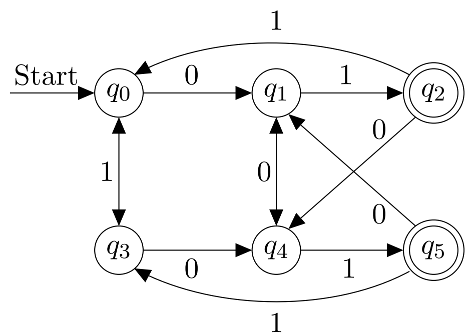
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

September 13th, 2023

1. **[6 points]** Consider the DFA A whose transition function is graphically represented as follows (arcs with double direction represent two arcs in opposite directions)



   (a) Apply to A the tabular algorithm presented in the textbook for detecting pairs of equivalent states, reporting all the **intermediate steps**.

   (b) Specify the minimal DFA equivalent to A.

2. **[7 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b\}$:

$$L_1 = \{a^m b a^n b a^p \mid m, n, p \geq 1,\ m < n < p\}$$
$$L_2 = \{a^m b a^n b a^p \mid m, n, p \geq 1,\ m + n < n + p\}$$

   For each of the above languages, state whether it belongs to the class CFL and provide a mathematical proof of your answer.

3. **[5 points]** With reference to the membership problem for context-free languages, answer the following two questions.

   (a) Specify the dynamic programming algorithm developed in the textbook for the solution of this problem.

   (b) Consider the CFG G defined by the following rules:

$$S \rightarrow BD$$
$$B \rightarrow BB \mid b$$
$$D \rightarrow DD \mid d$$

   Assuming as input the CFG G and the string $w = bbbbdd$, trace the application of the above algorithm.

(please see next page)

4. **[6 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) The concatenation of a regular language and a context-free language is never a regular language.

   (b) The concatenation of a regular language and a context-free language is always a regular language.

   (c) The concatenation of a regular language and a context-free language is always a context-free language.

   (d) The concatenation of a regular language and a language in $\mathcal{P}$ is always a language in $\mathcal{P}$ ($\mathcal{P}$ is the class of languages that can be recognized in polynomial time by a TM).

5. **[4 points]** Define the diagonalization language L_d. Show that L_d is not an RE language using the proof reported in the textbook.

6. **[6 points]** Recall that for two strings $x, w \in \Sigma^*$, we say that x is an **infix** of w is we can write $w = uxv$ for some strings $u, v \in \Sigma^*$. Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{RE}, \;\; \text{every string in } L \text{ has an infix } 01110\}$$

   and define $L_\mathcal{P} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$.

   (a) Use Rice's theorem to show that $L_\mathcal{P}$ is not in REC.

   (b) Prove that $L_\mathcal{P}$ is not in RE.