

$$1) \quad f(x) = \arctg\left(\frac{x}{x^2+1}\right)$$

arctangente  
è dispari

$$\text{D: } \mathbb{R}, \quad f(-x) = \arctg\left(\frac{-x}{(-x)^2+1}\right) = \arctg\left(-\frac{x}{x^2+1}\right) = -f(x)$$

$f \in \text{DISPARI}$

$$f(x) \geq 0 \Leftrightarrow \frac{x}{x^2+1} \geq 0 \Leftrightarrow x \geq 0$$

$$\lim_{x \rightarrow +\infty} \arctg\left(\frac{x}{x^2+1}\right) = \arctg 0 = 0 = \lim_{x \rightarrow -\infty} f(x)$$

$y = 0$  è AS. DR int  $a = \pm\infty$ .

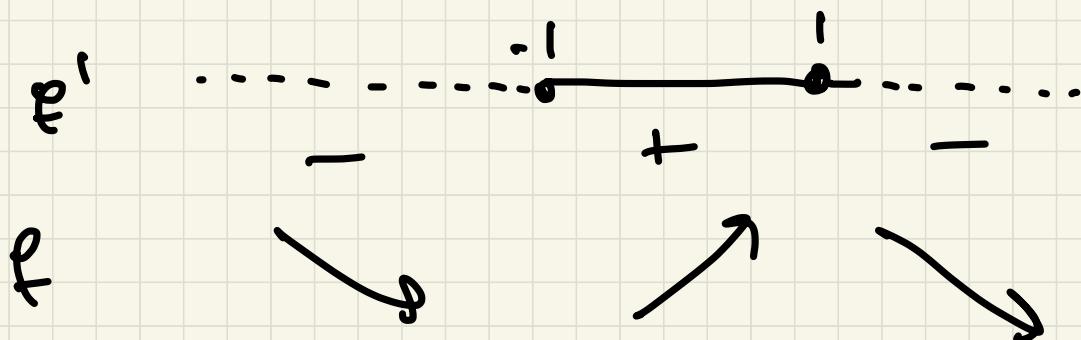
derivata arctangente

$$f'(x) = \frac{1}{1 + \left(\frac{x}{x^2+1}\right)^2} \cdot \frac{x^2+1 - x \cdot 2x}{(x^2+1)^2} = \frac{1}{1 + \left(\frac{x}{x^2+1}\right)^2} \cdot \frac{1-x^2}{(1+x^2)^2}$$

derivate analog.

$f$  è derivabile  $\forall x \in \mathbb{R}$

$$f'(x) \geq 0 \Leftrightarrow 1 - x^2 \geq 0 \Leftrightarrow x^2 - 1 \leq 0 \Leftrightarrow -1 \leq x \leq 1$$



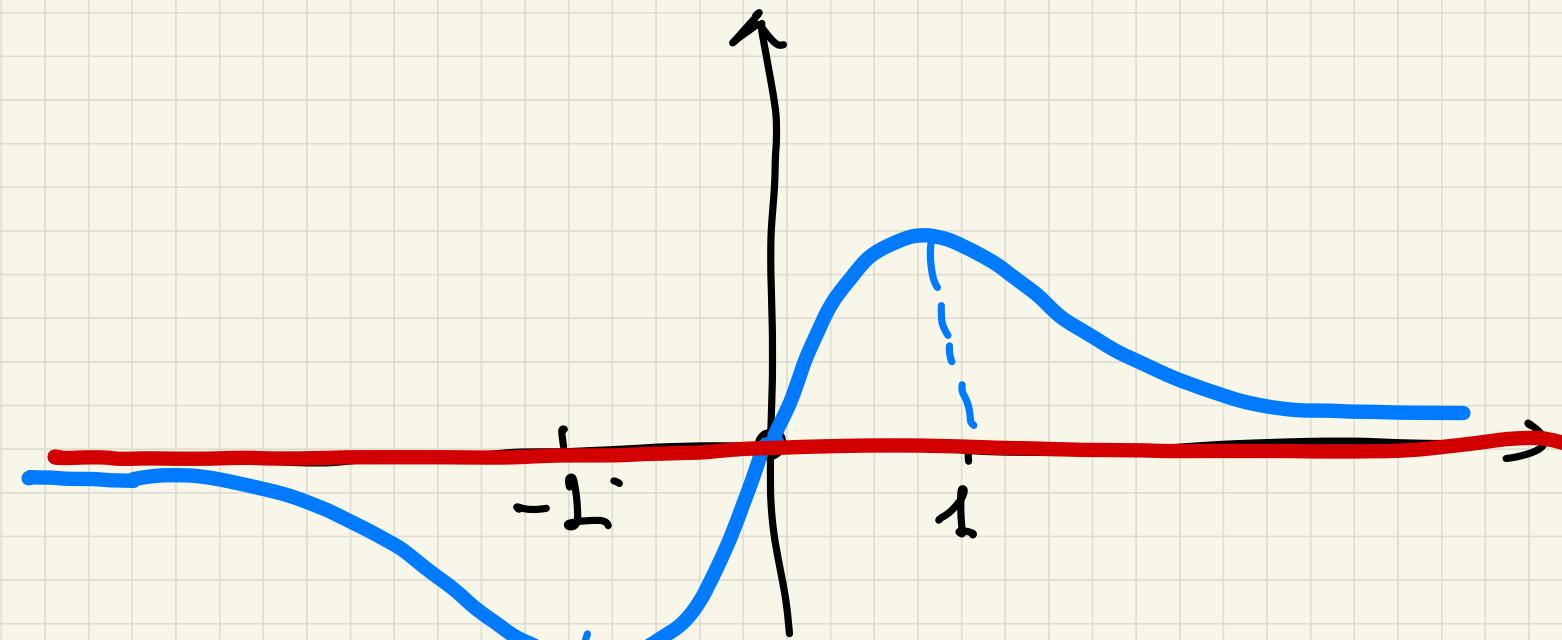
$x = -1$  pto di min. locale

$x = 1$  pto di max. locale

dato che  $f(x) \rightarrow 0$  per  $x \rightarrow \pm\infty$   $x = 1$  è anche pto

di  $\max$  assoluto e  $\max f = f(1) = \arctg \frac{1}{2}$

e  $x = -1$  è pto di min. assoluto  $\min f = f(-1) = -\arctg \frac{1}{2}$



$$2) f(x) = x e^{-\frac{1}{x}}$$

$$D = \{x \neq 0\} = (-\infty, 0) \cup (0, +\infty)$$

$f(-x) \neq f(x), -f(x)$  f wou ē pari reū dišperi

$$f(x) \geq 0$$

$$x e^{-\frac{1}{x}} \geq 0 \quad (\Rightarrow x \geq 0)$$

sempre

$$\lim_{x \rightarrow 0^+}$$

$$x \downarrow 0 \quad e^{-\frac{1}{x}} \uparrow +\infty = 0$$

$$\lim_{x \rightarrow 0^-} x \cdot e^{-\frac{1}{x}} = \begin{pmatrix} y = -\frac{1}{x} & x \rightarrow 0^- \\ y \rightarrow +\infty & \\ x = -\frac{1}{y} & \end{pmatrix} = \lim_{y \rightarrow +\infty} -\frac{1}{y} e^y = +\infty$$

per comportamento infinito

$$x=0 \text{ discount di 2^a specie}$$

$$\lim_{x \rightarrow +\infty}$$

$$x \downarrow 0 \quad e^{-\frac{1}{x}} \uparrow +\infty = +\infty$$

$$\lim_{x \rightarrow -\infty}$$

$$x \uparrow -\infty \quad e^{-\frac{1}{x}} \downarrow 1 = -\infty$$

non ci sono gr. omissibili.

cerco es. obliqui ( $a + \infty$  e  $a - \infty$ )

$$\lim_{x \rightarrow \pm\infty} \frac{x e^{-\frac{1}{x}}}{x} = \lim_{x \rightarrow \pm\infty} e^{-\frac{1}{x}} = 1 = m = \lim_{x \rightarrow \infty} \frac{x e^{-\frac{1}{x}}}{x}$$

$$\lim_{x \rightarrow +\infty} x e^{-\frac{1}{x}} - x = \begin{matrix} y = -\frac{1}{x} \\ x = -\frac{1}{y} \end{matrix} \quad \begin{matrix} x \rightarrow +\infty \\ y \rightarrow 0^- \end{matrix} = \lim_{y \rightarrow 0^-} -\frac{1}{y} e^y + \frac{1}{y} =$$

$$= \lim_{y \rightarrow 0^-} \frac{-e^y + 1}{y} = \lim_{y \rightarrow 0^-} -\left(\frac{e^y - 1}{y}\right) \xrightarrow{\text{limite notevole}} -1 = q$$

Stessa cosa  $x \rightarrow -\infty$ .  
( $y \rightarrow 0^+$ )

$$y = x - 1$$

ASINTOTO OBBLIQUO  
 $a + \infty$  e  $a - \infty$

$$f'(x) = 1 \cdot e^{-\frac{1}{x}} + \textcircled{x} \cdot \left( e^{-\frac{1}{x}} \right) \cdot \left( + \frac{1}{x^2} \right) = e^{-\frac{1}{x}} \left( 1 + \frac{1}{x^2} \right) =$$

$$= e^{-\frac{1}{x}} \left( \frac{x+1}{x} \right)$$

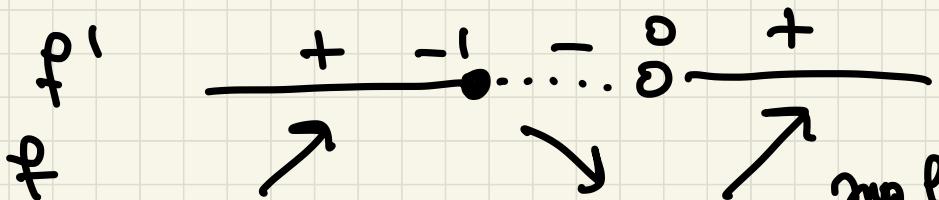
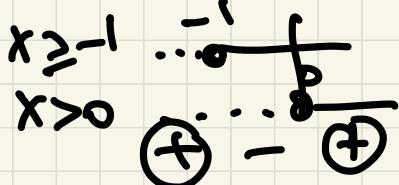
noslo f è derivabile  $\forall x \neq 0$   
n'slo che  $\lim_{x \rightarrow 0^+} f(x) = 0$  calcolo  $\lim_{x \rightarrow 0^+} f'(x)$

$$\lim_{x \rightarrow 0^+} e^{-\frac{1}{x}} \left( 1 + \frac{1}{x^2} \right) = y = -\frac{1}{x} \quad y \rightarrow -\infty = \lim_{y \rightarrow -\infty} e^y \left( 1 - y \right) = 0$$

in  $x=0^+$  la tangente è ORIZZONTALE

confronto  
tra infiniti

$$f'(x) \geq 0 \quad (\Rightarrow e^{-\frac{1}{x}} \left( \frac{x+1}{x} \right) \geq 0 \quad (\Rightarrow \frac{x+1}{x} \geq 0)$$



$x = -1$  è pto di max locale

$x = 0$  NON È NEL DOMINIO!  
 $\sup f = +\infty \quad \inf f = -\infty$

$$f'(x) = e^{-\frac{1}{x}} \frac{x+1}{x}$$

- derivative  $e^{-\frac{1}{x}}$

→ derivative  $\frac{x+1}{x}$

$$f'''(x) = e^{-1/x} \cdot \left( +\frac{1}{x^2} \right) \cdot \frac{x+1}{x} + e^{-1/x} \cdot \frac{\frac{1 \cdot x - (x+1) \cdot 1}{x^2}}{x^2} =$$

$$= e^{-1/x} \left[ \frac{+(x+1)}{x \cdot (x^2)} + \frac{-1}{x^2} \right] = e^{-1/x} \frac{+x+1-x}{x^3} = e^{-1/x} \frac{1}{x^3}$$

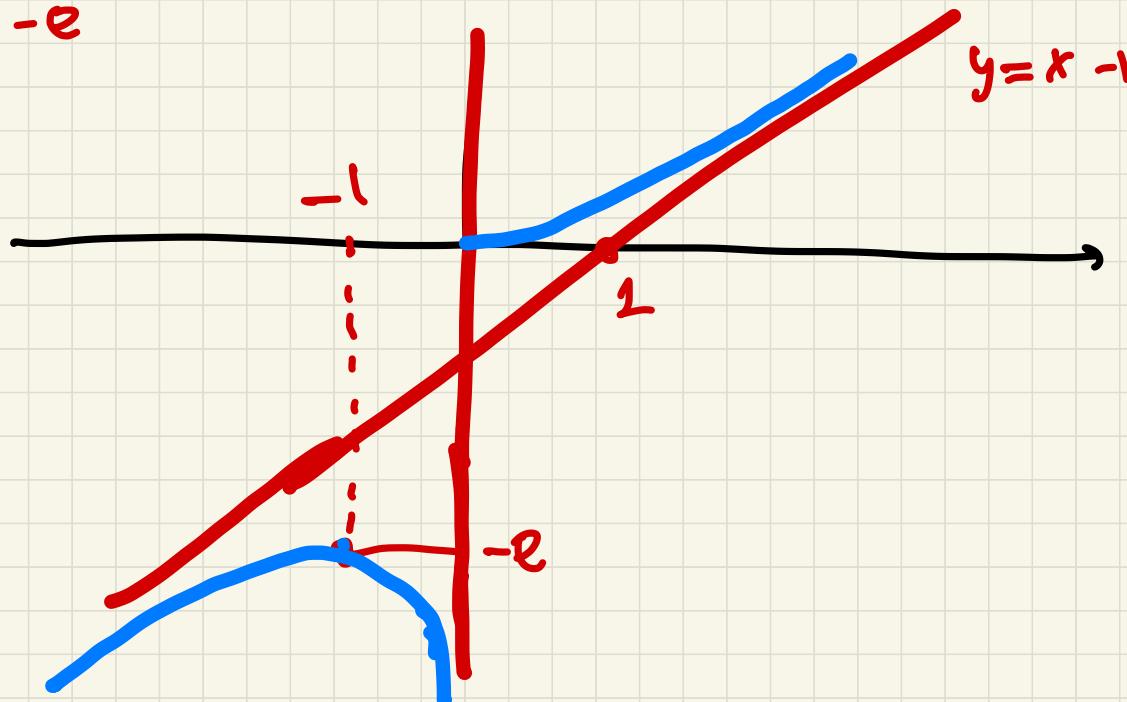
$$f''(x) \geq 0 \iff x > 0$$

$$f^u \quad \begin{array}{c} \text{---} \\ \vdots \end{array} \quad \begin{array}{c} + \\ - \\ 0 \end{array} \quad \begin{array}{c} + \\ \hline \end{array}$$

$f$  è concava per  $x > 0$  e convessa per  $x < 0$

( $x=0$  non ste vel dovezio!)

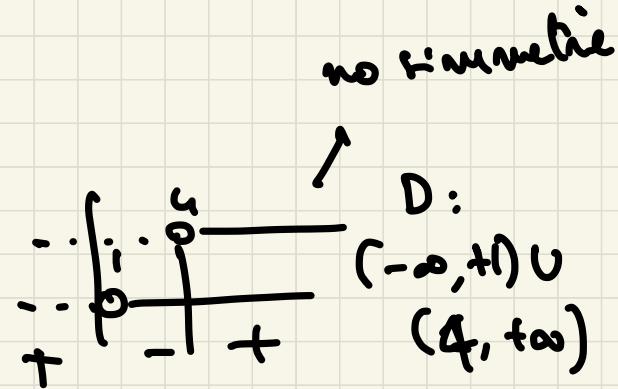
$$f(-1) = -e$$



3)  $f(x) = \lg\left(\frac{x-4}{x-1}\right)$

$$D: \frac{x-4}{x-1} > 0$$

$$\begin{aligned} x-4 &> 0 & x &> 4 \\ x-1 &> 0 & x &> 1 \end{aligned}$$



$$D: (-\infty, 1) \cup (4, +\infty)$$

$$f(x) \geq 0 \quad \lg\left(\frac{x-4}{x-1}\right) \geq 0 = \lg 1 \quad (\Leftrightarrow) \quad \frac{x-4}{x-1} \geq 1 \Leftrightarrow$$

$$\frac{x-4}{x-1} - 1 \geq 0 \quad \frac{x-4 - x+1}{x-1} \geq 0 \quad (\Leftrightarrow) \quad \frac{-3}{x-1} \geq 0 \quad (\Leftrightarrow) \quad \frac{3}{x-1} < 0$$

$$\Leftrightarrow x-1 < 0$$

line  
 $x \rightarrow 4^+$

$$\lg\left(\frac{x-4}{x-1}\right) = -\infty$$

$x=4$  AS. VERT (CALE DESTRO)

$$x < 1$$

line  
 $x \rightarrow 1^-$

$$\lg\left(\frac{x-1}{x-1}\right) = +\infty$$

$x=1$  AS. VERTICALE DESTRO

line  
 $x \rightarrow +\infty$

$$\lim_{x \rightarrow +\infty} \lg\left(\frac{x-4}{x-1}\right) = \lim_{x \rightarrow +\infty} \lg\left(\frac{x\left(1-\frac{4}{x}\right)}{x\left(1-\frac{1}{x}\right)}\right) = \lg 1 = 0$$

$y=0$  AS.  
 ORIZ q  
 $+\infty, -\infty$

$\lim_{x \rightarrow -\infty} \lg\left(\frac{x-4}{x-1}\right) = \lim_{x \rightarrow -\infty} \lg\left(\frac{x\left(1-\frac{4}{x}\right)}{x\left(1-\frac{1}{x}\right)}\right) = \lg 1 = 0$

$$f'(x) = \frac{1}{x-4} \cdot \frac{1 \cdot (x-1) - 1 \cdot (x-4)}{(x-1)^2} = \frac{x-1}{x-4} \cdot \frac{x-1-x+4}{(x-1)^2} = \frac{3}{(x-4)(x-1)}$$

$x-1$   
derivate logaritmo

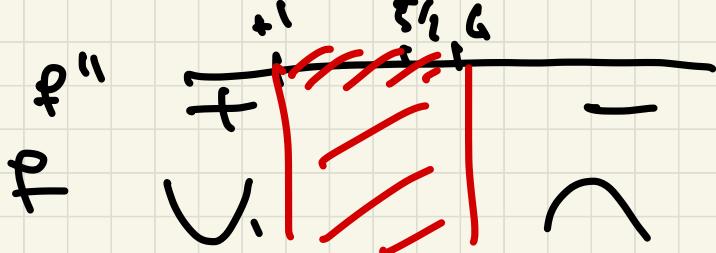
$$= \frac{3}{x^2 - 5x + 4}$$

dunque  $f'(x) \geq 0 \Leftrightarrow (x-4)(x-1) > 0 \Leftrightarrow x \in D!$

$f$  crescente nel dominio  $\sup f = +\infty \quad \inf f = -\infty$

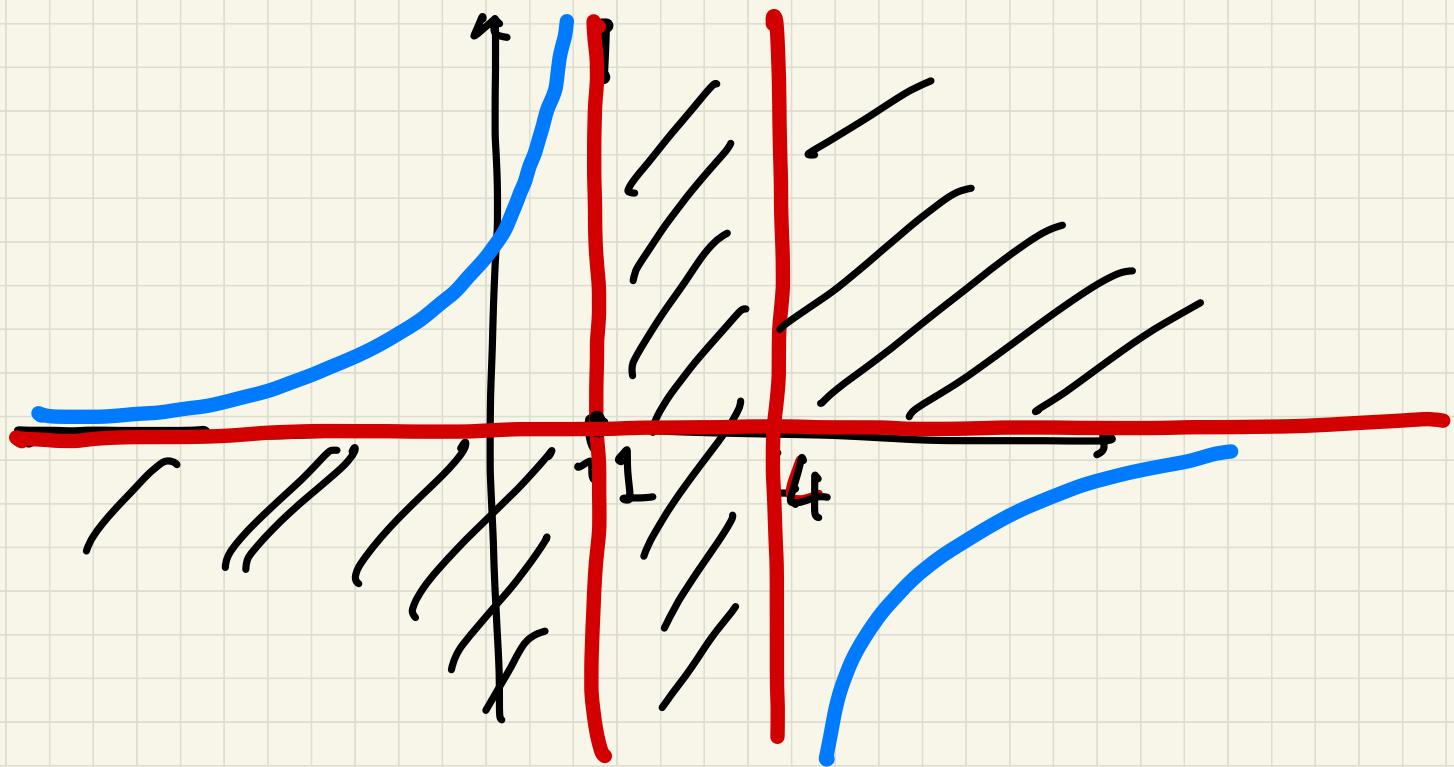
$$f''(x) = \frac{-3(2x-5)}{(x^2 - 5x + 4)^2} > 0 \quad (x \neq 1, 4)$$

$f''(x) \geq 0 \Leftrightarrow -3(2x-5) \geq 0 \Leftrightarrow 2x-5 \leq 0 \Leftrightarrow x \leq \frac{5}{2}$



$$x = 5/2 \neq 0$$

$f$  è convessa per  $x < 1$   
 $f$  è concava per  $x > 4$



$$(4) \quad f(x) = \arctan(\operatorname{sen} x)$$

$D = \mathbb{R}$     $f(x+2\pi) = \arctan(\operatorname{sen}(x+2\pi)) = \arctan(\operatorname{sen} x)$   
 $f$  è periodica di periodo  $2\pi$  → basta studiare in  $[-\pi, \pi]$

$$f(-x) = \arctg(\operatorname{sen}(-x)) = \arctg(-\operatorname{sen}x) = -\arctg(\operatorname{sen}x)$$

↗  
 sen è  
 dispari

↗  
 arctg è  
 dispari

$f \in$  dispari

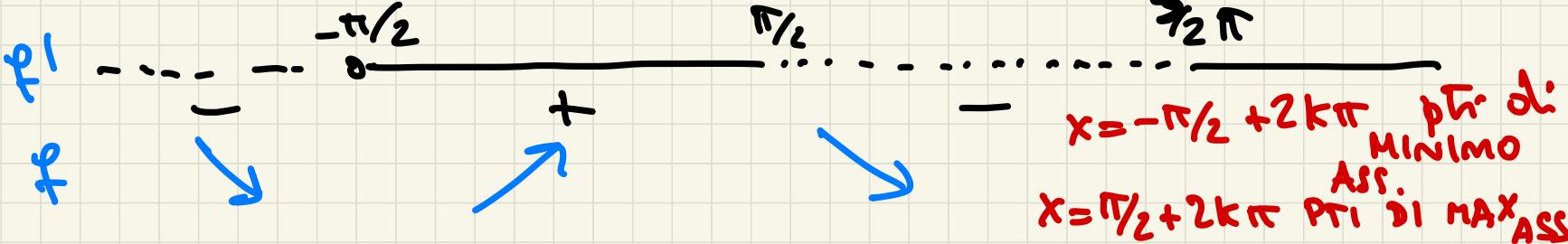
$$f(x) \geq 0 \Leftrightarrow \operatorname{sen}x \geq 0 \Leftrightarrow 2k\pi \leq x \leq \pi + 2k\pi$$

$f$  è periodica  $\Rightarrow$  NON HA LIMITI A  $\pm\infty$ !  
 derivata arcotangente

$$f'(x) = \frac{1}{1+(\operatorname{sen}x)^2} \cdot \cos x = \frac{1}{1+\operatorname{sen}^2 x} \cdot \cos x$$

$f \in$   
 deriv.  
 su IR

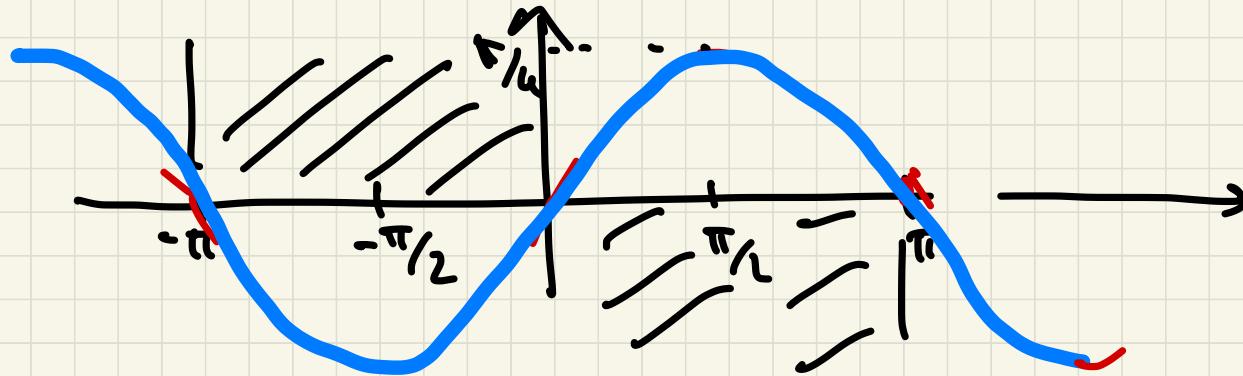
$$f'(x) \geq 0 \quad (\Leftrightarrow \cos x \geq 0) \quad (\Leftrightarrow) \quad -\frac{\pi}{2} + 2k\pi \leq x \leq \frac{\pi}{2} + 2k\pi$$



$x = -\pi/2 + 2k\pi$  p.ti di MINIMO  
 $x = \pi/2 + 2k\pi$  p.ti di MAXIMO

$$f\left(\frac{\pi}{2}\right) = \operatorname{arctg}(\operatorname{csc}\frac{\pi}{2}) = \operatorname{arctg} 1 = \pi/4$$

$$f(-\frac{\pi}{2}) = \operatorname{arctg}(\operatorname{csc}-\frac{\pi}{2}) = -\pi/4$$



$$5) \quad f(x) = \operatorname{arctg}\left(\frac{1}{|x-1|}\right)$$

$D : \{x \neq 1\} = (-\infty, 1) \cup (1 + \infty)$  und symmetrie

Glebt die  $\frac{1}{|x-1|} > 0 \quad \forall x \in D \Rightarrow f(x) > 0 \quad \forall x \in D$

$$\lim_{x \rightarrow 1} \operatorname{arctg} \frac{1}{|x-1|} = \pi/2$$

$\frac{1}{0^+} = +\infty$

$x=1$  è singolarità eliminabile

esteso per continuità in  $x=1$

$$\lim_{x \rightarrow +\infty} \operatorname{arctg} \frac{1}{|x-1|} = 0 = \lim_{x \rightarrow -\infty} \operatorname{arctg} \frac{1}{|x-1|}$$

$\frac{1}{+\infty} = 0$  ponendo  $f(1) = \pi/2$ .

$y=0$   
AS. ORIZZ  
 $a+\infty, -\infty$ .

derivate  $x > 1$   $f(x) = \operatorname{arctg} \left( \frac{1}{x-1} \right)$   $f'(x) = \frac{1}{1 + \left( \frac{1}{x-1} \right)^2} \cdot \frac{-1}{(x-1)^2} =$

$$= \frac{1}{\frac{(x-1)^2 + 1}{(x-1)^2}} \cdot \frac{(-1)}{(x-1)^2}$$

$$= \frac{-1}{(x-1)^2 + 1} < 0$$

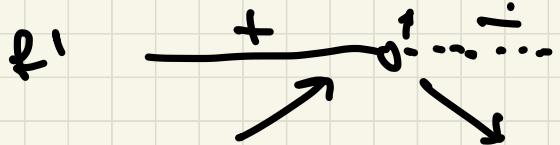
NON DERIV. IN  $x \neq 1$   
 $x=1$  PTO ANGOLOSO

$$x < 1 \quad f(x) = \operatorname{arctg} \frac{1}{1-x}$$

$$f'(x) = \frac{1}{1 + \left( \frac{1}{1-x} \right)^2} \cdot \frac{1}{(1-x)^2} = \frac{1}{(1-x)^2 + 1} > 0$$

$\lim_{x \rightarrow 1^+} f'(x) = \lim_{x \rightarrow 1^+} \frac{-1}{(x-1)^2 + 1} = -1$

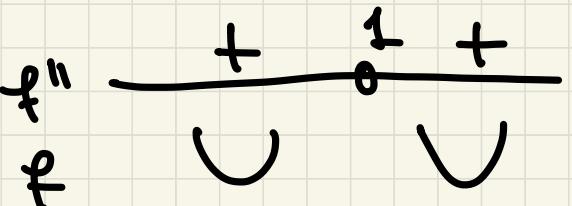
$\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} \frac{1}{(1-x)^2 + 1} = 1$



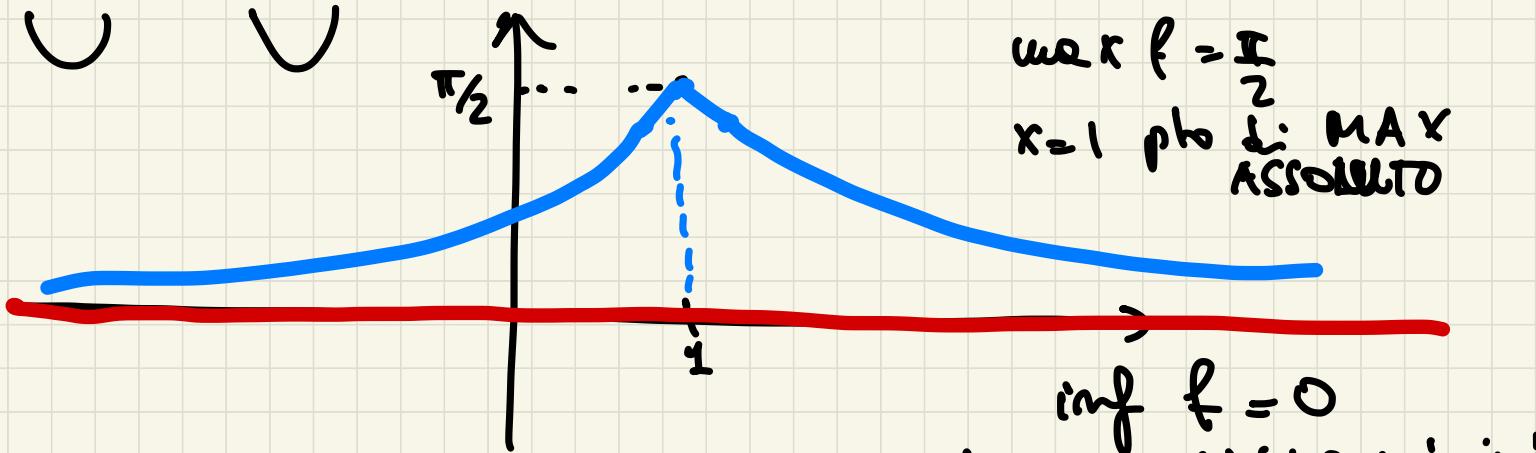
$x=1$  PTO DI MAX  
assoluto  $f(1) = \frac{\pi}{2} = \max f$

$$x > 1 \quad f'(x) = -\frac{1}{(x-1)^2 + 1} = -\frac{1}{x^2 - 2x + 2} \Rightarrow f''(x) = +\frac{2x-2}{(x^2 - 2x + 2)^2} = \frac{2(x-1)}{(x^2 - 2x + 2)^2}$$

$$x < 1 \quad f'(x) = \frac{1}{(1-x)^2 + 1} = \frac{1}{x^2 - 2x + 2} \Rightarrow f''(x) = -\frac{2x-2}{(x^2 - 2x + 2)^2} = \frac{-2(x-1)}{(x^2 - 2x + 2)^2}$$



$f$  è sempre convessa



metti  $f = \frac{\pi}{2}$

$x=1$  pto di MAX  
ASSOLUTO

$\inf f = 0$

ma non ci sono minimi

$$6) f(x) = \arcsin \sqrt{x} - \lg x$$

D:  $x > 0$  (per il logaritmo) ]  $\boxed{x > 0}$

$x \geq 0$  (per radice)

$-1 \leq \sqrt{x} \leq 1$  (per arco seno)  $\Rightarrow$

$$\sqrt{x} \leq 1 \rightarrow \boxed{x \leq 1}$$

D:  $\{0 < x \leq 1\} = (0, 1]$

$$\begin{aligned} f(1) &= \arcsin 1 - \lg 1 = \\ &= \pi/2 - 0 = \pi/2. \end{aligned}$$

NB  $\arcsin \sqrt{x} \geq 0$  perché  $\sqrt{x} \geq 0$ !

$\lg x \leq 0$  se  $x \in (0, 1] \Rightarrow -\lg x \geq 0$

$$f(x) = \underbrace{\arcsin \sqrt{x}}_{\text{VI}} - \underbrace{\lg x}_{\text{VII}}$$

$$f(x) \geq 0 \quad \forall x \in D.$$

$$\lim_{x \rightarrow 0^+} \underbrace{\arcsin \sqrt{x} - \lg x}_{\substack{\downarrow 0 \\ -(-\infty)}} = +\infty \quad x=0$$

AS. VERTICALE DESTRA

$$f'(x) = \frac{1}{1-(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{x} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{x} = \frac{\sqrt{x}-2\sqrt{1-x}}{2x\sqrt{1-x}}$$

$f'$   
esiste  
 $\forall x \in (0, 1)$

$f$  è derivabile per ogni  $0 < x < 1$

Studio derivabilità in  $x=1$  ( $x=1$  STA NEL DOM'NIG)

$$\text{lim}_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} \frac{\sqrt{x} - 2\sqrt{1-x}}{2x\sqrt{1-x}}$$

$\sqrt{x} - 2\sqrt{1-x} \rightarrow 1 - 2 \cdot 0 = 1$   
 $= +\infty$

$f$  in  $x=1$   
ha TANGENTE  
VERTICALE

$$f'(x) \geq 0$$

$$\frac{\sqrt{x} - 2\sqrt{1-x}}{2x\sqrt{1-x}} \geq 0 \Leftrightarrow \sqrt{x} - 2\sqrt{1-x} \geq 0$$

$$\Leftrightarrow (\sqrt{x})^2 \geq (2\sqrt{1-x})^2$$

$$5x \geq 4$$

$$x \geq \frac{4}{5}$$

$$x \geq 4(1-x)$$

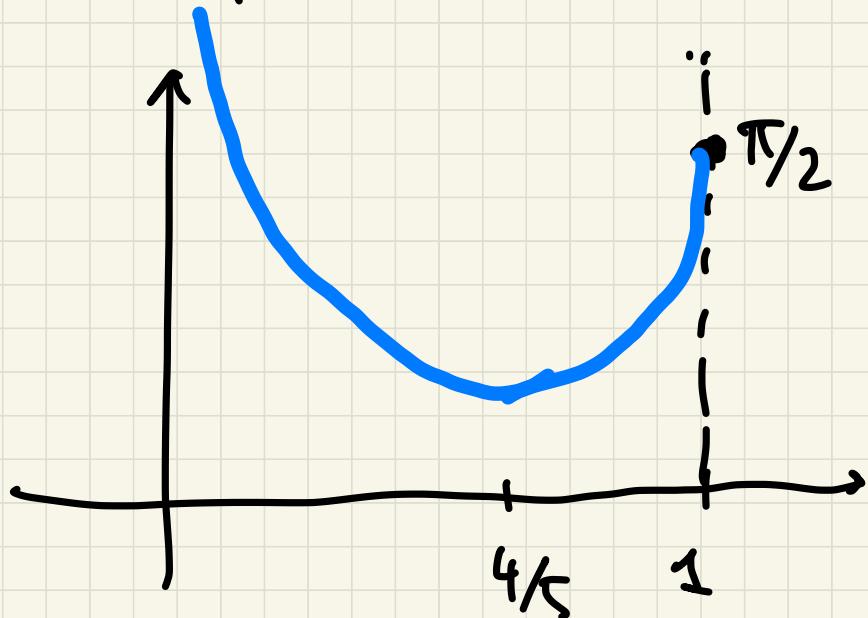
$$0 - \dots - \overset{4x}{\dots} + \overset{1}{0} \quad f'$$

$\downarrow$   
 $\downarrow$

$$x \geq 4-4x$$

$x = \frac{4}{5}$  pto di minimo locale (e anche  
ostacolo)

$x = 1$  pto di massimo locale



$$7) f(x) = \sqrt{\frac{x^2-1}{x-2}}$$

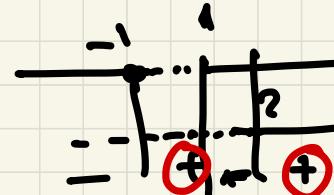
$$D: \frac{x^2-1}{x-2} \geq 0$$

$$x^2-1 \geq 0$$

$$x-2 > 0$$

$$x \geq 1 \quad x \leq -1$$

$$x > 2$$



$$D: \{x > 2, -1 \leq x \leq 1\} = [-1, 1] \cup (2, +\infty)$$

no symmetrie,  $f(x) \geq 0 \quad \forall x \in D$   $f(1) = f(-1) = 0$

$$\lim_{x \rightarrow 2^+} \sqrt{\frac{x^2-1}{x-2}} = +\infty$$

$x=2$  AS. VERT. DESTRO

NB  $\min f = 0$   
 $f(1) = f(-1) = \min f$

$$\lim_{x \rightarrow +\infty} \sqrt{\frac{x^2-1}{x-2}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2(1-\frac{1}{x^2})}{x(1-\frac{2}{x})}} = +\infty \quad (\text{confronto infiniti})$$

cresce as. oblique

$$\lim_{x \rightarrow +\infty} \frac{1}{x} \cdot \sqrt{\frac{x^2-1}{x-2}} = \lim_{x \rightarrow +\infty} \sqrt{\frac{x^2-1}{x^2(x-2)}} = 0 \quad (\text{confronto infiniti})$$

NON HO AS. ORIZZONTALI né OBLIQUE

$$f'(x) = \frac{1}{2\sqrt{\frac{x^2-1}{x-2}}} \cdot \frac{2x \cdot (x-2) - (x^2-1) \cdot 1}{(x-2)^2} = \frac{1}{2\sqrt{\frac{x^2-1}{x-2}}} \cdot \frac{2x^2 - 4x - x^2 + 1}{(x-2)^2} =$$

$$= \frac{1}{2\sqrt{\frac{x^2-1}{x-2}}} \frac{x^2 - 4x + 1}{(x-2)^2}$$

$$\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} \frac{1}{2\sqrt{\frac{x^2-1}{x-2}}}$$

$f'$  è derivabile  
per  $x \in (-1, 1) \cup (2, +\infty)$

$$\frac{x^2 - 4x + 1}{(x-2)^2} \xrightarrow{x \rightarrow 1^-} -\infty$$

in  $x = 1$   
TANGENTE  
VERTICALE

$$\lim_{x \rightarrow -1^+} f'(x) = \lim_{x \rightarrow -1^+} \frac{1}{2\sqrt{\frac{x^2-1}{x-2}}}$$

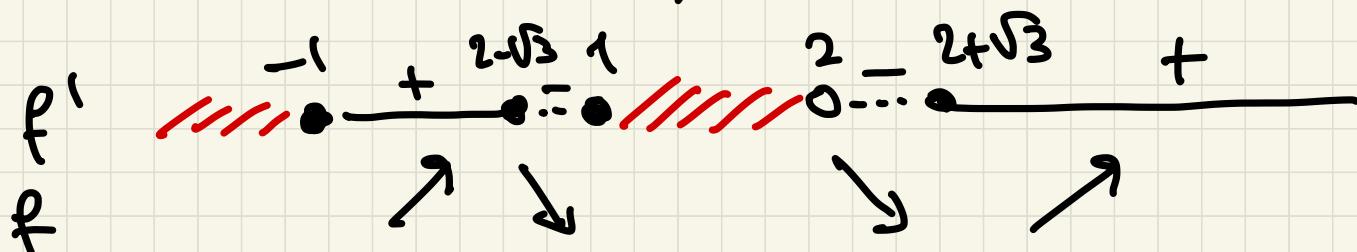
in  $x = -1$   
TANG VERTICALE

$$f'(x) \geq 0 \Leftrightarrow \frac{x^2 - 4x + 1}{x-2} \geq 0 \Leftrightarrow x^2 - 4x + 1 \geq 0$$

$$\left(2\sqrt{\frac{x^2-1}{x-2}}\right)^2 \geq 0$$

$$x_{1,2} = 2 \pm \sqrt{4-1} \\ = 2 \pm \sqrt{3}$$

$$\Leftrightarrow \begin{array}{l} x \leq 2-\sqrt{3} \\ \sim 0,3\dots \\ \in (-1,1) \end{array} \quad \begin{array}{l} x \geq 2+\sqrt{3} \\ \approx 3,7\dots > 2 \end{array} \quad \begin{array}{l} \sqrt{3} \sim 1,7\dots \end{array}$$

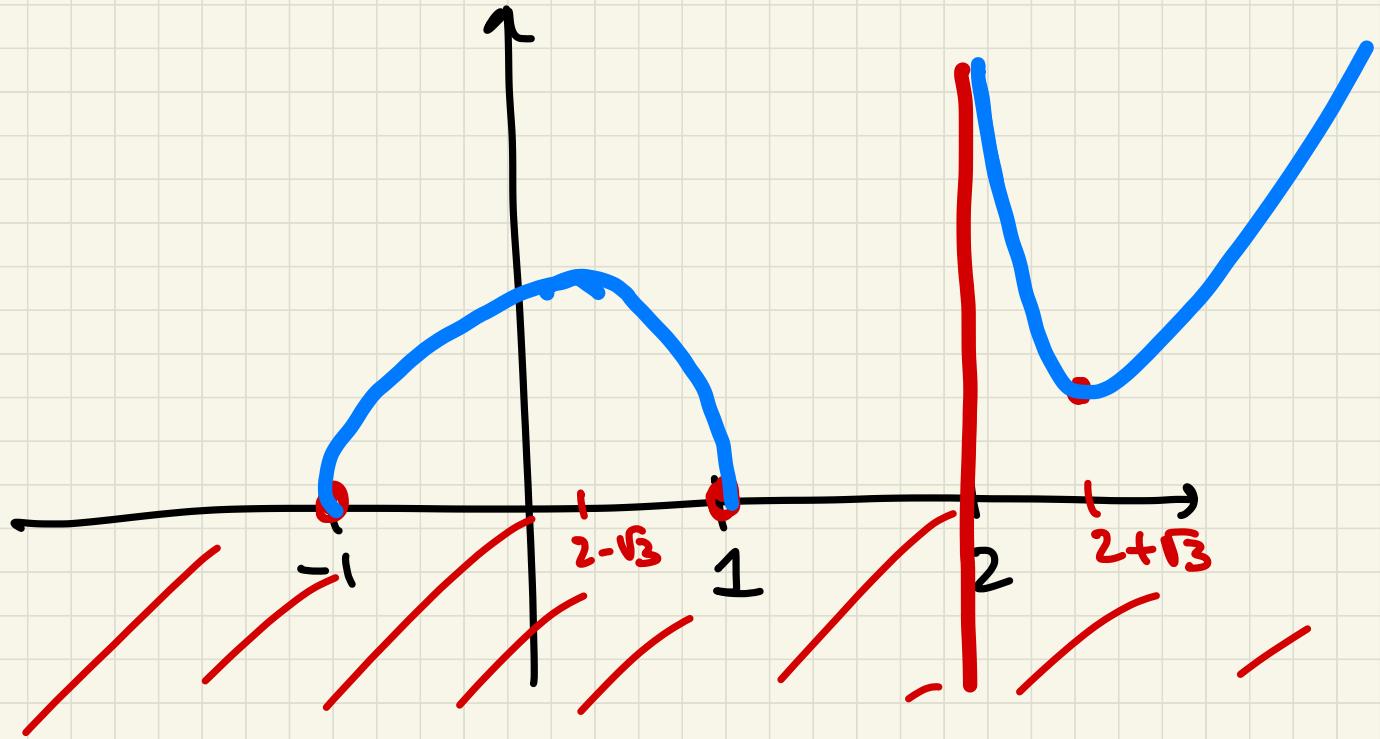


$x = 2 - \sqrt{3}$  pto di max locale (NON ASSOLUTO  $\Rightarrow \sup f = +\infty$ )

$x = -1$  pto di min. locale e ASSOLUTO  $f(x) \geq 0 = f(-1) = f(1)$

$x = 1$  pto di min. locale e ASSOLUTO  $\min f = 0$

$x = 2 + \sqrt{3}$  pto di min. locale NO N ASSOLUTO



8)  $f(x) = \sqrt[3]{x^2} e^x = \sqrt[3]{x^2} \sqrt[3]{e^x} = x^{2/3} e^{x/3}$   $D = \mathbb{R}$

dato che  $x^{2/3} e^{x/3} \geq 0 \Rightarrow f(x) \geq 0 \forall x \in \mathbb{R}$   $\boxed{f(0) = 0 = \min f}$

$$\lim_{x \rightarrow +\infty} \sqrt[3]{x^2 e^x} = +\infty$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt[3]{x^2 e^x}}{x} = \lim_{x \rightarrow +\infty} \frac{e^{x/3}}{x^{1/3}} = +\infty$$

non ci sono as. obliqui  
con orizzontali a  $+\infty$

$$\frac{x^{2/3} e^{x/3}}{x}$$

comportamento  
fra infiniti

$$\lim_{x \rightarrow -\infty} \sqrt[3]{x^2 e^{x/3}} = 0 \text{ per confronto infinito}$$

$y=0$  AS. ORIZZONTALE  $a = -\infty$

$$f'(x) = \frac{2}{3} x^{2/3-1} e^{x/3} + x^{2/3} e^{x/3} \cdot \frac{1}{3} = \frac{1}{3} e^{x/3} (2x^{-1/3} + x^{2/3}) =$$

$$= x^{-1/3} \frac{e^{x/3}}{3} (2+x) = \frac{e^{x/3}}{3x^{1/3}} (2+x)$$

definita  $\forall x \neq 0$  (NB  $x^{2/3} \cdot x^{1/3} = x$ )

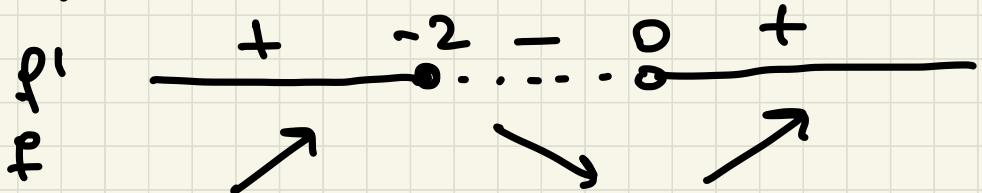
$$\lim_{x \rightarrow 0^+} \frac{e^{x/3} (2+x)}{3x^{1/3}} = +\infty$$

$$\lim_{x \rightarrow 0^-} \frac{e^{x/3} (2+x)}{3x^{1/3}} = -\infty$$

in  $x=0$   $f$  non è derivabile  
 $x=0$  PUNTO DI CUSPIDE

$$f'(x) \geq 0 \quad \text{oc } e^{x/3}(2+x) \geq 0$$

$$f'(x) \geq 0 \quad x \leq -2, \quad x > 0$$



$$\begin{aligned} 2+x &\geq 0 & x &\geq -2 & \therefore \begin{array}{c} -2 \\ \hline + \\ 0 \\ + \end{array} \\ x^{1/3} &> 0 & x &> 0 & \therefore \begin{array}{c} 0 \\ \hline - \\ 0 \\ + \end{array} \end{aligned}$$

$x = -2$  pto di max locale  
(NON ASSOLUTO)  $\sup f = +\infty$

$x = 0$  pto di min locale  
e ASSOLUTO  
 $f(0) = \min f = 0$

$$f''(x) = \frac{\left(\frac{1}{3}e^{x/3}(2+x) + e^{x/3} \cdot 1\right) 3x^{2/3} - e^{x/3}(2+x) \cdot 3 \cdot \frac{1}{3}x^{-2/3}}{9x^{2/3}} =$$

$$= \frac{(2+x)x^{1/3}e^{x/3} + 3x^{1/3}e^{x/3} - (2+x)e^{x/3}x^{-2/3}}{9x^{2/3}} =$$

$$= \frac{e^{x/3} \cdot x^{-2/3} [(2+x) \cdot x + 3x - (2+x)]}{9x^{2/3}} = \frac{e^{x/3}}{9x^{4/3}} (x^2 + 4x - 2)$$

$$f''(x) \geq 0 \Leftrightarrow x^2 + 4x - 2 \geq 0 \quad x = \frac{-4 \pm \sqrt{16+8}}{2} = \frac{-4 \pm 2\sqrt{6}}{2} = -2 \pm \sqrt{6}$$

$x \geq -2 + \sqrt{6} \approx 0, \dots$

