Sol

1) $$|(-1)^n \cos n| \leq 1$$ (successione limitata

$$-\frac{1}{n} \leq \frac{(-1)^n \cdot \cos n}{n} \leq \frac{1}{n} \Rightarrow \lim_n \frac{(-1)^n \cos n}{n} = 0$$

per criterio rapporto

2) $$a_n = \frac{n^n}{e^{n^2}} \qquad a_{n+1} = \frac{(n+1)^{n+1}}{e^{(n+1)^2}} = \frac{n^{n+1}\left(1+\frac{1}{n}\right)^{n+1}}{e^{n^2+2n+1}} = \frac{n^{n+1}\left(1+\frac{1}{n}\right)^{n+1}}{e^{n^2} e^{2n+1}}$$

$$\frac{a_{n+1}}{a_n} = a_{n+1} \cdot \frac{1}{a_n} = \frac{n^{\cancel{n+1}}\left(1+\frac{1}{n}\right)^{n+1}}{\cancel{e^{n^2}}\, e^{2n+1}} \cdot \frac{\cancel{e^{n^2}}}{\cancel{n^n}} = \frac{n \cdot \left(1+\frac{1}{n}\right)^{n+1}}{e^{2n+1}} \to 0$$

$\Rightarrow$ per il criterio del rapporto $\lim_n \frac{n^n}{e^{n^2}} = 0$.

3) $$\lim_{n} \frac{1+n^3 - n\sin n + n^2 \sin\frac{1}{n}}{\lg^4(n) + \sqrt{n^2+1}}$$

raccolgo n^3

$$\sqrt{n^2+1} = \sqrt{n^2\left(1+\frac{1}{n^2}\right)} = n\sqrt{1+\frac{1}{n^2}}$$

raccolgo n

$$= \lim_{n} \frac{n^{\cancel{3}\,2}\left[\underbrace{\frac{1}{n^3}}_{\to 0} + 1 - \underbrace{\frac{\sin n}{n^2}}_{\to 0} + \underbrace{\frac{\sin 1/n}{n}}_{\to 0}\right]}{\cancel{n}\left[\underbrace{\frac{\lg^4(n)}{n}}_{\to 0} + \underbrace{\sqrt{1+\frac{1}{n^2}}}_{\to 1}\right]} = +\infty$$

4) NUMERATORE $e^n\left(\frac{n^2}{e^n} + 1\right)$

DENOMINATORE : $n^n\left(\frac{n^5}{n^n} + 1\right)$

$$\lim_{n} \frac{e^n\left(\frac{n^2}{e^n}+1\right)}{n^n\left(\frac{n^5}{n^n}+1\right)} = 0$$

with $\frac{e^n}{n^n} \to 0$, $\left(\frac{n^2}{e^n}+1\right) \to 1$, $\left(\frac{n^5}{n^n}+1\right) \to 1$

5) applico il criterio del rapporto

$$\left(\frac{n^{2n}}{2^{n^2}}\right) = a_n \qquad a_{n+1} = \frac{(n+1)^{2(n+1)}}{2^{(n+1)^2}} = \frac{(n+1)^{2n}\cdot(n+1)^2}{2^{n^2}\cdot 2^{2n+1}}$$

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^{2n}\cdot(n+1)^2}{\cancel{2^{n^2}}\cdot 2^{n+1}} \cdot \frac{\cancel{2^{n^2}}}{n^{2n}} =$$

$$= \left(\frac{n+1}{n}\right)^{2n} \frac{(n+1)^2}{2^{n+1}} = 0$$

$$\left(\frac{n+1}{n}\right)^{2n} = \left[\left(1+\frac{1}{n}\right)^{n}\right]^2 \to e^2 \qquad \frac{(n+1)^2}{2^{n+1}} \to 0$$

$$\lim_n \frac{n^{2n}}{2^{n^2}} = 0$$

6) $\forall a > 0$ N. $n^a \left(1 - \frac{\cos n}{n^a} + \frac{\lg n}{n^a}\right)$ $\frac{\cos n}{n^a} \to 0$, $\frac{\lg n}{n^a} \to 0$

D: $3n^{1/3} + n^{1/2} = n^{1/2}\left(3\frac{n^{1/3}}{n^{1/2}} + 1\right)$ $\frac{n^{1/3}}{n^{1/2}} \to 0$

$$\lim_n \frac{n^a\left(1 - \frac{\cos n}{n^a} + \frac{\lg n}{n^a}\right)}{n^{1/2}\left[3\frac{n^{1/3}}{n^{1/2}} + 1\right]} =$$

$a = \frac{1}{2}$ $\lim = 1$

$a > \frac{1}{2}$ $\lim = +\infty$

$a < \frac{1}{2}$ $\lim = 0$

7) teorema ponte

$$\lim_{x \to +\infty} \left(1 - \frac{1}{2\sqrt{x}}\right)^{\sqrt{x}} = \lim_{y \to +\infty} \left(1 - \frac{1}{2y}\right)^y = e^{-1/2}.$$

$x \to +\infty$ $y = \sqrt{x} \to +\infty$