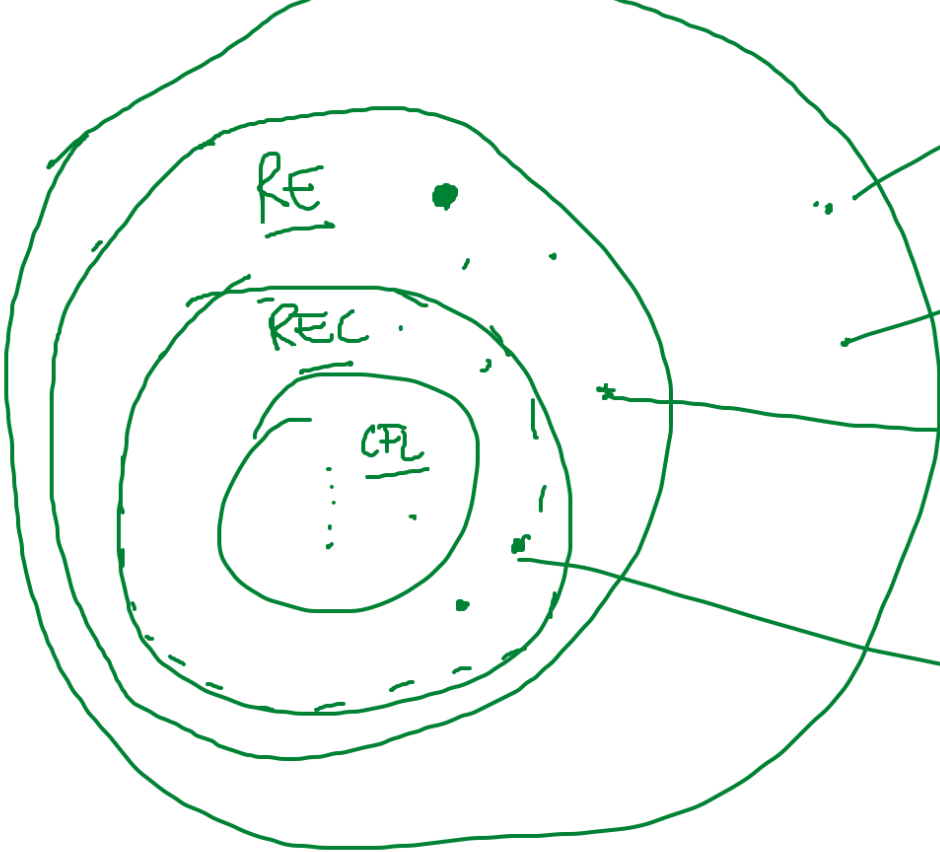


no way of recognizing

class of all possible languages

(lang. recog. by TM that <u>may not</u> halt for <u>NO answer.</u>)

(lang recog. by TMs that always halt (<u>yes</u>/<u>no</u>))

Exercise

Set of languages from RE

$\rightarrow \underline{P} = \{ L \mid L \in RE, \; |L| \geq 5 \}$

\# of strings

we need to encode languages into some finite representation

$L \dashrightarrow enc(M)$ s.t. $L(M) = L$

$\rightarrow L_P = \{ enc(M) \mid L(M) \in P \}$

string (finite)

L_ϕ encoding of ϕ

1) is L_ϕ in REC ? (is there an algo / TM for L_ϕ ?)

. YES : design TM M s.t. $L(M) = L_\phi$ and M always halts

. NO : use Rice's Theorem !

some methodology used for REG L. of CF L.

apply Rice's thm

We must show $\mathcal{P}$ not a trivial property

1) $\mathcal{P} \neq \emptyset$ $\quad L = \Sigma^*$ $\quad |L| \geq 5$ $\quad (\infty)$

$L \in \mathcal{P}$!!

alternative sol.
($L = \{ w \mid |w| \text{ even} \}$ over Σ^*)

2) $\mathcal{P} \neq RE$: find L in RE s.t. $L \notin \mathcal{P}$

$L = \emptyset$ (empty lang. $\equiv$ zero strings)

$L \in RE$, $\quad |L| = 0$ $\quad L \notin \mathcal{P}$!!

We conclude from 1 + 2 that $\mathcal{P}$ not in REC possible mistakes on Rice's thm

- misunderstanding of $\mathcal{P}$
- too complex languages for 1 / 2 :
  keep it simple !!

...

2) $L_\emptyset$ in RE ?

YES ---> design TM s.t. $L(M) = L_\emptyset$

means that for every $w \in L_\emptyset$, M halts with positive answer

NO ---> reduction .

pick L s.t. $L \notin RE$

show $L \leq_m L_\emptyset$ (meaning $L_\emptyset$ is more difficult than L)

$L_\emptyset \in RE$

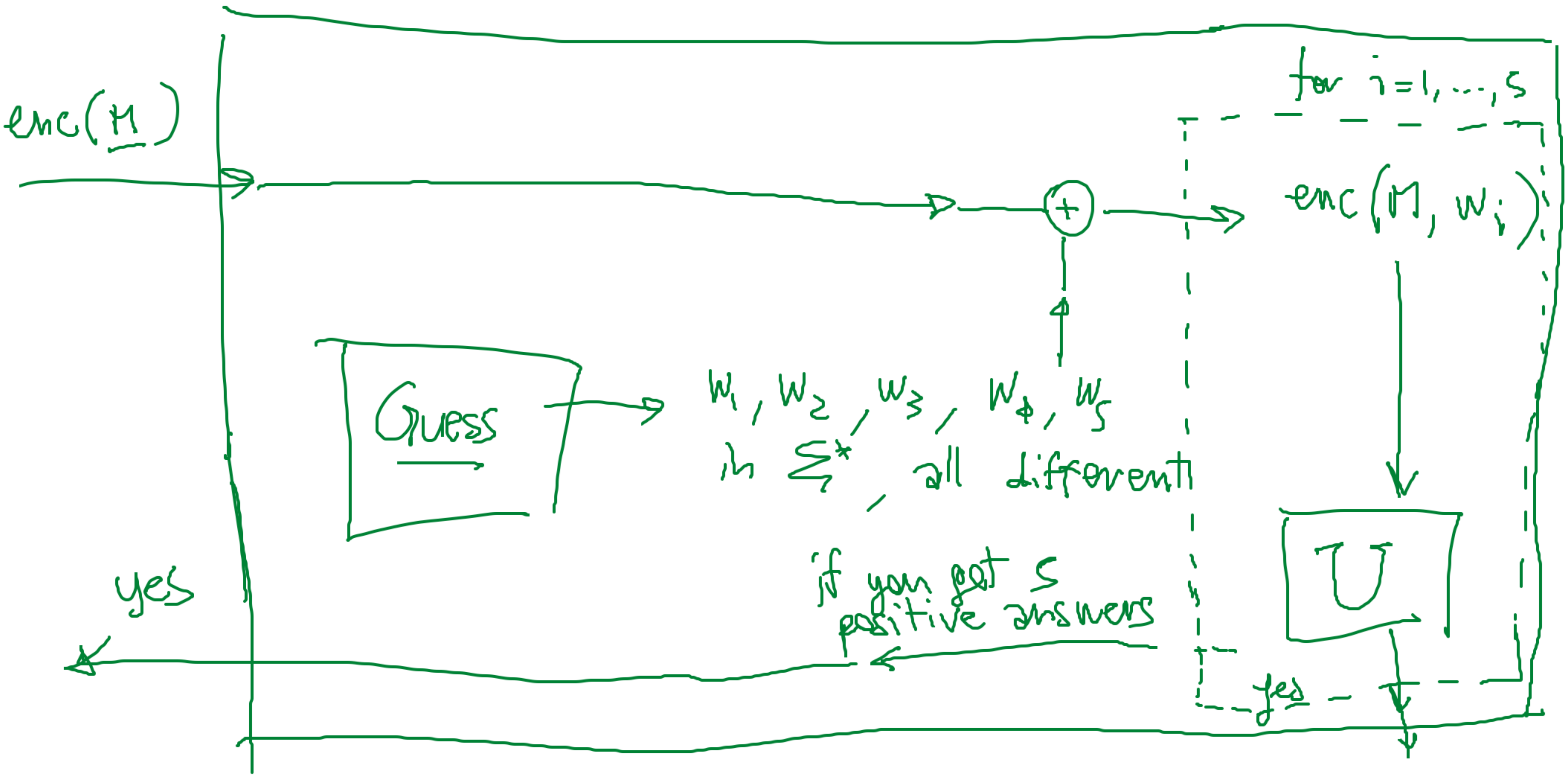
$L_\emptyset = \{ enc(M) \mid |L(M)| \geq 5 \}$

<u>design TM</u>

$$enc(M) \longrightarrow \boxed{TM\ ?} \xrightarrow{\text{yes}} \text{if } |L(M)| \geq 5$$

specify M through diffent steps or through blocks

design nondeterministic TM



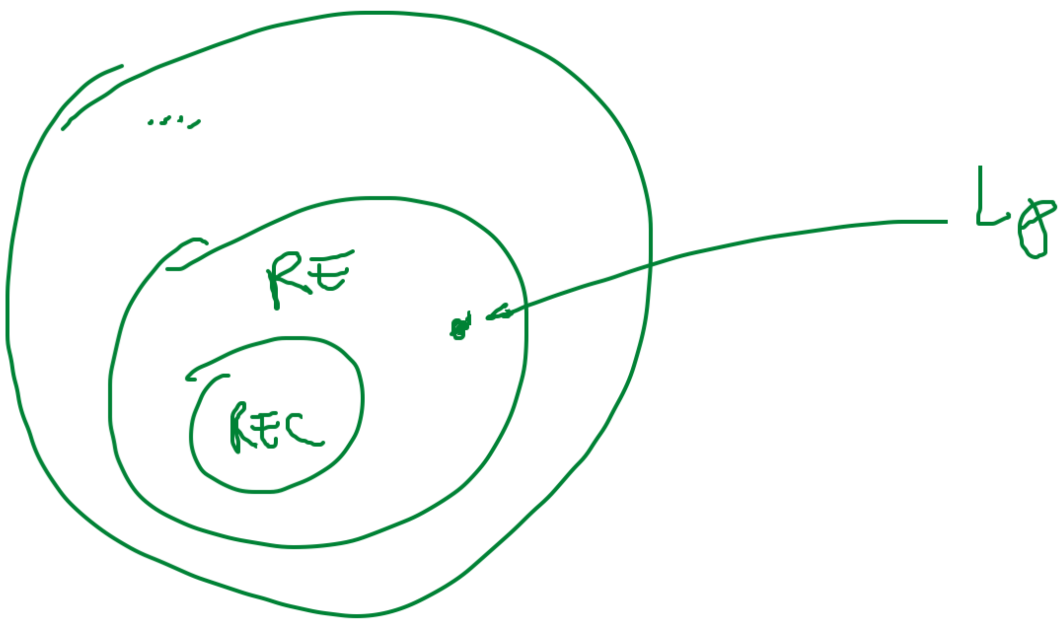
- if $L(M) \in P$, then our nondeterministic TM will go through a computation where S strings in $L(M)$ are guessed, and it will therefore provide positive answer and halt.

- if $L(M) \notin P$, then no guessing of S strings all accepted by M is ever possible. Therefore NTM will never accept / halt

We translate NTM into (deterministic) TM M

such that $L(M) = L_\emptyset$.

We conclude $L_\emptyset$ is in RE.



possible mistakes

$L_\phi \in RE$

(don't use reduction!)

wrong!!

don't

L_{ne}

$L_{ne} \leq_m L_\phi$

not REC

not REC

this would be alternative sol. for #1.

(which we have solve using Rice)