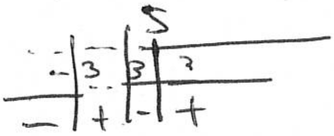
Soluzioni esercizi su asintoti

1) $f(x) = \dfrac{x-5}{x^2-9}$

Dominio $x^2-9 \neq 0 \Rightarrow x \neq \pm 3$ $\quad (-\infty,-3) \cup (-3,3) \cup (3,+\infty)$

Segno $\dfrac{x-5}{x^2-9} \geq 0$ $\quad x \geq 5$

$x < -3 \quad x > 3$



$f(x) \geq 0 \Leftrightarrow x \geq 5$

$-3 < x < 3$

Simmetrie $f(-x) = \dfrac{-x-5}{x^2-9} \neq f(x)$, $\neq -f(x)$ non ci sono simmetrie

Asintoti $\lim\limits_{x \to +\infty} \dfrac{x-5}{x^2-9} = \lim\limits_{x \to +\infty} \dfrac{x\,(1-5/x)}{x^2\,(1-9/x^2)} = 0$ $\quad (1-5/x) \to 1,\ (1-9/x^2) \to 1,\ \frac{x}{x^2} \to 0$

$\lim\limits_{x \to -\infty} \dfrac{x-5}{x^2-9} = \lim\limits_{x \to -\infty} \dfrac{x\,(1-5/x)}{x^{\cancel{2}}(1-9/x^2)} = 0$

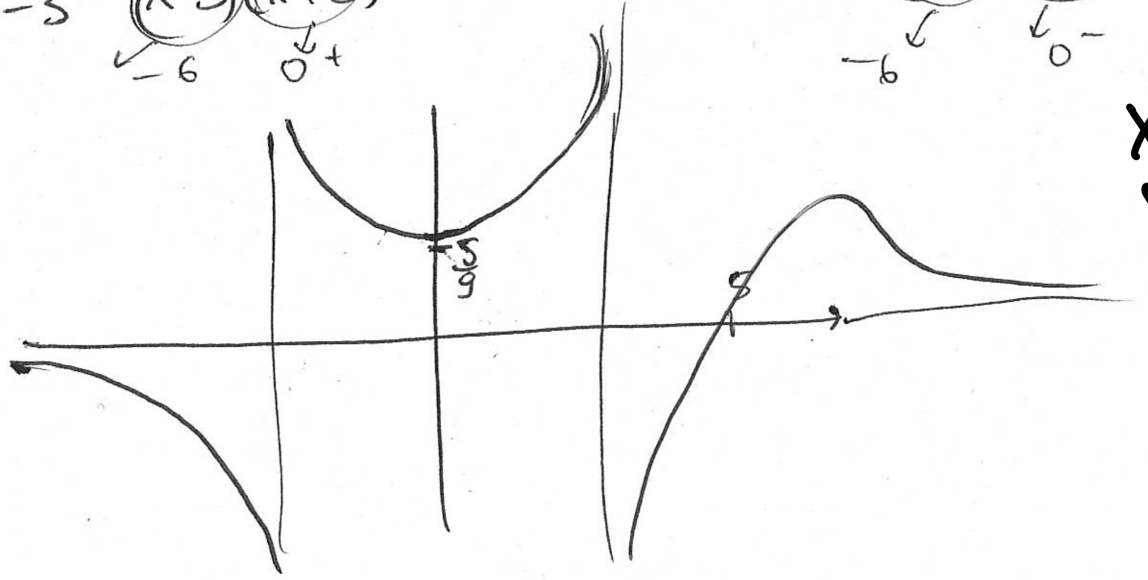
$y = 0$ asintoto orizzontale a $\pm\infty$.

$\lim\limits_{x \to 3^+} \dfrac{(x-5)}{(x-3)(x+3)} = -\infty$ $\quad (x-5) \to -2,\ (x-3) \to 0^+,\ (x+3) \to 6$

$\lim\limits_{x \to 3^-} \dfrac{(x-5)}{(x-3)(x+3)} = +\infty$ $\quad (x-5) \to -2,\ (x-3) \to 0^-,\ (x+3) \to 6$

$\lim\limits_{x \to -3^+} \dfrac{(x-5)}{(x-3)(x+3)} = +\infty$ $\quad (x-5) \to -8,\ (x-3) \to -6,\ (x+3) \to 0^+$

$\lim\limits_{x \to -3^-} \dfrac{(x-5)}{(x-3)(x+3)} = -\infty$ $\quad (x-5) \to -8,\ (x-3) \to -6,\ (x+3) \to 0^-$



$x = 3$

$x = -3$

ASINTOTI VERTICALI

2) $f(x) = \dfrac{x^3}{x^2-1}$

②

__dominio__ $x^2-1 \neq 0 \Rightarrow x \neq \pm 1 \quad (-\infty,-1) \cup (-1,1) \cup (1,+\infty)$

__segno__ $f(x) \geq 0 \quad x^3 \geq 0 \quad x \geq 0$

$x^2-1 > 0 \quad x<-1 \quad x>1$

$f(x) \geq 0 \iff x \geq 1, \ -1 < x \leq 0$

__simmetrie__ $f(-x) = \dfrac{(-x)^3}{(-x)^2-1} = -\dfrac{x^3}{x^2-1} = -f(x)$ __f dispari__

__asintoti__ $\lim_{x\to+\infty} \dfrac{x^{\cancel{3}}}{\cancel{x^2}(1-\frac{1}{x^2})} = +\infty \quad \lim_{x\to-\infty} \dfrac{x^{\cancel{3}}}{\cancel{x^2}(1-\frac{1}{x^2})} = -\infty$

non ci sono asintoti orizzontali.

cerco gli obliqui

$\lim_{x\to+\infty\,(-\infty)} \dfrac{f(x)}{x} = \lim_{x\to+\infty} \dfrac{x^3}{x(x^2-1)} = \lim_{x\to+\infty} \dfrac{\cancel{x^3}}{\cancel{x}\cdot\cancel{x^2}(1-\frac{1}{x^2})} = 1 = m$

$\lim_{x\to+\infty} f(x)-x = \lim_{x\to+\infty} \dfrac{\cancel{x^3}-\cancel{x^3}+x}{x^2-1} = \lim_{x\to+\infty\,(-\infty)} \dfrac{\cancel{x}}{x^{\cancel{2}}(1-\frac{1}{x^2})} = 0 = q$

$y=x$ asintoto obliquo a $+\infty$ e a $-\infty$.

$\lim_{x\to1^+} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ ($x^3 \to 1$, $x-1 \to 0^+$, $x+1 \to 2$)

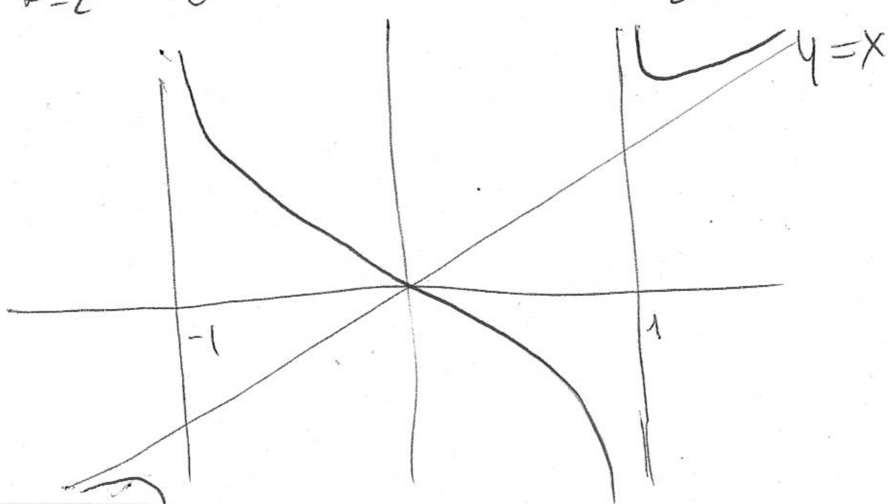
$\lim_{x\to1^-} \dfrac{x^3}{(x-1)(x+1)} = -\infty$ ($x^3 \to 1$, $x-1 \to 0^-$, $x+1 \to 2$)

$x=1$ asintoto verticale

$\lim_{x\to-1^+} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ ($x^3 \to -1$, $x-1 \to -2$, $x+1 \to 0^+$)

$\lim_{x\to-1^-} \dfrac{x^3}{(x-1)(x+1)} = +\infty$ ($x^3 \to -1$, $x-1 \to -2$, $x+1 \to 0^+$)

$x=-1$ as. verticale



3) $f(x) = \lg(\cosh x)$ NB $\cosh x = \frac{e^x + e^{-x}}{2} = e^x \frac{(1+e^{-2x})}{2} = e^{-x}\frac{(e^{2x}+1)}{2}$ (3)

<u>dominio</u> $\cosh x > 0 \ \forall x$ quindi
$D = \mathbb{R}$

<u>segno</u> $\cosh x \geq 1 \ \forall x$ quindi $f(x) \geq 0$ e $f(x) = 0 \iff x = 0$.

<u>simmetrie</u> $\cosh(-x) = \cosh x \Rightarrow f(x) = f(-x)$ f <u>pari</u>

<u>asintoti</u> $\lim_{x\to+\infty} f(x) = +\infty$.

cerco as. obliqui $\lim_{x\to+\infty} \frac{\lg(\cosh x)}{x} = \lim_{x\to+\infty} \frac{1}{x}\lg\left(\frac{e^{x}(1+e^{-2x})}{2}\right)$

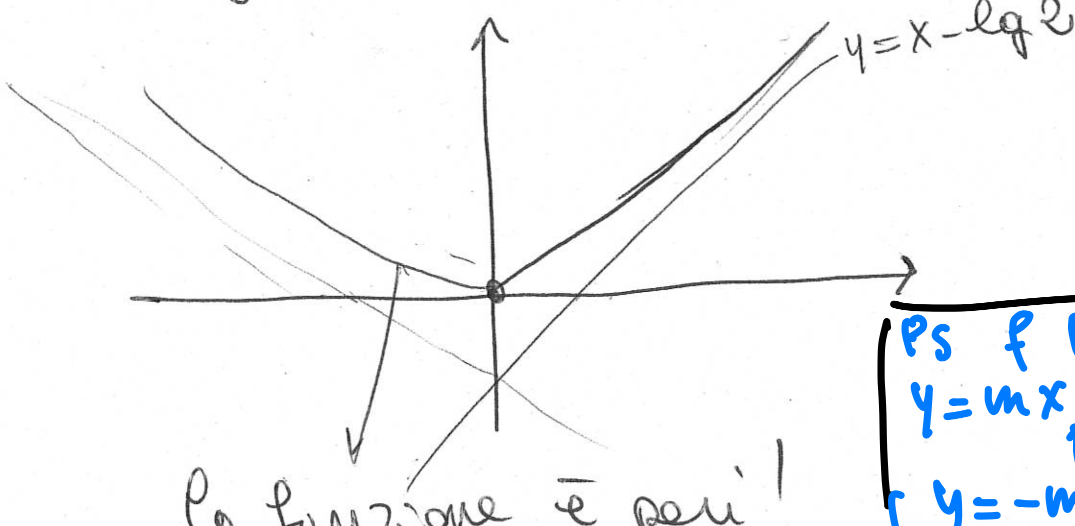
$= \lim_{x\to+\infty} \frac{x + \lg\left(\frac{1+e^{-2x}}{2}\right)}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\left(1 + \frac{\lg\left(\frac{1+e^{-2x}}{2}\right)}{x}\right)}{\cancel{x}} = 1 = a$

$\downarrow 0$

$\lim_{x\to+\infty} f(x) - x = \lim_{x\to+\infty} \lg\left(e^{x}\left(\frac{1+e^{-2x}}{2}\right)\right) - x =$

$= \lim_{x\to+\infty} \cancel{x} + \lg\left(\frac{1+e^{-2x}}{2}\right) - \cancel{x} = \lg\left(\frac{1}{2}\right) = -\lg 2 = b$ ($e^{-2x} \to 0$)

$y = x - \lg 2$ è asintoto obliquo (a $+\infty$)



la funzione è pari!

$y = -x - \lg 2$ è asintoto obliquo a $-\infty$.

PS f PARI
$y = mx + q$ AS. a $+\infty$
$\Updownarrow$
$y = -mx + q$ AS a $-\infty$

Es 4 $\quad f(x) = \lg (e^{2x}-1)^2$ (4)

$D: (e^{2x}-1)^2 \neq 0 \Rightarrow e^{2x} \neq 1 \Rightarrow x \neq 0$

$(-\infty, 0) \cup (0, +\infty)$

simmetrie $\quad f(-x) = \lg (e^{-2x}-1)^2 \neq f(x)$ NO SIMM.

$\neq -f(x)$

segno $\quad \lg (e^{2x}-1)^2 \geq 0 = \lg 1$

$(e^{2x}-1)^2 \geq 1 \Rightarrow e^{4x} - 2e^{2x} + 1 \geq 1 \Rightarrow$

$\Rightarrow e^{4x} - 2e^{2x} \geq 0 \Rightarrow e^{2x}(e^{2x}-2) \geq 0$

$e^{2x} > 0 \ \forall x \quad \boxed{e^{2x}-2 \geq 0} \Leftrightarrow e^{2x} \geq 2 = e^{\lg 2} \Leftrightarrow 2x \geq \lg 2$

$x \geq \lg 2 / 2$

$f(x) \geq 0 \iff x \geq \frac{\lg 2}{2}$

limiti:

$\lim_{x \to 0} \lg \underbrace{(e^{2x}-1)^2}_{0^+} = -\infty$ $x=0$ AS. VERTICALE

$\lim_{x \to -\infty} \lg (\underbrace{e^{2x}-1}_{})^2 = \lg(1) = 0$ $y=0$ AS. ORIZZONTALE a $-\infty$

$(-1)^2 = 1$

$\lim_{x \to +\infty} \lg \underbrace{(e^{2x}-1)}_{+\infty}{}^2 = +\infty$ NON ci sono as. orizz a $+\infty$

cerco as. obliquo

cerco as. obliquo

$$\lim_{x\to+\infty} \frac{\lg(e^{2x}-1)^2}{x} \underset{*}{=} \lim_{x\to+\infty} \frac{\cancel{x}\left(4+\frac{1}{x}\lg\left(1-\frac{1}{e^{2x}}\right)^2\right)}{\cancel{x}} = 4 = m$$

$\frac{1}{x} \to 0$ $\quad \lg(1) = 0$

⑥

$$(e^{2x}-1)^2 = \left[e^{2x}\left(1-\frac{1}{e^{2x}}\right)\right]^2 = (e^{2x})^2\left(1-\frac{1}{e^{2x}}\right)^2 =$$

$$= e^{4x}\left(1-\frac{1}{e^{2x}}\right)^2$$

$$\lg(e^{2x}-1)^2 = \lg\left(e^{4x}\left(1-\frac{1}{e^{2x}}\right)^2\right) = \lg e^{4x} + \lg\left(1-\frac{1}{e^{2x}}\right)^2 =$$

$$= 4x + \lg\left(1-\frac{1}{e^{2x}}\right)^2 = \underbrace{x\left(4+\frac{1}{x}\lg\left(1-\frac{1}{e^{2x}}\right)^2\right)}_{*}$$

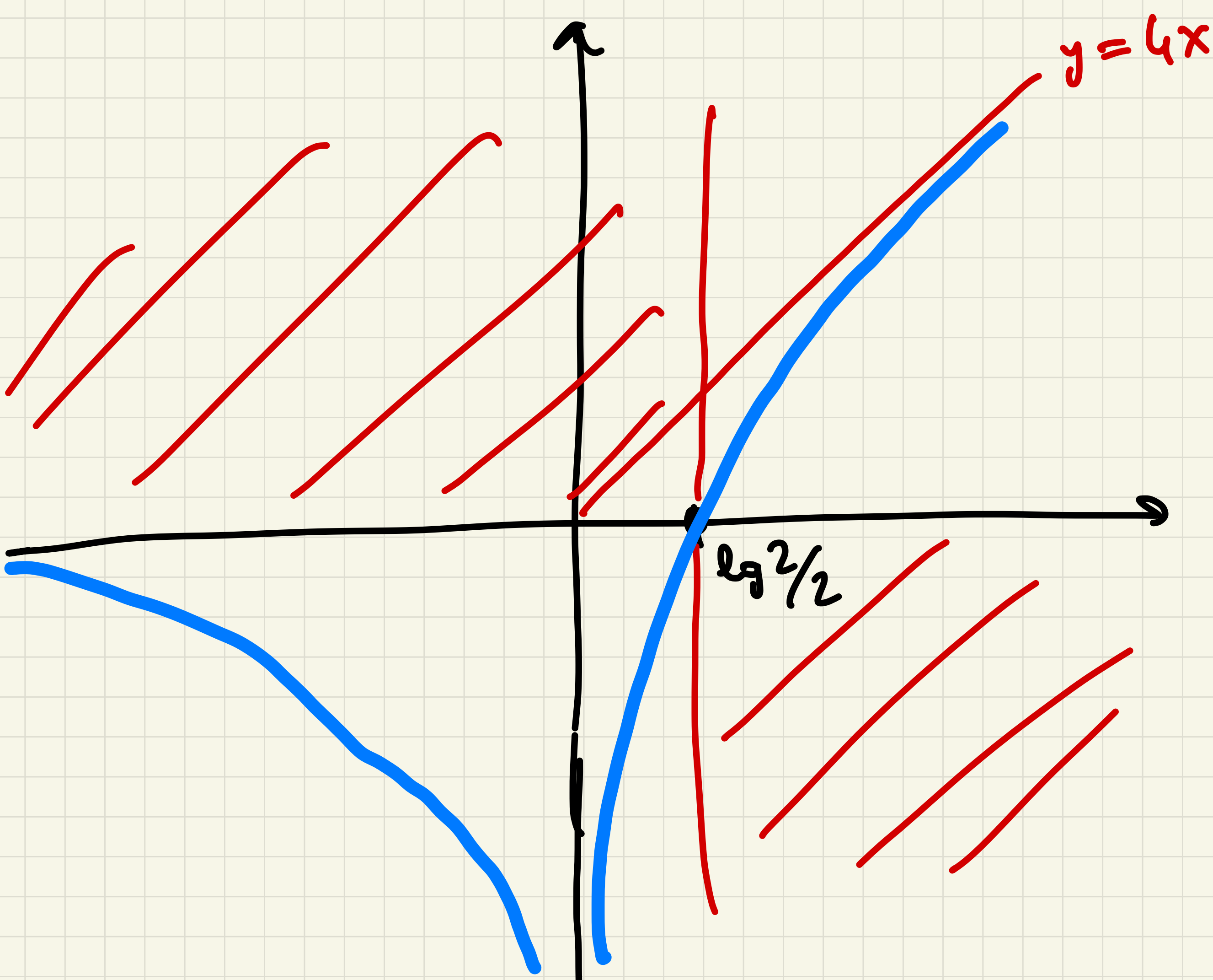
7

$$\lim_{x\to+\infty} f(x) - mx = \lim_{x\to+\infty} \lg\left(e^{2x}-1\right)^2 - 4x =$$

$$= \left(\text{ricordo } (\ast) \text{ pagina prec.}\right) = \lim_{x\to+\infty} \cancel{4x} + \lg\left(1-\frac{1}{e^{2x}}\right)^2 - \cancel{4x}$$

$$= \lg 1 = 0$$

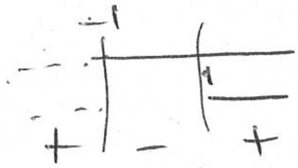
$y = 4x$ è AS. OBLIQUO a $+\infty$.



$y = 4x$
$\lg 2/2$

5) $f(x) = \lg\left(\frac{x+1}{x-1}\right)$

Dominio $\quad \frac{x+1}{x-1} > 0 \qquad x > -1 \qquad x > 1$



$x > 1$ e $x < -1$

$(-\infty, -1) \cup (1, +\infty)$.

segno $\quad f(x) \geq 0 \qquad \frac{x+1}{x-1} \geq 1 \iff \frac{x+1-(x-1)}{x-1} \geq 0 \qquad \frac{2}{x-1} \geq 0 \Rightarrow \boxed{x > 1}$

simmetrie $\quad f(-x) = \lg\left(\frac{-x+1}{-x-1}\right) = \lg\left(\frac{x-1}{x+1}\right) = -\lg\left(\frac{x+1}{x-1}\right) = -f(x)$

f è dispari.

asintoti $\quad \lim_{x \to \pm\infty} \lg\left(\frac{x+1}{x-1}\right) = \lim_{x \to \pm\infty} \lg\left(\frac{\cancel{x}\overbrace{\left(1+\frac{1}{x}\right)}^{\to 1}}{\cancel{x}\underbrace{\left(1-\frac{1}{x}\right)}_{\to 1}}\right) = \lg 1 = 0$

$y = 0$ è asintoto orizzontale a $+\infty$ e $-\infty$.

$\lim_{x \to 1^+} \lg\left(\frac{\overbrace{x+1}^{\to 2}}{\underbrace{x-1}_{\to 0^+}}\right) = +\infty \qquad x = 1$ asintoto verticale destro

$\lim_{x \to -1^-} \lg\left(\frac{\overbrace{x+1}^{\to 0^-}}{\underbrace{x-1}_{\to -2}}\right) = -\infty \qquad x = -1$ asintoto verticale sinistro

7) $f(x) = e^{\frac{x^2-4}{x-1}}$

Dominio $x \neq 1$ $\quad (-\infty,1) \cup (1,+\infty)$

segno $f(x) > 0 \quad \forall x$

simmetrie $f(-x) = e^{\frac{x^2-4}{-x-1}} \neq f(x)$, $\neq -f(x)$ no simmetrie (dominio NON E' SIMM.)

asintoti $\lim_{x\to+\infty} e^{\frac{x^2-4}{x-1}} = \lim_{x\to+\infty} e^{\frac{\cancel{x^2}(1-4/x^2)}{\cancel{x}(1-x)} \to +\infty} = +\infty$

$\lim_{x\to-\infty} e^{\frac{\cancel{x^2}(1-4/x^2)}{\cancel{x}(1-1/x)} \to -\infty} = 0 \qquad y=0$ asintoto orizzontale a $-\infty$.

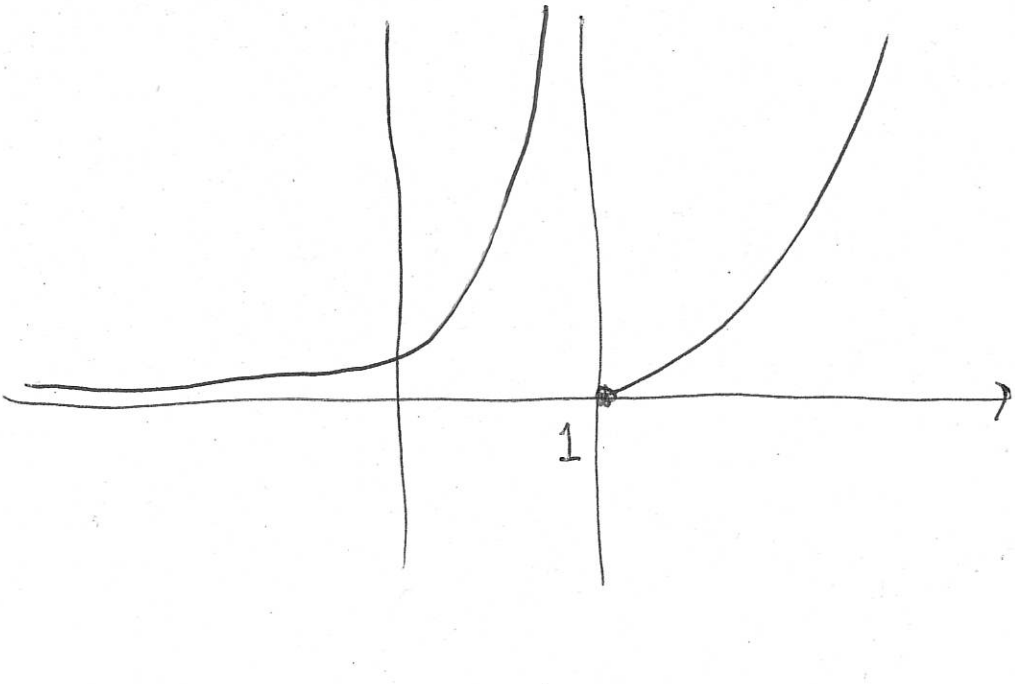
cerco as. obliquo a $+\infty$

$\lim_{x\to+\infty} \frac{e^{x\frac{(1-4/x^2)}{(1-1/x)}}}{x} = +\infty$ (per confronto infiniti)

non ci sono asintoti obliqui.

$\lim_{x\to1^+} e^{\left[\frac{x^2-4 \,\to -3}{x-1 \,\to 0^+}\right] \to -\infty} = 0$

$\lim_{x\to1^-} e^{\left[\frac{x^2-4 \,\to -3}{x-1 \,\to 0^-}\right] \to +\infty} = +\infty \qquad x=1$ è asintoto verticale sinistro



8) $f(x) = 5x+2-\sqrt{25x^2+12x}$

<u>dominio</u> $25x^2+12x \geq 0 \iff x(25x+12)\geq 0$

$x \geq 0$ e $x \leq -\frac{12}{25}$

$(-\infty, -\frac{12}{25}] \cup [0, +\infty)$

$x \geq 0$
$x \geq -\frac{12}{25}$ + − +

<u>segno</u> $5x+2 \geq \sqrt{25x^2+12x}$

se $5x+2<0$ non è mai verificata.
se $5x+2 \geq 0$ elevo entrambi al quadrato

$$\begin{cases} (5x+2)^2 \geq 25x^2+12x \\ 5x+2 \geq 0 \end{cases} \qquad \begin{cases} \cancel{25x^2}+20x+4 \geq \cancel{25x^2}+12x \\ x \geq -\frac{2}{5} \end{cases}$$

$$\begin{cases} x \geq -\frac{4}{8} = -\frac{1}{2} \\ x \geq -\frac{2}{5} \end{cases}$$

$-1/2$, $-2/5$ $\Rightarrow x \geq -\frac{2}{5}$

$x \in D \Rightarrow \boxed{x \geq 0}$

$-2/5$, $-12/25$, 0

Dunque $f(x) \geq 0 \Rightarrow x \geq 0$.

<u>simmetrie</u> non ci sono simmetrie.

$f(0) = 2 \quad f(-\frac{12}{25}) = -\frac{12}{5}+2 = -\frac{2}{5}$.

$$\lim_{x\to+\infty} \underset{+\infty \ -\infty}{\left(5x+2-\sqrt{25x^2+12x}\right)} \cdot \frac{\left(5x+2+\sqrt{25x^2+12x}\right)}{\left(5x+2+\sqrt{25x^2+12x}\right)} =$$

$$= \lim_{x\to+\infty} \frac{(5x+2)^2-(25x^2+12x)}{5x+2+\sqrt{x^2}\sqrt{25+\frac{12}{x}}} = \lim_{x\to+\infty} \frac{8x+4}{x\left[5+\frac{2}{x}+\sqrt{25+\frac{12}{x}}\right]}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\overbrace{\left(8+\frac{4}{x}\right)}^{\to 8}}{\cancel{x}\underbrace{\left[5+\frac{2}{x}+\sqrt{25+\frac{12}{x}}\right]}_{\to 10}} = \frac{8}{10} = \frac{4}{5}$$

$y = \frac{4}{5}$ è asintoto orizzontale a $+\infty$.

$$\lim_{x\to-\infty} 5x-2-\sqrt{25x^2+12x} = -\infty$$

cerco asintoto obliquo a $-\infty$

$-x = \sqrt{x^2}$!

$$\lim_{x\to-\infty} \frac{5x+2-\sqrt{25x^2+12x}}{x} = \lim_{x\to-\infty} \frac{5x+2-\sqrt{x^2}\sqrt{25+12/x}}{x} =$$

$$= \lim_{x \to -\infty} \frac{\cancel{x}\left(5 + \frac{2}{x} + \sqrt{25 + \frac{12}{x}}\right)}{\cancel{x}} = 5+5 = 10 = m$$

$$\lim_{x \to -\infty} 5x + 2 - \sqrt{25x^2 + 12x} - 10x = \lim_{x \to -\infty} \underbrace{-5x+2}_{\downarrow \atop +\infty} - \underbrace{\sqrt{25x^2+12x}}_{-\infty} =$$

f.i.

$$= \lim_{x \to -\infty} \left(-5x+2 - \sqrt{25x^2+12x}\right) \cdot \frac{\left(-5x+2+\sqrt{25x^2+12x}\right)}{\left(-5x+2+\sqrt{25x^2+12x}\right)} =$$

$$= \lim_{x \to -\infty} \frac{(-5x+2)^2 - (25x^2 + 12x)}{-5x+2+\sqrt{x^2}\sqrt{25+\frac{12}{x}}} = \lim_{x \to -\infty} \frac{\cancel{25x^2} - 20x + 4 - \cancel{25x^2} - 12x}{x\left[-5 + \frac{2}{x} - \sqrt{25 + \frac{12}{x}}\right]}$$

$$= \lim_{\cancel{x} \to -\infty} \frac{\cancel{x}\left(-32 + \frac{4}{x}\right)}{\cancel{x}\left(-5 + \frac{2}{x} - \sqrt{25 + \frac{12}{x}}\right)} = \frac{-32}{-10} = \frac{16}{5} = q$$

$y = 10x + \frac{16}{5}$ è AS. OBLIQUO a $-\infty$.

$f(0) = +2$

$f\left(-\frac{12}{5}\right) = -10$