LCD (08/04/2024)

Weak Bisimilarity

* Weak Bisimulation : $\mathcal{R} \subseteq Proc \times Proc$ s.t. whenever $P \mathcal{R} Q$

  (i) if $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Longrightarrow} Q'$ and $P' \mathcal{R} Q'$

  (ii) if $Q \xrightarrow{\alpha} Q'$ then $P \overset{\alpha}{\Longrightarrow} P'$ and $P' \mathcal{R} Q'$

* Weak Bisimilarity : $P \approx Q$ if there exists $\mathcal{R}$ weak bisimulation such that $P \mathcal{R} Q$

EXAMPLE (FIFO Buffer)

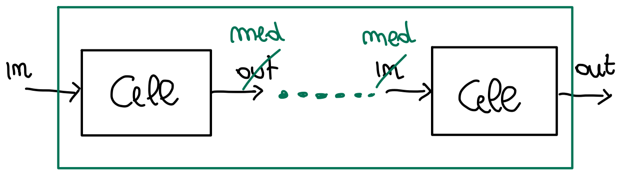
$$\begin{cases} Cell = in(x).\ C(x) \\ C(x) = \overline{out}(x).\ Cell \end{cases}$$

realise a specification

$$F_2 = in(x).\ F_1(x)$$

$$F_1(x) = \overline{out}(x).\ F_2 + in(y).\ F_0(x,y)$$

$$F_0(x,y) = \overline{out}(x).\ F_1(y)$$



$$(Cell[{}^{med}/_{out}] \mid Cell[{}^{med}/_{in}]) \setminus med$$

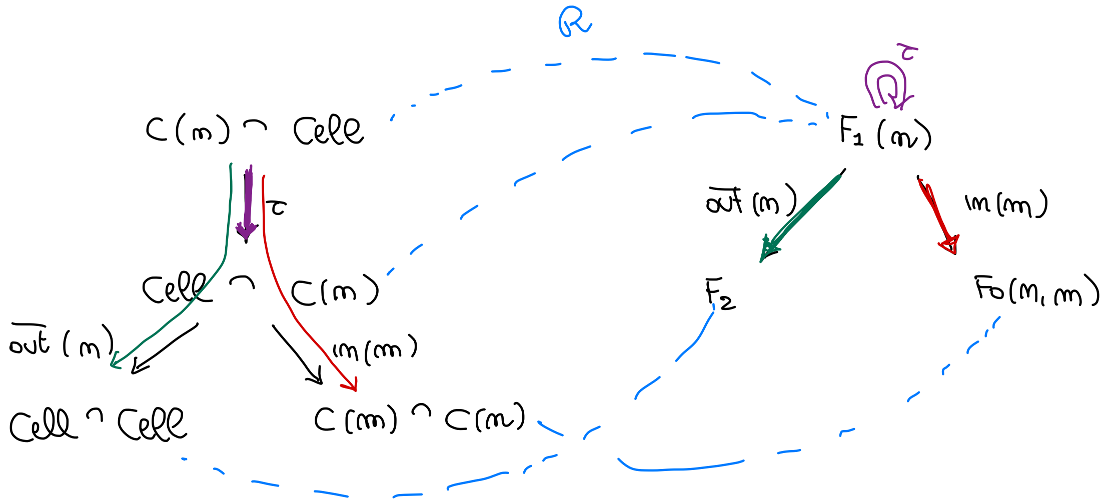
$$\approx$$

$$F_2$$

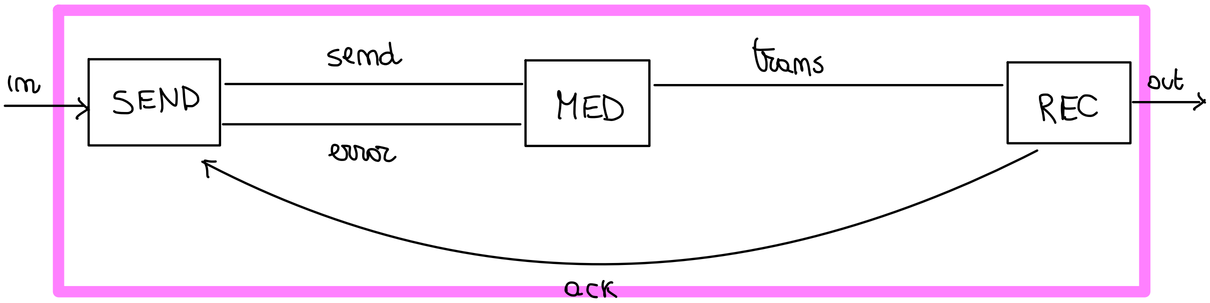
notation: $A \frown B = (A[{}^{med}/_{out}] \mid B[{}^{med}/_{in}]) \setminus med$

weak bisimulation $\mathcal{R}$ s.t. $Cell \frown Cell \ \mathcal{R}\ F_2$

$$\mathcal{R} = \{ (Cell \frown Cell,\ F_2) \} \cup \{ (C(m) \frown Cell,\ F_1(m)),\ (Cell \frown C(m),\ F_1(m)),\ (C(m) \frown C(n),\ F_0(n,m)) \mid m,n \in \mathbb{N} \}$$



EXAMPLE: Non Lossy-Channel over a Lossy one



$SEND = in(x).\ SENDING(x)$

$SENDING(x) = send(x).\ WAIT(x)$

$WAIT(x) = error.\ SENDING(x) + ack.\ SEND$

$REC = trans(x).\ DEL(x)$

$DEL(x) = \overline{out}(x).\ ACK$

$ACK = \overline{ack}.\ REC$

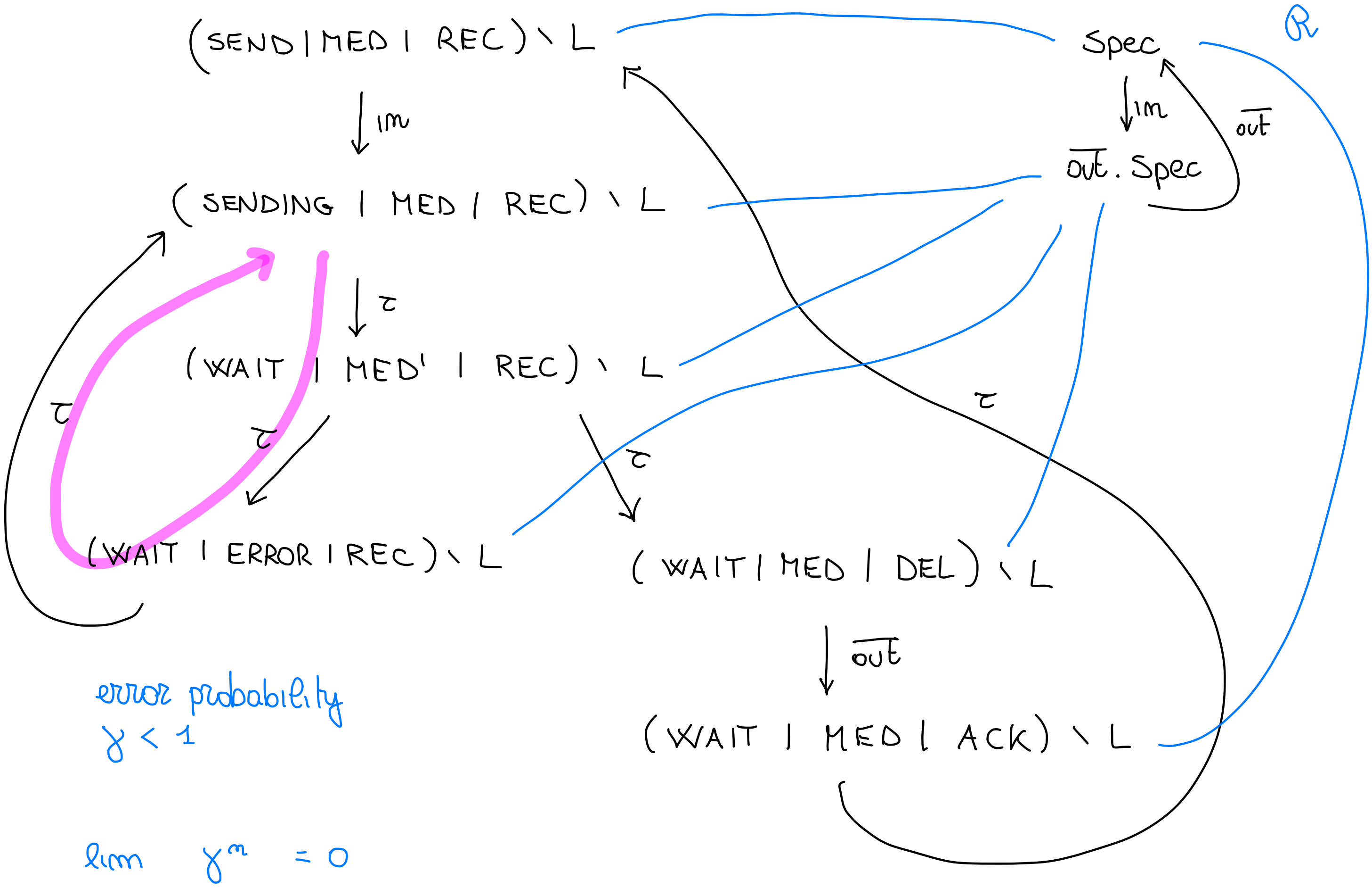
$MED = send(x).\ MED'(x)$

$MED'(x) = \tau.\ ERROR + \overline{trans}(x).\ MED$

$ERROR = \overline{error}.\ MED$

$(SEND \mid MED \mid REC) \setminus L \quad \approx \quad Spec = in(x).\ \overline{out}(x).\ Spec$

$L = \{ send, trans, error, ack \}$



error probability
$\gamma < 1$

$\lim_{n \to \infty} \gamma^n = 0$

* Weak Bisimilarity
  * equivalence
  * (largest) weak bisimulation
  * $\approx$ satisfies

$P \approx Q$ iff for all $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \approx Q'$
& dual

proof

weak bisimilarity is bisimilarity over $\Rightarrow$ (weak transitions)

* Congruence ?

if $P \approx Q$ then $C[P] \approx C[Q]$ for all $C[\,]$ context ?

NO

$$\underset{P}{\overset{\shortparallel}{0}} \approx \underset{Q}{\overset{\shortparallel}{\tau.0}}$$

$$C[\;] = a.0 + \_$$

then $C[P] = a.0 + 0 \not\approx C[Q] = a.0 + \tau.0$

$$a.0 + 0 \overset{\tau}{\Downarrow} a.0 + 0 \xrightarrow{a} 0$$

$$a.0 + \tau.0 \xrightarrow{\tau} 0 \not\xrightarrow{a}$$

<u>OBSERVATION</u> : If $P \approx Q$, R process then

(i) $\alpha.P \approx \alpha.Q$

(ii) $P \mid R \approx Q \mid R$

(iii) $P \setminus L \approx Q \setminus L$

(iv) $P[f] \approx Q[f]$

EXERCISE

<u>proof</u>

(i) $R = \{(\alpha.P, \alpha.Q)\} \cup \approx$ weak bisimulation

(ii) need to show

$$\frac{P \overset{\alpha}{\Rightarrow} P'}{P \mid Q \overset{\alpha}{\Rightarrow} P' \mid Q}$$

after this, show

$R = \{(P' \mid R', Q' \mid R') \mid P' \approx Q', R' \text{ proc}\}$ weak bisimulation

⋮

Non Deterministic Choice

We would like to show that $P \approx Q$ then $P + R \approx Q + R$

try $\mathcal{R} = \{ (P+R, Q+R) \} \cup \approx$

weak bisimulation

i.e.

if $P + R \xrightarrow{\alpha} \sqcup$ then $Q + R \xRightarrow{\alpha} \sqcup$

$\mathcal{R}$

Assume $P + R \xrightarrow{\alpha} \sqcup$, 2 possibilities

(1) $\dfrac{R \xrightarrow{\alpha} R'}{P + R \xrightarrow{\alpha} R'}$ then $\dfrac{R \xrightarrow{\alpha} R'}{Q + R \xrightarrow{\alpha} R'}$ $R' \approx R'$

$R' \approx R' \Rightarrow R' \; \mathcal{R} \; R'$

(2) $\dfrac{\overbrace{P \xrightarrow{\alpha} P'}^{(*)}}{P + R \xrightarrow{\alpha} P'}$

we know $P \approx Q$, hence $(*)$ implies $Q \xRightarrow{\alpha} Q'$

& $\boxed{P' \approx Q'}$

$\downarrow$

$P' \; \mathcal{R} \; Q'$

then $\boxed{\dfrac{Q \xRightarrow{\alpha} Q'}{Q + R \xRightarrow{\alpha} Q'}}$ No! if $Q \xRightarrow{\tau} Q'$

Q could be idle

in counterexample

$P = \tau.0 \approx Q = 0$

$R = a.0$

$$\frac{P \xrightarrow{\tau} 0}{P+R \xrightarrow{\tau} 0}$$

since $P \approx Q$ $\quad Q \stackrel{\tau}{\Rightarrow} Q \quad \not\Rightarrow \quad \dfrac{Q \stackrel{\tau}{\Rightarrow} Q}{Q+R \stackrel{\tau}{\Rightarrow} Q}$ NO

Solutions for getting back to a congruence

1 Guarded Sum

$$P, Q ::= k \mid \sum_{i \in I} \alpha_i . P_i \mid P|Q \mid P \setminus L \mid P[f]$$

Exercise: in CCS with guarded sum, $\approx$ is a congruence

2 Observational Congruence

$\approx$ not a congruence $\quad 0 \not\approx \tau.0$

we define the contextual closure of $\approx$

$P \hat{\approx} Q$ if for all $C[\,]$ $\quad C[P] \approx C[Q]$

one can prove

1. $\hat{\approx}$ equivalence
2. $\hat{\approx}$ congruence
3. $\hat{\approx} \subseteq \approx$ (if $P \hat{\approx} Q$ then $P \approx Q$)
4. it is the coarsest equivalence which refines $\approx$ and is a congruence

(if $\equiv$ is a congruence and $\equiv \subseteq \approx$ then $\equiv \subseteq \hat{\approx}$)

Alternative characterisation

Define $\overset{}{\Rightarrow}_c$

$P \overset{\alpha}{\Rightarrow}_c P'$ if $P \xrightarrow{\tau}{}^{*} \xrightarrow{\alpha} \xrightarrow{\tau}{}^{*} P'$

at least one action (also when $\alpha = \tau$)

$0 \overset{\tau}{\Rightarrow} 0 \qquad 0 \overset{\tau}{\not\Rightarrow}_c 0$

Observational congruence

$P \approx_c Q$ if

- if $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Rightarrow}_c Q'$ and $P' \approx Q'$
- dual

you can verify:

(a) $\approx_c$ equivalence

(b) $\approx_c$ congruence

(c) $\approx_c$ refines $\approx$ (if $P \approx_c Q$ then $P \approx Q$)

(d) $\approx_c = \hat{\approx}$ $\qquad \approx_c \subseteq \hat{\approx}$

exercise for exam

$\approx_c \supseteq \hat{\approx}$

DIFFICULT

EXERCISE 3.27

$P, Q \qquad P \xrightarrow{\tau}{}^{*} Q \qquad Q \xrightarrow{\tau}{}^{*} P$ then $P \approx Q$

EXERCISE 3.29

$P \setminus A \approx 0$