Master Degree in Computer Engineering

Final Exam for Automata, Languages and Computation

June 28th, 2021

1. **[6 points]** With reference to finite state automata, answer the following two questions.
   (a) Specify the construction developed in class for transforming a NFA N into a DFA D equivalent to N.
   (b) Formally prove the equivalence $L(D) = L(N)$.

2. **[8 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b, c\}$

$$\begin{aligned} L_1 &= \{a^n a^m b^n c^m \mid n, m \geq 1\} , \\ L_2 &= \{a^n a^n b^n c^n \mid n \geq 1\} . \end{aligned}$$

   State whether L_1 and L_2 are context-free languages, and provide a mathematical proof of your answers.

3. **[4 points]** Let L be an arbitrary language.
   (a) Provide the mathematical definition of language L^*.
   (b) Provide a rigorous proof that $(L^*)^* = L^*$.

(please see next page)

4. **[7 points]** Considering the membership problem for context-free languages, answer the following two questions.

   (a) Specify the dynamic programming algorithm developed in class for the solution of this problem.

   (b) Consider the CFG G, in Chomsky normal form, defined by the following productions:

$$\begin{aligned} S &\to AC \mid BC \\ A &\to BA \mid a \\ B &\to BB \mid b \\ C &\to BC \mid c \end{aligned}$$

   Trace the application of the above algorithm for string $w = bbbabbbc$

5. **[8 points]** Let L_R be an arbitrary regular language defined over the alphabet $\Sigma = \{0, 1\}$.

   (a) Consider the following property of the RE languages defined over Σ

$$\mathcal{P} \;=\; \{L \mid L \in \text{RE},\ L \cup L_R = \Sigma^*\}\ .$$

   and let

$$L_1 = \{\mathsf{enc}(M) \mid L(M) \in \mathcal{P}\},$$

   where $\mathsf{enc}(M)$ is a binary string representing a fixed encoding of M. Assess whether L_1 belongs to the class REC, and provide a mathematical proof of your answer.

   (b) Consider also the following language

$$L_2 \;=\; \{\mathsf{enc}(M, M') \mid L(M) \cup L(M') \cup L_R = \Sigma^*\}$$

   where M, M' are generic TMs accepting languages defined over Σ, and $\mathsf{enc}(M, M')$ is a binary string representing a fixed encoding of M, M'. Assess whether L_2 belongs to the class REC, and provide a mathematical proof of your answer.