Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

July 4th, 2024

1. **[4 points]** Consider the regular expression $R = (\boldsymbol{ab} + \boldsymbol{ba})^*\emptyset(\boldsymbol{aa})$. Convert R into an equivalent ϵ-NFA using the construction provided in the textbook, and report all the intermediate steps.
Important: do not simplify the regular expression R before applying the construction.

2. **[9 points]** Let $\Sigma = \{a, b, c\}$. For $w \in \Sigma^*$ and $X \in \Sigma$, we write $\#_X(w)$ to denote the number of occurrences of X in w. Consider the following languages

$$\begin{aligned} L_1 &= \{w \mid w \in \Sigma^*,\ \#_a(w) = \#_b(w) = \#_c(w)\} ; \\ L_2 &= \{w \mid w \in \Sigma^*,\ \#_a(w) = \#_c(w)\} . \end{aligned}$$

   (a) Prove that L_1 is outside of CFL.
   (b) Prove that L_2 is in CFL.
   (c) Prove that L_2 is not in REG.

3. **[5 points]** Consider the CFG G implicitly defined by the following productions:

$$\begin{aligned} &S \to AAB \mid ABB \mid BBB \\ &A \to aAB \mid bBB \\ &B \to b \mid \varepsilon \end{aligned}$$

Perform on G the transformations indicated below, that have been specified in the textbook, in the given order. Report the CFGs obtained at each of the intermediate steps.

   (a) Eliminate the ε-productions
   (b) Eliminate the unary productions
   (c) Eliminate the useless symbols
   (d) Produce a CFG in Chomsky normal form equivalent to G.

(please turn to the next page)

4. **[5 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) Let L_1 be a language in REG with L_1 non-finite, and let L_2 be a language in CFL$\setminus$REG. The language $L_1 \cap L_2$ may be in CFL$\setminus$REG.

   (b) Let L_1 be a language in REG with L_1 non-finite, and let L_2 be a language in CFL$\setminus$REG. The language $L_1 \cap L_2$ may be in REG.

   (c) Let L_1, L_2 be languages in CFL. The language $L_1 \cap L_2$ belongs to $\mathcal{P}$, the class of languages that can be recognized in polynomial time by a TM.

   (d) Let R be the string reversal operator, which we extend to languages. Let L be a language in REC. Then L^R belongs to REC.

5. **[4 points]** Define the diagonalization language L_d. Show that L_d is not an RE language, using the proof reported in the textbook.

6. **[6 points]** Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{RE, every string in } L \text{ has even length}\}$$

   and define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Use Rice's theorem to show that $L_{\mathcal{P}}$ is not in REC.

   (b) Prove that $L_{\mathcal{P}}$ is not in RE.