Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

July 3rd, 2023

1. **[4 points]** Consider the regular expression $R = (\mathbf{1} + \epsilon)(\mathbf{0}\mathbf{0}^*\mathbf{1})^*$. Convert R into an equivalent ϵ-NFA using the construction provided in the textbook, and report the intermediate steps.

2. **[7 points]** Consider the following languages, defined over the alphabet $\{a, b\}$:

$$\begin{aligned} L_1 &= \{ba^{\frac{n}{2}}ba^nb \mid n \geq 0,\ n \text{ even}\}\ ; \\ L_2 &= \{ba^{\frac{n}{2}}a^nb \mid n \geq 0,\ n \text{ even}\}\ ; \\ L_3 &= L_2L_2\ . \end{aligned}$$

   For each of the above languages, state whether it belongs to REG or else CFL$\smallsetminus$REG, and provide a mathematical proof for all of your answers.

3. **[6 points]** Consider the CFG G implicitly defined by the following productions:

$$\begin{aligned} S &\rightarrow BAB \mid BBB \\ A &\rightarrow aB \\ B &\rightarrow bA \mid \varepsilon \end{aligned}$$

   Perform on G the following transformations that have been specified in the textbook, in the given order.

   (a) Eliminate the ε-productions.
   (b) Eliminate the unary productions.
   (c) Eliminate the useless symbols.
   (d) Produce a CFG G' in Chomsky normal form such that $L(G') = L(G) \smallsetminus \{\epsilon\}$.

   Discuss each intermediate step, reporting the obtained CFGs.

(please see next page)

4. **[8 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) Let L be a language in CFL and let L' be a finite language. Then the language $L \smallsetminus L'$ is always in CFL.

   (b) Let L be a language in CFL and let L' be an infinite language. Then the language $L \smallsetminus L'$ is always in CFL.

   (c) Let L be a language in REC$\smallsetminus$CFL. For every natural number n, there exists a string $w \in L$ such that $|w| \geq n$.

   (d) If an NP-complete problem is in $\mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$.

5. **[5 points]** Define the language L_{ne} which we have studied in class. Prove that L_{ne} does not belong to the class REC, using the proof presented in the textbook.

6. **[3 points]** Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{CFL}\}$$

   and define $L_{\mathcal{P}} = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\}$. Use Rice's theorem to show that $L_{\mathcal{P}}$ is not in REC.