DISEQUAZIONI

1) $x < x^2$

R. $x(x-1) > 0 \Rightarrow x < 0$ o $x > 1$

2) $3x^2 + 2x - x^2 < 1 + 2x^2$

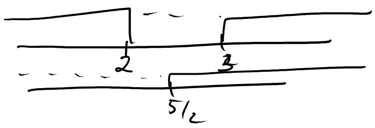
R. $2x - 1 < 0 \Rightarrow x < 1/2$

3) $(x^2 - 5x + 6)(2x - 5) \geq 0$

R. $x^2 - 5x + 6 \geq 0 \qquad x \geq 3$ o $x \leq 2$

$2x - 5 \geq 0 \qquad x \geq 5/2$

$\Rightarrow \; x \geq 3$ o $2 \leq x \leq 5/2$

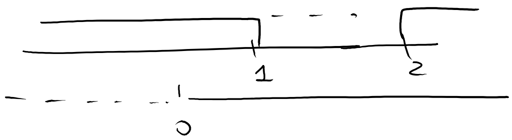


4) $\dfrac{x-2}{x} \leq x - 2$

R. $x \neq 0 \qquad \dfrac{x-2}{x} - x + 2 \leq 0 \qquad \dfrac{x - 2 - x^2 + 2x}{x} \leq 0$

$\dfrac{-x^2 + 3x - 2}{x} \leq 0 \qquad \dfrac{x^2 - 3x + 2}{x} \geq 0 \longrightarrow x \leq 1$ o $x \geq 2$

$\Rightarrow \; 0 < x \leq 1, \; x \geq 2$



5) $\dfrac{1}{\alpha x} - 2x > 0, \qquad \alpha \in \mathbb{R}, \alpha \neq 0$

$\dfrac{1 - 2\alpha x^2}{\alpha x} > 0 \qquad 1 - 2\alpha x^2 = 0 \qquad x^2 = \dfrac{1}{2\alpha} \qquad$ se $\alpha < 0$

$1 - 2\alpha x^2 > 0$

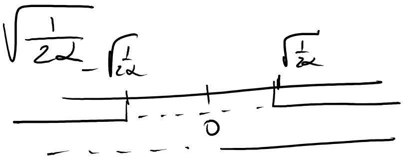
quindi $\alpha < 0 \qquad \dfrac{(\;)}{(\;)} > 0 \Leftrightarrow x < 0$

se $\alpha > 0 \qquad 1 - 2\alpha x^2 = 0 \Leftrightarrow x = \pm\sqrt{\dfrac{1}{2\alpha}}$

$1 - 2\alpha x^2 > 0 \Leftrightarrow x < -\sqrt{\dfrac{1}{2\alpha}}$ o $x > \sqrt{\dfrac{1}{2\alpha}}$

$\alpha x > 0 \Leftrightarrow x > 0$ perché $\alpha > 0$

quindi $\alpha > 0 \quad -\sqrt{\dfrac{1}{2\alpha}} < x < 0, \; x > \sqrt{\dfrac{1}{2\alpha}}$



se $\alpha < 0$ per $x < 0$.

6) $|x - 1| < |2x - 3|$

a) $x \geq \frac{3}{2} \qquad |x - 1| < 2x - 3$ cioè

$-(2x-3) \underset{①}{\leq} x - 1 \underset{②}{\leq} 2x - 3$

① $x - 1 > -2x + 3 \Rightarrow 3x > 4 \qquad x > 4/3$

② $x - 1 < 2x - 3 \Rightarrow x > 2$

$\Rightarrow \{x > 2\} \cap \{x \geq \frac{3}{2}\}$

$\Rightarrow x > 2$

b) $x < \frac{3}{2} \qquad |x - 1| < 3 - 2x$ cioè

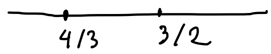
$2x - 3 \underset{①}{\leq} x - 1 \underset{②}{\leq} 3 - 2x$

① $2x - 3 < x - 1 \Rightarrow x < 2$

② $x - 1 < 3 - 2x \Rightarrow 3x < 4 \Rightarrow x < 4/3$

$\Rightarrow \{x < 4/3\} \cap \{x < \frac{3}{2}\}$

$\Rightarrow x < 4/3$



Quindi $x<\frac{4}{3}$ o $x>2$ (faccio l'unione delle soluzioni di a) e b))

7) $|x|x+5x-6>0$

- $x>0$ $\quad x^2+5x-6>0$ $\quad x=\frac{-5\pm\sqrt{25+24}}{2}=\frac{-5\pm 7}{2}=\begin{cases}-6\\1\end{cases}$

$x>1$ o $x<-6$ e poiché $x>0$
$\Rightarrow x>1$

- $x<0$ $\quad -x^2+5x-6>0$
$x^2-5x+6<0$ $\quad x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm 1}{2}=\begin{cases}3\\2\end{cases}$

$2<x<3$ ma poiché $x<0$ mai!

Quindi $x>1$.

8) $|5x-1|\geq x$ $\Rightarrow$ $5x-1\geq x$ o $5x-1\leq -x$

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

$4x\geq 1$ $\qquad$ $6x\leq 1$

$x\geq 1/4$ o $x\leq 1/6$

9) $x^2+|x+1|\leq 4$

$x\geq -1$ $\Rightarrow$ $x^2+x-3\leq 0$ $\quad x=\frac{-1\pm\sqrt{1+12}}{2}=\frac{-1\pm\sqrt{13}}{2}$

$\frac{-1-\sqrt{13}}{2}\leq x\leq\frac{-1+\sqrt{13}}{2}$ ma poiché $x\geq -1$

$\Downarrow$

$-1\leq x\leq\frac{-1+\sqrt{13}}{2}$

$x<-1$ $\quad x^2-x-5\leq 0$ ..... le soluzioni vanno $\cap$ con $x<-1$

10) $\left|\frac{x-3}{x-5}\right|>2x$

R. $x\neq 5$ $\qquad \frac{x-3}{x-5}>2x$ o $\frac{x-3}{x-5}<-2x$

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

$\frac{x-3}{x-5}-2x>0$ $\qquad$ $\frac{x-3}{x-5}+2x<0$

$\frac{x-3-2x(x-5)}{x-5}>0$ $\qquad$ $\frac{x-3+2x(x-5)}{x-5}<0$

$\frac{-2x^2+11x-3}{x-5}>0$ $\qquad$ $\frac{2x^2-9x-3}{x-5}<0$

$\vdots$ $\qquad\qquad$ $\vdots$

e si fa l'$\cup$ delle soluzioni.

Disequazioni

1) $\boxed{x+|x|>1}$

R. $x>0 \Rightarrow \quad 2x>1 \Rightarrow \quad x>\frac{1}{2}$

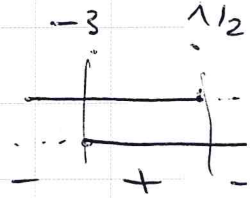
$x<0 \Rightarrow \quad x-x>1$ nessuna sol.

$\Big\} \; x>\frac{1}{2}$

2) $\boxed{\left|\frac{6-5x}{3+x}\right| \leq 1}$

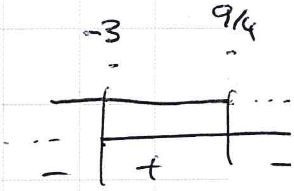
R. $-1 \leq \frac{6-5x}{3+x} \leq 1 \Rightarrow \begin{cases} \frac{6-5x}{3+x} \leq 1 \\ \frac{6-5x}{3+x} \geq -1 \end{cases}$

$\frac{6-5x}{3+x} \leq 1 \Rightarrow \frac{6-5x-3-x}{3+x} \leq 0 \Rightarrow \frac{3-6x}{3+x} \leq 0$



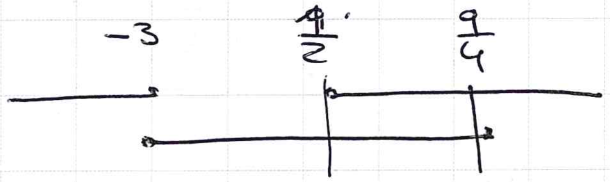
$\left[\begin{array}{l} x<-3 \\ x\geq \frac{1}{2} \end{array}\right.$

$\frac{6-5x}{3+x} \geq -1 \Rightarrow \frac{6-5x+3+x}{3+x} \geq 0 \Rightarrow \frac{9-4x}{3+x} \geq 0$



$-3 < x \leq \frac{9}{4}$

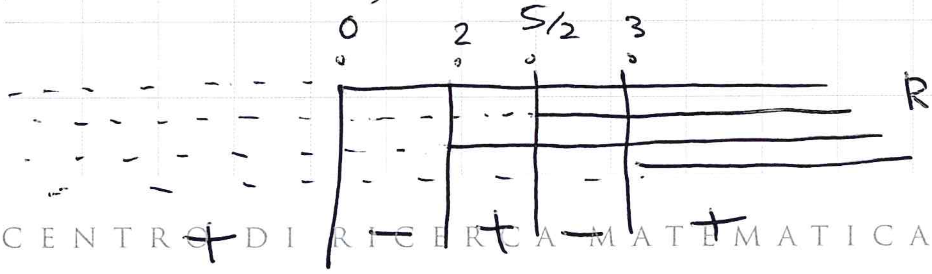
A sistema



R: $\quad \frac{1}{2} \leq x \leq \frac{9}{4}$

3) $\boxed{\frac{x}{x-3} + \frac{x}{x-2} \leq 0}$

$\frac{x[x-2+x-3]}{(x-3)(x-2)} \leq 0 \qquad \frac{x(2x-5)}{(x-3)(x-2)} \leq 0$



R $\boxed{\begin{array}{l} 0 \leq x < 2 \\ \frac{5}{2} \leq x < 3 \end{array}}$

4) $\boxed{\sqrt{x+1} \leq x-1}$

$x+1 \geq 0$ condizioni esistenza $x \geq -1$

inoltre $\sqrt{x+1} \geq 0 \Rightarrow x-1 \geq 0$ altrimenti non ho soluzioni

- se $-1 \leq x < 1$ nessuna soluzione

$x \geq 1$ $\sqrt{x+1} \leq x-1$ elevo al quadrato

$$x+1 \leq (x-1)^2$$

$$x+1 \leq x^2-2x+1$$

$$x^2-3x \geq 0 \Rightarrow x(x-3) \geq 0$$

$x \geq 3$

non accettab. $\leftarrow x \leq 0$

$\Rightarrow$ R $x \geq 3$

5) $\boxed{x+1 < \sqrt{x^2-3x+2}}$

$x^2-3x+2 \geq 0$ condizioni esistenza $\Rightarrow$

$x^2-3x+2=0$

$x=1$, $x=2$

$\Rightarrow$ $\boxed{x \geq 2, \ x \leq +1}$

Elevo al quadrato

$$\cancel{x^2+2x+1}\ (x+1)^2 < \left(\sqrt{x^2-3x+2}\right)^2$$

$$x^2+2x+1 < x^2-3x+2$$

$$\Rightarrow 5x-1 < 0 \Rightarrow \boxed{x < \frac{1}{5}}$$

A sistema con condiz. esistenza

1/5 1 2

R $\boxed{x < \frac{1}{5}}$