

Soluzioni foglio 3 tris

$$3) f(x) = \lg(e^{2x} - 4e^x + 4)$$

$$D = (-\infty, \lg 2) \cup (\lg 2, +\infty)$$

f è continua in D .

Calcolo limiti in $\lg 2, +\infty, -\infty$.

$$\lim_{x \rightarrow \lg 2} \lg(e^{2x} - 4e^x + 4) = -\infty$$

infatti: $e^{2x} - 4e^x + 4 = (e^x - 2)^2$

$$\begin{aligned} x &\rightarrow \lg 2 \\ e^x &\rightarrow 2 \\ (e^x - 2)^+ &\rightarrow 0^+ \end{aligned}$$

$$\lim_{x \rightarrow +\infty} \lg(e^{2x} - 4e^x + 4)$$

$\infty - \infty$ fa!

$$\lim_{x \rightarrow +\infty} e^{2x} - 4e^x + 4 =$$

raccoglio
 l'infinito di
 ordine MAGGIORE

$$= \lim_{x \rightarrow +\infty} e^{2x} \left(1 - \frac{4}{e^x} + \frac{4}{e^{2x}} \right)$$

tra e^{2x} e e^x è
 maggiore e^{2x} .

NB $e^{2x} = e^x \cdot e^x!$

$$e^{2x} = (e^2)^x = e^x \cdot e^x$$

$$\begin{array}{l} \downarrow \quad \downarrow \\ \infty \quad 1 \\ \hline = \infty \cdot 1 = \infty \end{array}$$

$$\lim_{x \rightarrow +\infty} \lg(e^{2x} - 4e^x + 4) = \lim_{y \rightarrow +\infty} \lg y = +\infty$$

$$\lim_{x \rightarrow -\infty} \lg \left(\underbrace{e^{2x} - 4e^x + 4}_{0+0+4} \right) = \lg 4$$

$\lg x$ è continuo!

$$4) f(x) = \arcsin\left(\frac{|x+2|}{x}\right)$$

$$D = (-\infty, -1]$$

oss f è continua in tutto il suo dominio e anche in -1 (continua a sinistra)

$$\lim_{x \rightarrow -1^-} \arcsin\left(\frac{|x+2|}{x}\right) = \arcsin\left(\frac{|1|}{-1}\right) = -\frac{\pi}{2}$$

Calcolo limite a $-\infty$

$$\lim_{x \rightarrow -\infty} \arcsin\left(\frac{|x+2|}{x}\right)$$

$$x \rightarrow -\infty$$

$$x+2 < 0$$

$$|x+2| = -x-2$$

$$= \lim_{x \rightarrow -\infty} \arcsin\left(\frac{-x-2}{x}\right)$$

$$\lim_{x \rightarrow -\infty} \frac{-x-2}{x} =$$

Indet!
 $\frac{+\infty}{-\infty}$

(raccolgo
infinito di
ordine
maggiore)

$$= \lim_{x \rightarrow -\infty} \frac{\cancel{x}(-1 - \frac{2}{x})}{\cancel{x}} = \lim_{x \rightarrow -\infty} -1 - \frac{2}{x} = -1$$

$$\lim_{x \rightarrow -\infty} \arcsin \frac{|x+2|}{x} = \lim_{y \rightarrow -1} \arcsin y = \arcsin(-1) = -\frac{\pi}{2}$$

Ess

$$f(x) = \frac{1}{|x+1|-2}$$

$$D: x \neq 1, -3$$

$$D = (-\infty, -3) \cup (-3, 1) \cup (1, +\infty)$$

f è continua in D

calcoliamo i limiti a $\pm\infty$

$$\text{Nota che se } x \rightarrow +\infty \quad |x+1|-2 \rightarrow +\infty$$

$$\text{se } x \rightarrow -\infty \quad |x+1|-2 \rightarrow +\infty$$

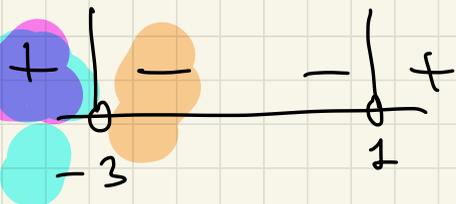
$$\text{dunque } \lim_{x \rightarrow +\infty} \frac{1}{|x+1|-2} = 0 = \lim_{x \rightarrow -\infty} \frac{1}{|x+1|-2}$$

ne calcoliamo i limiti in $x=1$ e $x=-3$

$$\text{se } x \rightarrow 1 \quad |x+1|-2 \rightarrow 0$$

$$\text{se } x \rightarrow -3 \quad |x+1|-2 \rightarrow 0$$

Ricordiamo anche $|x+1|-2 > 0 \Leftrightarrow x > 1$
oppure $x < -3$



quindi se
 $x \rightarrow 1^+$

$$|x+1|-2 \rightarrow 0 \quad \text{e}$$

$$|x+1|-2 > 0$$

mentre

$$\text{se } x \rightarrow 1^-$$

$$|x+1|-2 \rightarrow 0 \quad \text{e}$$

$$|x+1|-2 < 0$$

$$\lim_{x \rightarrow 1^+} \frac{1}{|x+1|-2} = +\infty$$

$$\lim_{x \rightarrow 1^-} \frac{1}{|x+1|-2} = -\infty$$

Analogamente.

$$\lim_{x \rightarrow -3^-} \frac{1}{|x+1|-2} = +\infty$$

$$\lim_{x \rightarrow -3^+} \frac{1}{|x+1|-2} = -\infty$$

$$11) f(x) = 2x - \sqrt{|x^2 - 4x + 3|}$$

$$D = \mathbb{R}.$$

Calcolo limiti $\infty + \infty, -\infty$

$$\lim_{x \rightarrow -\infty} 2x - \sqrt{|x^2 - 4x + 3|} = -\infty - \infty = -\infty$$

$$\lim_{x \rightarrow +\infty} 2x - \sqrt{|x^2 - 4x + 3|} = \text{f. indet. } +\infty - \infty.$$

NB $x \rightarrow +\infty$ $|x^2 - 4x + 3| = x^2 - 4x + 3$
TOLGO IL MODULO.

per risolvere f. irred. applico trucco della
razionalizzazione

$$(2x - \sqrt{x^2 - 4x + 3}) \cdot \frac{2x + \sqrt{x^2 - 4x + 3}}{2x + \sqrt{x^2 - 4x + 3}} =$$

$$= \frac{4x^2 - (x^2 - 4x + 3)}{2x + \sqrt{x^2 - 4x + 3}} =$$

$$= \frac{4x^2 - x^2 + 4x - 3}{2x + \sqrt{x^2 - 4x + 3}} = \frac{3x^2 + 4x - 3}{2x + \sqrt{x^2 - 4x + 3}}$$

NUMERATOR

$$3x^2 + 4x - 3 =$$

$$x^2 \left(3 + \frac{4}{x} - \frac{3}{x^2} \right)$$

Annotations: A bracket under the terms $3 + \frac{4}{x} - \frac{3}{x^2}$ is labeled with a downward arrow and the number 3. A bracket under the term $\frac{4}{x}$ is labeled with a downward arrow and x^{-1} . A bracket under the term $-\frac{3}{x^2}$ is labeled with a downward arrow and x^{-2} . Arrows point from the superscripts 0 and -2 to the terms 3 and $-\frac{3}{x^2}$ respectively.

DENOMINATOR

$$2x + \sqrt{x^2 - 4x + 3} = 2x + \sqrt{x^2 \left(1 - \frac{4}{x} + \frac{3}{x^2} \right)} =$$

$$= 2x + x \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} = x \left(2 + \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} \right)$$

Annotations: A bracket under the terms $1 - \frac{4}{x} + \frac{3}{x^2}$ is labeled with a downward arrow and x^{-1} . A bracket under the terms $1 - \frac{4}{x} + \frac{3}{x^2}$ is labeled with a downward arrow and $2 + 1 = 3$. Arrows point from the superscripts 0 and -2 to the terms 1 and $-\frac{4}{x}$ respectively.

$\sqrt{x^2} = x$
 $x > 0!$

forma al limite

$$\lim_{x \rightarrow +\infty} 2x - \sqrt{x^2 - 4x + 3} =$$

$x \rightarrow +\infty$

$$= \lim_{x \rightarrow +\infty}$$

$$\frac{x^{\cancel{2}} \left(3 + \frac{4}{x} - \frac{3}{x^2} \right)}{\cancel{x} \left(2 + \sqrt{1 - \frac{4}{x} + \frac{3}{x^2}} \right)} = +\infty$$

Handwritten annotations: A red circle around the 2 in the numerator's $x^{\cancel{2}}$ term. A red arrow points from the 3 in the numerator to the 3 in the denominator's radical. A red arrow points from the 3 in the denominator's radical to the 3 in the denominator's constant term.

