cases}$$

$$\begin{cases} A = -1/4 \\ B = -\frac{7}{4} \end{cases}$$

$$\int_{-1}^{0} \frac{1-2x}{x^2+2x-3}\, dx = \int_{-1}^{0} \frac{-1/4}{x-1}\, dx + \int_{-1}^{0} \frac{-7/4}{x+3}\, dx =$$

$$= -\frac{1}{4}\left[\lg|x-1|\right]_{-1}^{0} - \frac{7}{4}\left[\lg|x+3|\right]_{-1}^{0} = \cancel{-\frac{1}{4}\lg 1} + \frac{1}{4}\lg 2$$

$$-\frac{7}{4}\lg 3 + \frac{7}{4}\lg 2 = 2\lg 2 - \frac{7}{4}\lg 3.$$

6) $\displaystyle\int_2^3 \frac{x}{2x^2-x-1}\,dx$

$2x^2-x-1 = 2(x-1)\left(x+\frac{1}{2}\right)$

fratti semplici

$$\frac{x}{2x^2-x-1} = \frac{1}{2}\left[\frac{A}{x-1} + \frac{B}{x+\frac{1}{2}}\right] = \frac{(A+B)x + \frac{1}{2}A - B}{2(x-1)\left(x+\frac{1}{2}\right)}$$

$$\begin{cases} A+B=1 \\ \frac{1}{2}A - B = 0 \end{cases} \quad \begin{cases} 3B=1 \\ A=2B \end{cases} \quad \begin{matrix} B=\frac{1}{3} \\ A=\frac{2}{3} \end{matrix}$$

$$\Delta = \frac{1}{2}\left[\int_2^3 \frac{\frac{2}{3}}{x-1}\,dx + \int_2^3 \frac{\frac{1}{3}}{x+\frac{1}{2}}\,dx\right] = \frac{1}{2}\left[\frac{2}{3}\lg|x-1| + \frac{1}{3}\lg\left|x+\frac{1}{2}\right|\right]_2^3 =$$

$$= \frac{1}{2}\left[\frac{2}{3}\lg 2 + \frac{1}{3}\lg\frac{7}{2} - \cancel{\frac{2}{3}\lg 1} - \frac{1}{3}\lg\frac{5}{2}\right] =$$

$$= \frac{1}{2}\left[\frac{1}{3}\lg 2^2 + \frac{1}{3}\lg\frac{7}{2} - \frac{1}{3}\lg\frac{5}{2}\right] = \frac{1}{6}\lg\frac{4\cdot 7/2}{5/2} =$$

$$= \frac{1}{6}\lg\left(\frac{28}{5}\right).$$

7) $$\int_1^2 \frac{2e^{-x}}{e^{-2x}-3e^{-x}+2}\,dx =$$

$y = e^{-x}$ $\quad dy = -e^{-x}\,dx$

$x=1 \to y = e^{-1}$

$x=2 \to y = e^{-2} < e^{-1}$

$$= \int_{e^{-1}}^{e^{-2}} \frac{2}{y^2-3y+2}\cdot(-1)\,dy = \int_{e^{-2}}^{e^{-1}} \frac{2}{y^2-3y+2}\,dy$$

$y^2-3y+2 =$
$=(y-1)(y-2)$

fratti semplici $$\frac{2}{y^2-3y+2} = \frac{A}{y-1} + \frac{B}{y-2} = \frac{(A+B)y-2A-B}{(y-1)(y-2)}$$

$$\begin{cases} A+B=0 \\ -2A-B=2 \end{cases}$$

$$\begin{cases} A=-B \\ B=2 \end{cases} \quad A=-2$$

$$= \int_{e^{-2}}^{e^{-1}} \frac{-2}{y-1}\,dy + \int_{e^{-2}}^{e^{-1}} \frac{2}{y-2}\,dy = -2\left[\lg|y-1|\right]_{e^{-2}}^{e^{-1}} + 2\left[\lg|y-2|\right]_{e^{-2}}^{e^{-1}}$$

$$= -2\lg\left(1-\frac{1}{e}\right) + 2\lg\left(1-\frac{1}{e^2}\right) + 2\lg\left(2-\frac{1}{e}\right) - 2\lg\left(2-\frac{1}{e^2}\right).$$

8) $$\int_{\pi/6}^{\pi/4} \frac{1}{\operatorname{tg} x \,\lg(\sin x)}\,dx = \int_{\pi/6}^{\pi/4} \frac{\cos x}{\sin x \,\lg(\sin x)}\,dx =$$

$$\frac{1}{\operatorname{tg} x} = \frac{\cos x}{\sin x}$$

$= \quad y = \sin x \qquad dy = \cos x\,dx \qquad \sin\frac{\pi}{6} = \frac{1}{2} \qquad \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$$= \int_{\sin \pi/6}^{\sin \pi/4} \frac{1}{y \lg y}\,dy = \begin{matrix} z = \lg y \\ dz = \frac{1}{y}\,dy \end{matrix} = \int_{\lg(\frac{1}{2})}^{\lg(\frac{\sqrt{2}}{2})} \frac{1}{z}\,dz =$$

$$= \left[\lg|z|\right]_{\lg(1/2)}^{\lg(\sqrt{2}/2)} = \lg\left|\lg\frac{\sqrt{2}}{2}\right| - \lg\left|\lg\frac{1}{2}\right| = \lg(\lg\sqrt{2}) - \lg(\lg 2)$$

9) $\int_0^1 \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$ $\quad y=e^{-x} \quad dy=-e^{-x}dx \quad =$

$= \int_1^{e^{-1}} \frac{(-1)\cdot}{\sqrt{1-y^2}}\,dy = +\int_{e^{-1}}^{1} \frac{1}{\sqrt{1-y^2}}\,dy = \left[\arcsin y\right]_{e^{-1}}^{1} =$

$= \arcsin 1 - \arcsin e^{-1} = \frac{\pi}{2} - \arcsin \frac{1}{e}.$

10) $\int_{\sqrt{3}}^{2} x\sqrt{x^2-3}\,dx$ $\quad y=x^2-3 \quad dy=2x\,dx \quad =$

$x=\sqrt{3} \quad y=0 \quad x=2 \quad y=1$

$= \int_0^1 \sqrt{y}\cdot\frac{1}{2}\,dy = \frac{1}{2}\cdot\left[\frac{1}{1+\frac{1}{2}}\,y^{1+\frac{1}{2}}\right]_0^1 = \frac{1}{\cancel{2}}\,\frac{1}{\frac{3}{\cancel{2}}}\left[y^{3/2}\right]_0^1 = \frac{1}{3}$

11) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{\cos^2 x+\cos x-2}\,dx =$ $\quad y=\cos x \quad dy=-\sin x\,dx =$

$$= \int_{\cos \pi/4}^{\cos \pi/2} \frac{(-1)}{y^2+y-2}\, dy = \int_{\frac{\sqrt{2}}{2}}^{0} \frac{(-1)\, dy}{y^2+y-2} = \int_{0}^{\sqrt{2}/2} \frac{1}{y^2+y-2}\, dy$$

$y^2+y-2 = (y-1)(y+2)$

fratti semplici

$$\frac{A}{y-1} + \frac{B}{y+2} = \frac{1}{y^2+y-2} = \frac{(A+B)y + 2A-B}{(y-1)(y+2)}$$

$$\begin{cases} A+B=0 \\ 2A-B=1 \end{cases} \qquad \begin{cases} -B=A \\ A=\frac{1}{3} \end{cases} \qquad B=-\frac{1}{3}$$

$$= \int_{0}^{\sqrt{2}/2} \frac{1/3}{y-1}\, dy + \int_{0}^{\sqrt{2}/2} \frac{-1/3}{y+2}\, dy = \frac{1}{3}\left[\lg|y-1|\right]_0^{\sqrt{2}/2} +$$

$$-\frac{1}{3}\left[\lg|y+2|\right]_0^{\sqrt{2}/2} = \frac{1}{3}\lg\left|\frac{\sqrt{2}}{2}-1\right| - \cancel{\frac{1}{3}\lg 1} - \frac{1}{3}\lg\left|\frac{\sqrt{2}}{2}+2\right| + \frac{1}{3}\lg 2$$

12) $\displaystyle\int_{1}^{e} \frac{\lg x}{x(\lg^2 x - 3\lg x - 4)}\, dx$

$y = \lg x \qquad dy = \frac{1}{x}\, dx$

$x=1 \to y = \lg 1 = 0$

$x=e \to y = \lg e = 1$

$$= \int_0^1 \frac{y}{y^2-3y-4}\,dy$$

$$y^2-3y-4 = (y+1)(y-4)$$

fratti semplici

$$\frac{y}{y^2-3y-4} = \frac{A}{y+1} + \frac{B}{y-4} = \frac{(A+B)y - 4A + B}{(y+1)(y-4)}$$

$$\begin{cases} A+B=1 \\ -4A+B=0 \end{cases} \qquad \begin{matrix} 5A=1 \\ B=4A \end{matrix} \qquad \begin{matrix} A=1/5 \\ B=4/5 \end{matrix}$$

$$= \int_0^1 \frac{1/5}{y+1}\,dy + \int_0^1 \frac{4/5}{y-4}\,dy = \frac{1}{5}\Big[\lg|y+1|\Big]_0^1 + \frac{4}{5}\Big[\lg|y-4|\Big]_0^1 =$$

$$= \frac{1}{5}\lg 2 - \cancel{\frac{1}{5}\lg 1} + \frac{4}{5}\lg 3 - \frac{4}{5}\lg 4$$