1) $f(x) = \dfrac{x-5}{x^2-9}$ $\quad$ ($D = \{x \neq \pm 3\} = (-\infty,-3) \cup (-3,3) \cup (3,+\infty)$

$$f'(x) = \frac{1(x^2-9)-(x-5)\cdot 2x}{(x^2-9)^2} = \frac{x^2-9-2x^2+10x}{(x^2-9)^2} = \frac{-x^2+10x-9}{\underbrace{(x^2-9)^2}_{>0}}$$

$$f'(x) \geq 0 \iff -x^2+10x-9 \geq 0 \Rightarrow x^2-10x+9 \leq 0$$

$$x = \frac{10 \pm \sqrt{100-36}}{2} = \frac{10 \pm 8}{2} \begin{cases} 9 \\ 1 \end{cases}$$

$1 \leq x \leq 9$

segno $f'$: $\quad - \quad 1 \quad + \quad 9 \quad -$

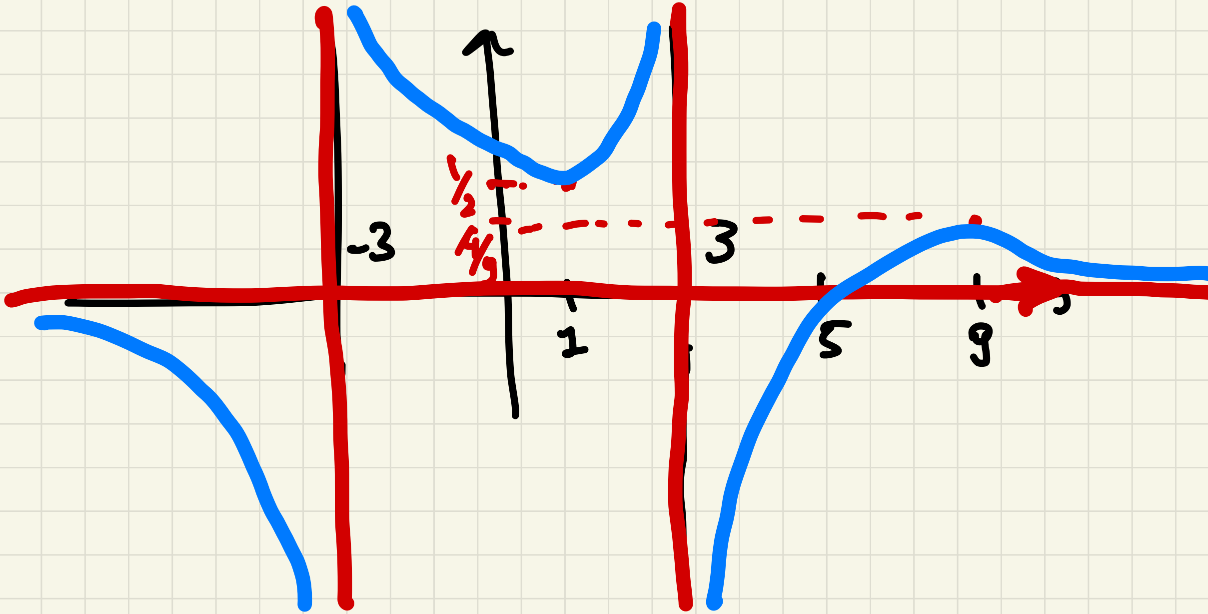
monotonia $f$: $\quad \searrow \quad \nearrow \quad \searrow$

$x=1$ pto di MINIMO LOCALE

$x=9$ pto di MAX LOCALE

$\Rightarrow$ $f \to +\infty$ per $x \to 3^-$

$f \to -\infty$ per $x \to 3^+$ $\Rightarrow$ $f$ NON HA MAX e MIN.




$$f(9) = \frac{9-5}{9^2-9} = \frac{4}{9}$$

$$f(1) = \frac{1-5}{1-9} = \frac{4}{8} = \frac{1}{2}$$

2) $f(x) = \dfrac{x^3}{x^2-1}$ $\qquad$ D: $x \neq \pm 1$

$$f'(x) = \frac{3x^2(x^2-1) - x^3(2x)}{(x^2-1)^2} = \frac{3x^4 - 3x^2 - 2x^4}{(x^2-1)^2} =$$

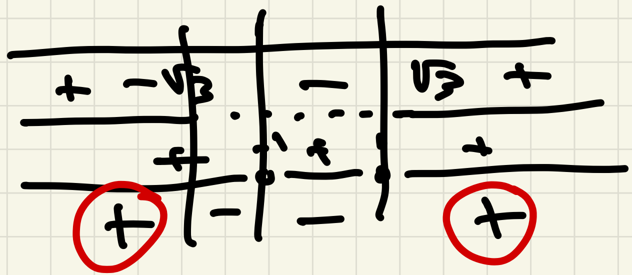
$$= \frac{x^4 - 3x^2}{(x^2-1)^2} = \frac{x^2(x^2-3)}{(x^2-1)^2}$$

$f'(x) \geq 0$ $\qquad \dfrac{x^2(x^2-3)}{(x^2-1)^2} \geq 0$

$x^2 \geq 0 \quad \forall x$

$x^2-3 \geq 0 \qquad x \geq \sqrt{3}, \quad x \leq -\sqrt{3}$

$(x^2-1)^2 > 0 \quad \forall x \neq \pm 1$



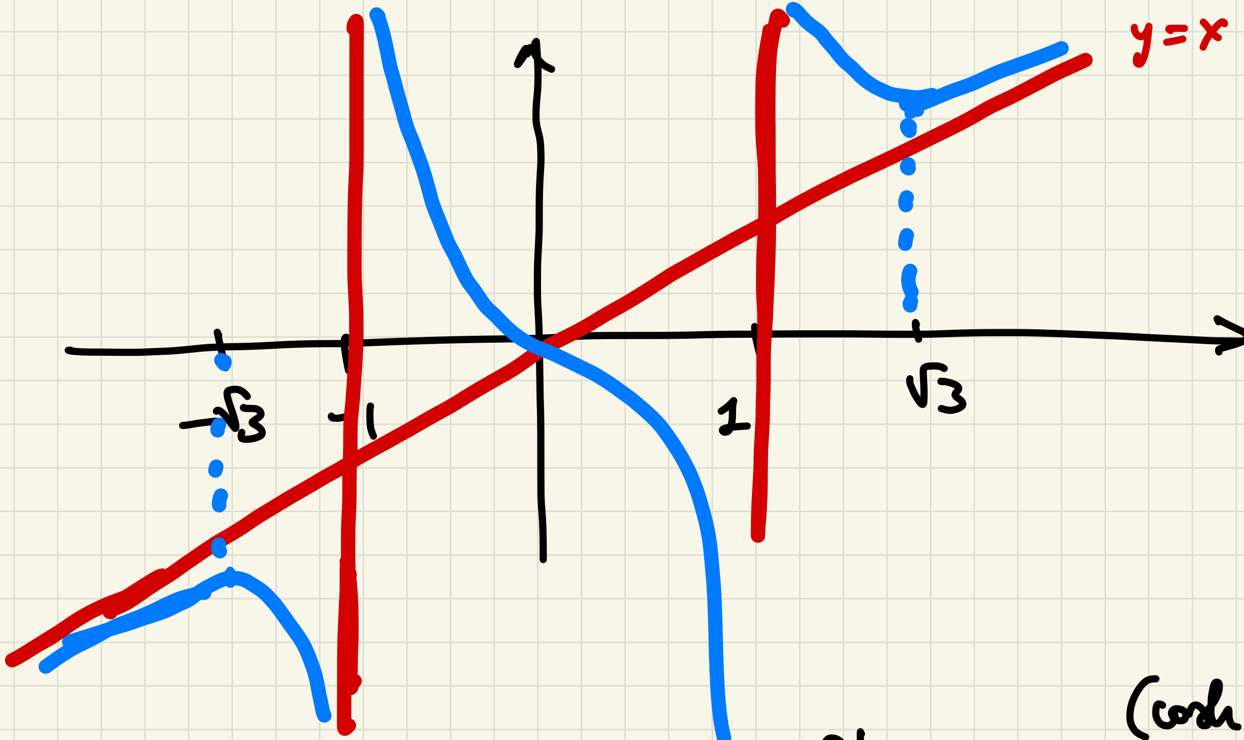

$x \leq -\sqrt{3}, \quad x \geq \sqrt{3}$

| | $-\sqrt{3}$ | $-1$ | $1$ | $\sqrt{3}$ | |
|---|---|---|---|---|---|
| segno $f'$ | + | − | − | − | + |
| monotonia $f$ | $\nearrow$ | $\searrow$ | | $\searrow$ | $\nearrow$ |

$x = -\sqrt{3}$ pto di max locale $\quad f(-\sqrt{3}) = \dfrac{-3\sqrt{3}}{(3-1)} = -\dfrac{3}{2}\sqrt{3} < -\sqrt{3}$

$x = +\sqrt{3}$ pto di min locale $\quad f(\sqrt{3}) = \dfrac{3\sqrt{3}}{3-1} = \dfrac{3}{2}\sqrt{3} > \sqrt{3}$

$f$ NON HA MAX e MIN !




3) $f(x) = \lg(\cosh x)$

$$f'(x) = \underbrace{\frac{1}{\cosh x}}_{\text{derivata } \log x = \frac{1}{x}} \cdot \underbrace{\operatorname{senh} x}_{\text{derivata } \cosh x}$$

$(\cosh x > 0 \ \forall x!)$

$f'(x) \geq 0 \iff \sinh x \geq 0$

$\iff x \geq 0$

$$\begin{array}{ccccc} - & & 0 & & + & f' \\ \searrow & & & & \nearrow & f \end{array}$$

$x=0$ pto di min locale.

inoltre $f(x) \geq 0$ e $f(0)=0 \Rightarrow$ $x=0$ pto di min assoluto

min $f = 0$

(grafico era già preciso)

4) $f(x) = \lg (e^{2x}-1)^2$

DOMINIO $x \neq 0$

$x=0$ AS. VERTICALE!

derivata argomento

$$f'(x) = \underbrace{\frac{1}{(e^{2x}-1)^2}}_{\text{derivata logaritmo}} \cdot \underbrace{2(e^{2x}-1)}_{\text{derivata elevam. al quadrato}} \cdot \underbrace{e^{2x}}_{\text{derivata esponenziale}} \cdot 2 = \frac{4e^{x}\cancel{(e^{2x}-1)}}{(e^{2x}-1)^{\cancel{2}}}$$

$$f'(x) \geq 0 \Leftrightarrow \frac{4e^{x}}{e^{2x}-1} > 0 \Leftrightarrow e^{2x}-1>0 \Leftrightarrow e^{2x} > 1 = e^0$$

$4e^{x} > 0 \ \forall x$

$\Leftrightarrow 2x > 0$ $x>0$

| | | | |
|---|---|---|---|
| $f'$ | $-$ | 0 | $+$ |
| $f$ | $\searrow$ | | $\nearrow$ |

NB $x=0$ NON E' PTO DI MIN NON STA NEL DOMINIO!

(grafico era già preciso).

5) $\lg\left(\frac{x+1}{x-1}\right)$ $\quad D: \{x>1,\ x<-1\}$

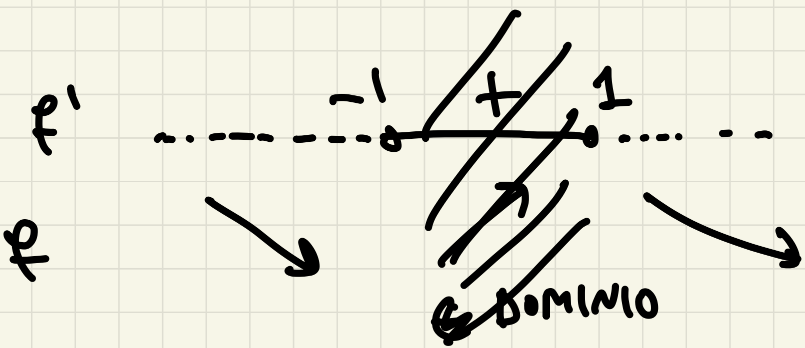
$$f'(x) = \frac{1}{\frac{x+1}{\cancel{x-1}}} \cdot \frac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^{\cancel{2}}} = \frac{x-1-x-1}{(x+1)\,(x-1)} = \frac{-2}{(x+1)(x-1)}$$

$$f'(x) \geq 0 \iff \frac{-2}{(x+1)(x-1)} \geq 0 \iff \frac{2}{(x+1)(x-1)} \leq 0$$

$$\iff -1<x<1$$

$x+1>0 \quad x>-1$

$x-1>0 \quad x>1$


f è sempre decrescente nel dominio

($x=1,-1$ non stanno nel dominio!)

grafico più preciso

6) $f = e^{\frac{x^2-4}{x-1}}$ $\qquad D = \{x \neq 1\}$

$$f'(x) = e^{\frac{x^2-4}{x-1}} \cdot \frac{2x\cdot(x-1) - (x^2-4)\cdot 1}{(x-1)^2} = \frac{2x^2-2x-x^2+4}{(x-1)^2}\, e^{\frac{x^2-4}{x-1}}$$

$$= \frac{x^2-2x+4}{(x-1)^2} \cdot e^{\frac{x^2-4}{x-1}}$$

$f'(x) \geq 0$ $\qquad x^2-2x+4 \geq 0 \Rightarrow \Delta = 4-8 < 0 \Rightarrow \forall x \in \mathbb{R}$

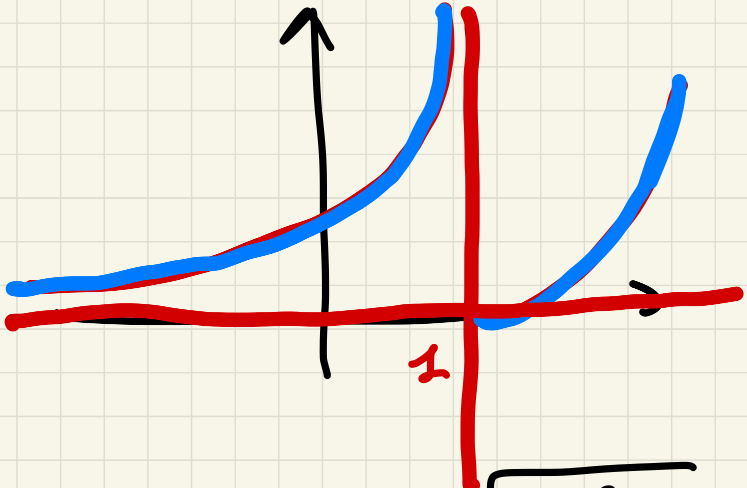
$\qquad (x-1)^2 > 0 \iff x \neq 1$

$\Rightarrow f'(x) \geq 0 \quad \forall x \in D.$ (la funzione è crescente nel dominio)

dato che $\lim_{x \to 1^+} f(x) = 0 \in \mathbb{R}$ calcolo $\lim_{x \to 1^+} f'(x) = 0$

per confronto infiniti

in $x = 1^+$ ho tang. ORIZZONTALE




g) $f(x) = 5x+2-\sqrt{25x^2+12x}$ $\qquad D=\left(-\infty, -\frac{12}{25}\right] \cup [0,+\infty)$

$$f'(x) = 5 - \frac{1}{2\sqrt{25x^2+12x}}\cdot(25\cdot 2x+12\cdot 1) = 5 - \frac{\cancel{2}(25x+6)}{\cancel{2}\sqrt{25x^2+12x}} =$$

$$= \frac{5\sqrt{25x^2+12x}-25x-6}{\sqrt{25x^2+12x}}$$

$\sqrt{25x^2+12x} > 0$

$$f'(x)\geq 0 \iff 5\sqrt{25x^2+12x}-25x-6\geq 0$$

$$\frac{5\sqrt{25x^2+12x}}{5}\geq \frac{25x+6}{5}$$

$$\underbrace{\sqrt{25x^2+12x}}_{\geq 0}\geq 5x+\frac{6}{5}$$

$$-\frac{6}{25}\geq -\frac{12}{25}\,!$$

1) se $5x+\frac{6}{5}\leq 0$ sempre vero $\boxed{x\leq -\frac{6}{25}}$

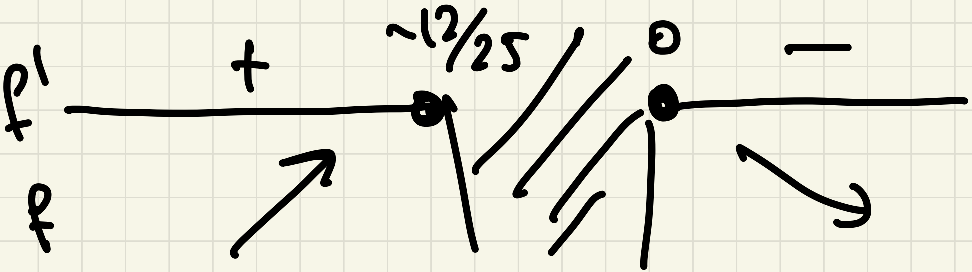
2) se $5x+\frac{6}{5}\geq 0$ elevo al quadrato

$$\left(\sqrt{25x^2+12x}\right)^2\geq\left(5x+\frac{6}{5}\right)^2$$

$$\cancel{25x^2}+\underbrace{\cancel{12x}}\geq \cancel{25x^2}+\frac{36}{25}+\underbrace{2\cdot\cancel{5}x\cdot\frac{6}{\cancel{5}}}$$

$$0\geq\frac{36}{25}\quad \text{FALSO}\quad \nexists x$$

dunque $f'(x)\ge 0 \iff x\le -\frac{12}{25}$ $\left(x \le -\frac{6}{25} \text{ e } x\in D!\right)$




$x=-\frac{12}{25}$ è pto di max locale

$x=0$ è pto di min locale

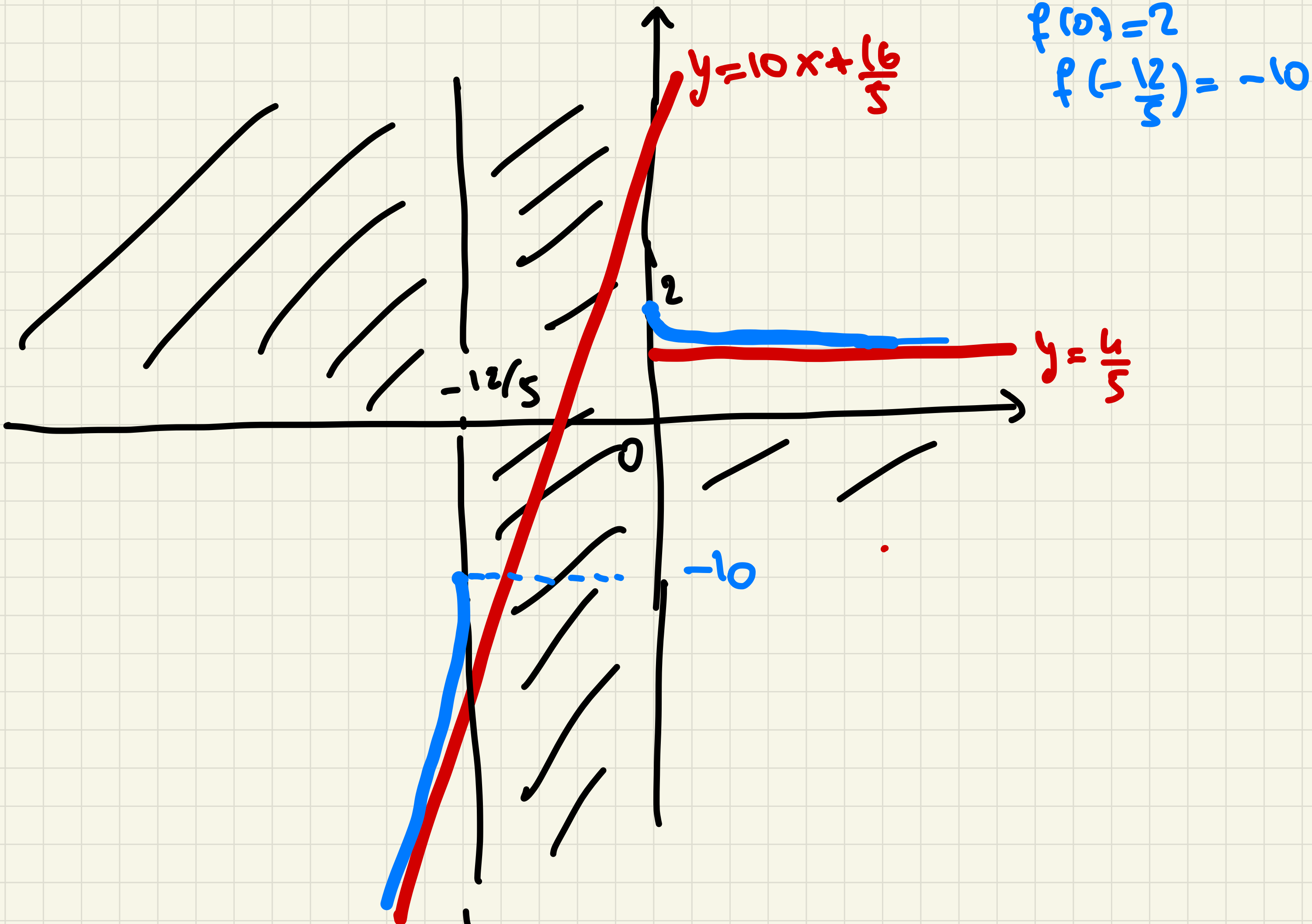
(NB $x=0$, $x=-\frac{12}{5}$ APPARTENGONO AL DOMINIO!)

calcolo $\displaystyle\lim_{x\to0^+} f'(x) = \lim_{x\to0^+} \frac{\overbrace{5\sqrt{25x^2+12x}-25x-6}^{\to -6}}{\underbrace{\sqrt{25x^2+12x}}_{\to 0^+}} = -\infty$

↘ in $x=0$ TANG. VERTICALE

$\displaystyle\lim_{x\to-\frac{12}{25}^-} f'(x) = \lim_{x\to\left(-\frac{12}{25}\right)^-} \frac{5\overbrace{\sqrt{25x^2+12x}}^{\to 0}\;\overbrace{-25x-6}^{\to +6}}{\underbrace{\sqrt{25x^2+12x}}_{0^+}} = +\infty$

$x=-\frac{12}{25}$ TANG. VERT.


$y = 10x + \frac{16}{5}$
$f(0) = 2$
$f(-\frac{12}{5}) = -10$
2
$y = \frac{4}{5}$
$-12/5$
0
$-10$