Reduction

$L \leq_m L'$

we know L is not in, say, RE

this is the language under investigation

L' turns out to be outside of RE, because of the reduction

mistakes :

$L' \leq_m L$

??

not in RE

X Don't do this !!

A <u>reduction</u> is a sort of <u>algorithm</u>

$L \leq_m L'$

transform M into M'

input : instance of L

<u>enc(M)</u>

output : instance of L'

<u>enc(M')</u>

<u>y2y / n2n relation</u> :

→ if $enc(M) \in L$ then $enc(M') \in L'$

if $enc(M) \notin L$ then $enc(M') \notin L'$

Let M_1, M_2 be TM. We write $\underline{enc(M_1, M_2)}$ = encoding of the two TMs.

$$L = \{ enc(M_1, M_2) \mid \overline{L(M_1)} = L(M_2) \}$$

$\underline{\text{string}}$

Show that L is not in RE.

L_d, $\underline{L_e}$ $\qquad L_e \le_m L$

Therefore, for some thm, L is not in RE.

specify the reduction

$L_e \leq_m L$

source lang.

target lang.

input : $enc(M)$

→ output : $enc(M_1, M_2)$ ← these TMs depend on M !!

possible mistakes : M_1, M_2 independent of M

Don't do !!

We want : if $L(M) = \emptyset$ then $\overline{L(M_1)} = L(M_2)$

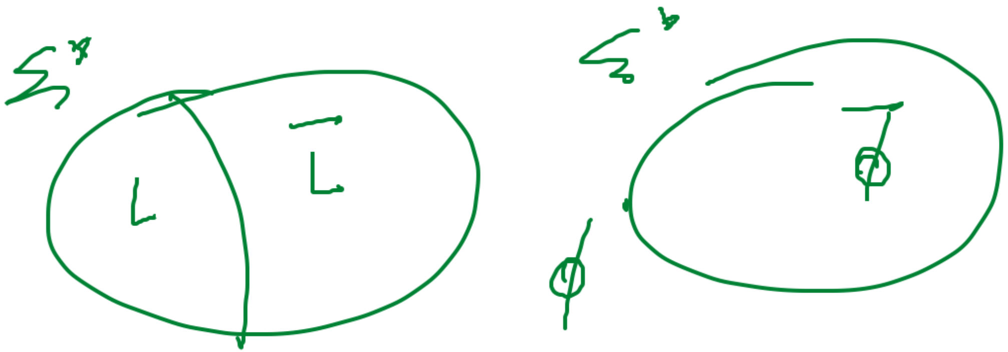
this is the property defining the source lang.

this is the property defining L_1 (target lang.)

Idea set $M_1 = M$

if M recognizes empty language, $L(M) = \emptyset$

then $\overline{L(M_1)} = \Sigma^*$



$\Sigma^* = \overline{\emptyset}$

set $\underline{M_2 = M_{\Sigma^*}}$ where $L(M_{\Sigma^*}) = \Sigma^*$

M_2 does not depend on M

but M_1 depends on M

input	output

$M \xrightarrow{\text{copy}} M_1$

$M_2 = M_{\Sigma^*} \rightarrow L(M_{\Sigma^*}) = \Sigma^*$

A y2y

if $L(M) = \emptyset$ then $L(M_1) = \emptyset$

this means $\mathrm{enc}(M) \in L_e$

then $\overline{L(M_1)} = \Sigma^*$

then $\overline{L(M_1)} = L(M_2)$

then $\mathrm{enc}(M_1, M_2) \in L$

B n2n

if $enc(M) \notin L_e$ then $L(M) \neq \emptyset$

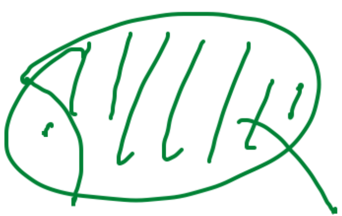
then $L(M_1) \neq \emptyset$

then $\overline{L(M_1)} \neq \Sigma^*$

then $\overline{L(M_1)} \neq L(M_2)$ because M_2 recog. Σ^*

then $enc(M_1, M_2) \notin L$

Σ^*



$\overline{L(M_1)} \neq \Sigma^*$

$L(M_1)$

We have shown y2y $\leq$ n2n property

Therefore the reduction is valid