



# ICT for HEART MONITORING

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# PART 3

# ECG RECONSTRUCTION

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Reconstructing a full 12-lead ECG from a reduced lead set is fundamental for diagnosing CADs without clinical equipment

- Limb leads recording can be achieved with minimal invasive technology
- Precordial leads recording is more difficult, but at least a precordial lead is necessary to obtain high reconstruction performance

# ECG RECONSTRUCTION

Limb leads are linearly dependent!

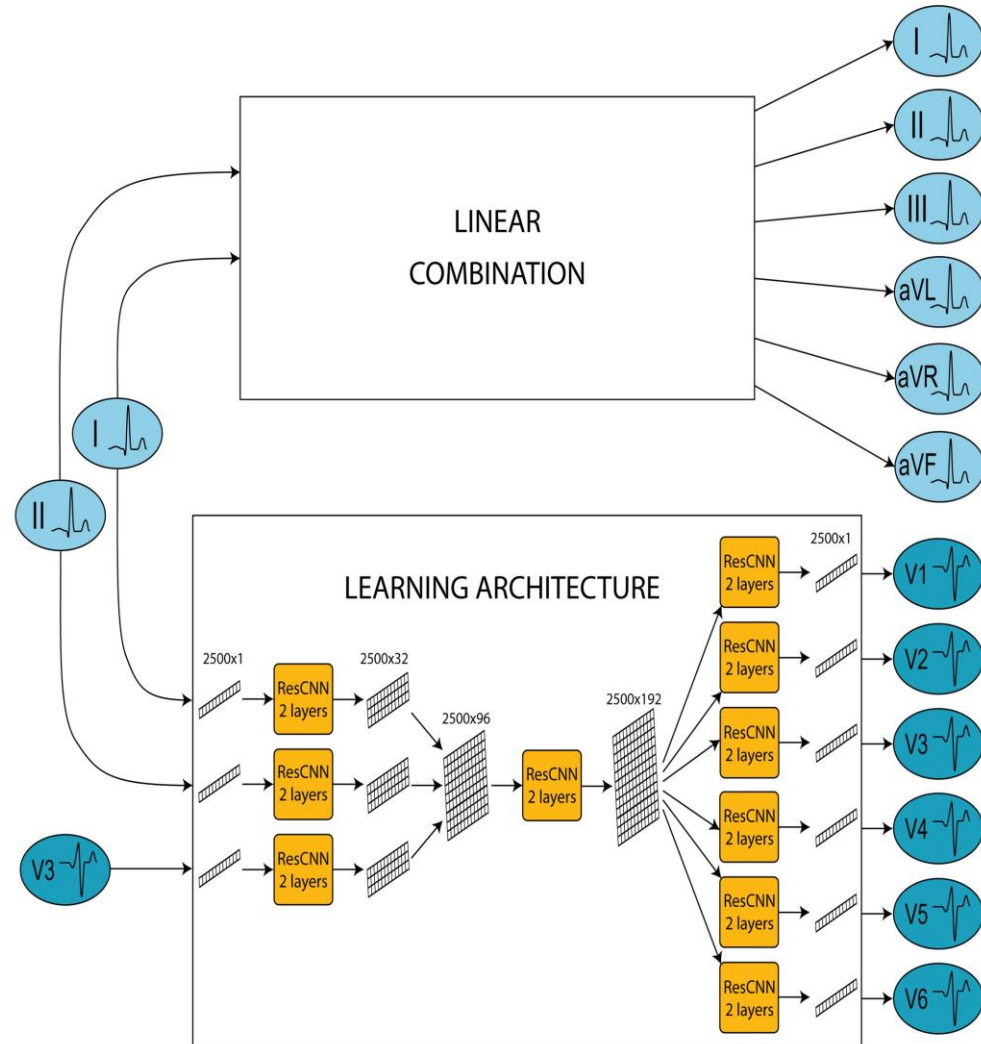
$$III = II - I$$

$$aVR = -\frac{I + II}{2}$$

$$aVL = \frac{I - III}{2}$$

$$aVF = \frac{I + II}{2}$$

$$V1 = \frac{V4 + V5 + V6}{3}$$



# LINEAR REGRESSION (I)

We assume that the missing precordial leads are given by a linear combination of the known leads

$$y(n) = \beta_0 + \beta_1 x_1(n) + \beta_2 x_2(n) + \cdots + \beta_k x_k(n) + \varepsilon(n)$$

where  $y(n)$  is the  $n^{\text{th}}$  sample of the reconstructed lead, while  $x_1(n)$ ,  $x_2(n)$ , ..., and  $x_k(n)$  are the  $n^{\text{th}}$  samples of the input leads

Considering multiple samples and ECG recordings, we obtain a system of  $N$  equations

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

## LINEAR REGRESSION (II)

- $\mathbf{y}$  is the  $N$ -dimensional vector of the reconstructed samples, i.e., the vector of the target variables
- $\mathbf{X}$  is the  $N \times (k + 1)$  matrix having as row the  $(k + 1)$ -dimensional vectors of the input samples, i.e., the vectors of the independent variables
- $\boldsymbol{\beta}$  is the  $(k + 1)$ -dimensional vector of the regression coefficients, where  $\beta_0$  is the intercept
- $\boldsymbol{\varepsilon}$  is the  $N$ -dimensional vector of the error terms

The goal is to minimize the error terms:

$$\min_{\boldsymbol{\beta}} \|\mathbf{X}\boldsymbol{\beta} - \mathbf{y}\|$$

# DEEP LEARNING (I)

To better capture the non-linearity that characterized lead relationship, we can design a Deep Learning (DL) architecture that generate a full 12-lead ECG from a subset of the signal leads

$$y = DL(x_1, x_2, \dots, x_k)$$

where  $y$  is the reconstructed lead, while  $x_1, x_2, \dots$ , and  $x_k$  are the input leads

We exploit also past and future input samples for reconstructing any missing sample  $y(n)$

## DEEP LEARNING (II)

**Feed-Forward Neural Networks (FNNs)** require to define a bias and a weight for each input sample and lead, involving a significant computational cost

- Assuming to implement a single layer, we have a total of  $(2N)^2$  parameters to be trained

**In Convolutional Neural Networks (CNNs)**, the number of training parameters depends on the kernel configuration and not on the input dimensionality

- We must tune the kernel size in order to ensure a sufficient high receptive field



# PERFORMANCE: QUANTITATIVE MEASURES

A naïve loss function for DL model reconstructing the missing leads is the Mean Squared Error (MSE)

$$MSE(y) = \sum_{i \in (V1, V2, V3, V4, V5, V6)} \frac{1}{6} \sum_{n=1}^K \frac{|\hat{y}_i(n) - y_i(n)|^2}{K}$$

The coefficient of determination (R2) is a standardized version of MSE being independent from the scale of the reconstructed signals

$$R2(y) = \sum_{i \in (V1, V2, V3, V4, V5, V6)} \frac{1}{6} \sum_{n=1}^K \left( 1 - \frac{|\hat{y}_i(n) - y_i(n)|^2}{|y_i(n) - \bar{y}_i|^2} \right),$$

# PERFORMANCE: QUALITATIVE MEASURES

Can a cardiologist use a synthesized ECG for diagnose CADs or other heart-related diseases?

- Does a classification algorithm (identifying CADs or other heart-related diseases) obtain the same performance when taking reconstructed and original ECGs as input?
- Can we optimize the reconstruction system towards the maximization of the classification instead of the reconstruction performance?



# Thank you!

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