Master Degree in Computer Engineering

Final Exam for Automata, Languages and Computation

September 3rd, 2021

1. **[4 points]** Consider the regular expression $r = (\mathbf{0}+\mathbf{1})^*\emptyset(\epsilon+\mathbf{01})$. Convert r into an equivalent ϵ-NFA using the construction we have presented in class. **Important**: do not simplify the regular expression before applying the construction, use r as is.

2. **[9 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b, c\}$

$$
\begin{aligned}
L_1 &= \{w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Z,\ |u| = |v|\} \ ; \\
L_2 &= \{w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Y = Z\} \ ; \\
L_3 &= \{w \mid w = XuYvZ,\ X, Y, Z \in \Sigma,\ u, v \in \Sigma^*,\ X = Y = Z,\ |u| = |v|\} \ .
\end{aligned}
$$

   State whether the above are regular languages, and provide a mathematical proof of your answers.

3. **[6 points]** Assess whether the following statements are true or false, providing motivations for all of your answers.

   (a) If $L_1, L_2 \in \text{CFL}$ then $L_1 \cap L_2 \in \text{REC}$;

   (b) If $L_1 \in \text{REG}$ and $L_2 \in \text{CFL} \setminus \text{REG}$ then $L_1 \cdot L_2$ is never in REG;

   (c) If $L_1 \cdot L_2 \in \text{REG}$ then $L_1, L_2 \in \text{REG}$.

(please see next page)

4. **[6 points]** Consider the language $L = \{a^n b^m \mid n, m \geq 0\}$ and the context-free grammar $G = (\{S, A\}, \{a, b\}, P, S)$, where P contains the following rules

$$
\begin{aligned}
S &\rightarrow aS \mid A \\
A &\rightarrow bA \mid \varepsilon
\end{aligned}
$$

Using mutual induction, we want to construct a mathematical proof that $L(G) = L$. In order to do this, for each variable $X \in \{S, A\}$ define a property $\mathcal{P}_X$ over strings $x \in \{a, b\}^*$ such that $\mathcal{P}_X(x)$ holds if and only if there is a derivation in G that starts with X and produces x. **Important**: develop your proof only for property $\mathcal{P}_S$.

5. **[8 points]** Define the notion of property of the languages generated by TMs and state Rice's theorem. Provide the proof of Rice's theorem that we have developed in class.