LCD (23/04/2024)

* PI - CALCULUS

customer $\quad C = \overline{askPizza} . \overline{pay} \quad pizza . 0$

pizza place $\quad P = askPizza . pay . \overline{pizza} . P$

CCS Value passing

$C = \overline{askPizza}(4\ stagioni) . \overline{pay}(7) . change(z) . (pizza . 0 + fail . 0)$

$P = askPizza(x) . pay(y) .$ if $(price(x) \leq y)$

then $\overline{change}(y - price(x)) . \overline{pizza} . P$

else $\overline{change}(y) . \overline{fail} . P$

system

$(C \mid P) \setminus \{askPizza, pay, change, \ldots\} \quad \mid C$

PROBLEM: only interactions on known channels, statically determined and shared

New Example: pizza place with home delivery

channel

$C = \overline{askPizza}(home) . \overline{pay} . \quad home(x) . \overline{eat}(x) . 0$

$P = askPizza(y) . pay . (\nu\ pizza) \underbrace{(\overline{y}(pizza) . P)}_{(*)}$

two interpretation

→ pizza is restricted to be used only in (*)

$(\quad) \setminus pizza$

→ pizza is a newly created channel

$$C \mid P \xrightarrow{\tau} .\ \overline{pay}\ .\ home(x).\ \overline{eat}(x).\ 0 \quad \mid$$

$$pay\ .\ (\nu\, pizza)(\overline{home}(pizza).\ P)$$

$$\xrightarrow{\tau} home(x).\ \overline{eat}(x).\ 0 \quad \mid$$

$$(\nu\, pizza)(\overline{home}(pizza).\ P)$$

$$\xrightarrow{\tau} (\nu\, pizza)\ (\overline{eat}(pizza).0 \mid P)$$

$$\xrightarrow{\overline{eat}(..)} (\nu\, pizza)\ (\cancel{0 \mid} P)$$

$$\equiv (\nu\, pizza)\ (ask\ Pizza(y).\ pay.\ (\nu\, pizza)\ (\bar{y}(pizza).\ P))$$

$$\equiv ask\ Pizza(y)\ .\ pay\ .\ (\nu\, pizza)\ (\bar{y}(pizza).\ P)$$

$\boxed{\text{PI - CALCULUS}}$ $\equiv$ CCS +

- channel creation
- channels passed over channels

* <u>Syntax</u>

set channels $\mathcal{N}$ $\quad x, y, z \qquad a, b, c$

- create new x in P
- x is local to P

process

$$P ::= S \mid P_1 \mid P_2 \mid (\nu x)\ P$$

$$S ::= 0 \mid \pi.\ P \mid S_1 + S_2$$

$$\pi ::= x(z) \mid \bar{x}(z) \mid \tau \mid [x = y]\ \pi$$

binders

complication

$$x(z).\ P = x(w).\ P\{^{w}/_{z}\}$$

$$(\nu x)\ P = (\nu w)\ P\{^{w}/_{x}\}$$

α - equivalence

careful in the choice of w

$$y(z).\,\bar{z}(x).\,0 \quad \cancel{\equiv_\alpha} \quad y(x).\,\bar{x}(x)$$

$$\equiv_\alpha$$

$$y(w).\,\bar{w}(x).0$$

replace only the free occurences....

$$y(z).\,\bar{z}(x).\,0\ \{z/x\} \;=\; y(z).\,\bar{z}(z).0$$

$$y(w).\,\bar{w}(x).0\ \{z/x\} \;=\; y(w).\,\bar{w}(z).0 \quad \not\equiv_\alpha$$

need of capture free substitution

Operational Rules

$$P \xrightarrow{\quad\quad} P'$$

$$\frac{}{a(x).P \xrightarrow{ab} P\{b/x\}} \qquad \frac{}{\bar{a}(b).P \xrightarrow{\bar{a}b} P}$$

capture free substitution

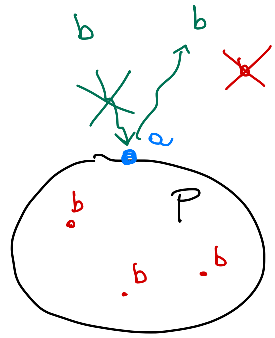
$$\frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q}$$

$$\frac{P \xrightarrow{\bar{a}b} P' \qquad Q \xrightarrow{ab} Q'}{P|Q \xrightarrow{\tau} P'|Q'}$$

interaction between ν & communication

$$\frac{P \xrightarrow{\alpha} P'}{(\nu z)P \xrightarrow{?} ?}$$

INPUT		OUTPUT
$\dfrac{P \xrightarrow{ab} P'}{(\nu z)P \xrightarrow{ab} (\nu z)P'}$		$\dfrac{P \xrightarrow{\bar{a}b} P'}{(\nu z)P \xrightarrow{\bar{a}b} (\nu z)P'}$
YES	$z \neq a, b$	YES
NO	$z = a$	NO
NO	$z = b$ $z \neq a$	



$$\frac{P \xrightarrow{\bar{a}b} P'}{(\nu b)P \xrightarrow{\bar{a}(\nu b)} P'} \quad \text{(OPEN)}$$

$$\frac{P \xrightarrow{\bar{a}(\nu b)} P' \qquad Q \xrightarrow{ab} Q'}{P \mid Q \xrightarrow{\tau} (\nu b)(P' \mid Q')} \quad \text{(close)}$$

→ behavioural equivalence (weak , late/early , -----)

→ logic (nominal logics)

⋮