LCD (26/03/2024)

* **Bisimulation up to Bisimilarity**

$\mathcal{R} \subseteq Proc \times Proc$ such that $P \,\mathcal{R}\, Q$

(i) if $P \xrightarrow{\alpha} P'$ then $Q \xrightarrow{\alpha} Q'$ and $P' \sim P'' \,\mathcal{R}\, Q'' \sim Q'$

(ii) dual

it holds: if $\mathcal{R}$ is a bisimulation up to $\sim$ and $P \mathcal{R} Q$ then $P \sim Q$

EXERCISE: Buffer with capacity 2

$Cell = in(x).\ C(x)$

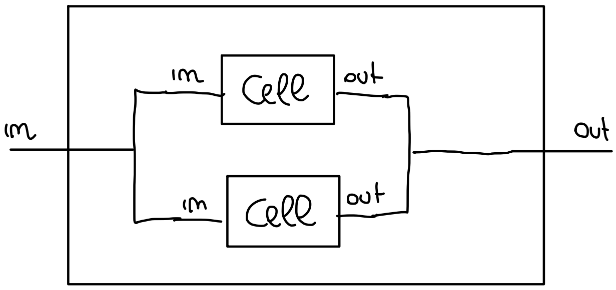
$C(x) = \overline{out}(x).\ Cell$

use it to build buffer with capacity 2

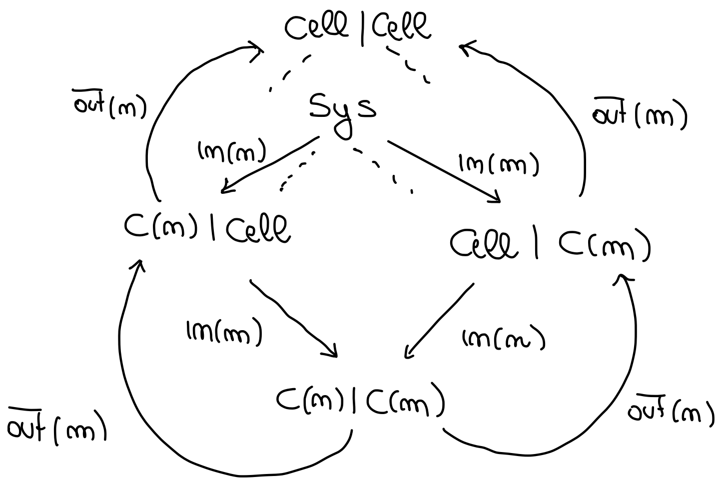
$$\left.\begin{aligned} B_2 &= in(x).\ B_1(x) \\ B_1(x) &= \overline{out}(x).\ B_2 + in(y).\ B_0(x,y) \\ B_0(x,y) &= \overline{out}(x).\ B_1(y) + \overline{out}(y).\ B_1(y) \end{aligned}\right\} \text{spec}$$

solution:

$Sys \stackrel{def}{=} Cell \mid Cell$



$Sys \overset{?}{\sim} B_2$



$$\mathcal{R} = \{(Sys, B_2)\} \cup \{(C(m) \mid Cell, B_1(m)) \mid m \in \mathbb{N}\}$$

$$\{(\cancel{Cell \mid Cell, B_2})\} \cup \{(Cell \mid C(m), \cancel{B_1(m)}) \mid m \in \mathbb{N}\}$$

$$\cup \; \{(C(n) \mid C(m), B_0(n,m)) \mid m,n \in \mathbb{N}\}$$

$$\cup \; \{(C(m) \mid C(n), \cancel{B_0(n,m)}) \mid m,n \in \mathbb{N}\}$$

$$(C(1) \mid C(2), B_0(1,2)) \qquad m = 1 \qquad n = 2$$

$$(C(2) \mid C(1), B_0(1,2)) \qquad m = 2 \qquad n = 2$$

<u>EXAMPLE</u> :

$$\begin{cases} C_0 = inc.\, C_1 \\ C_{n+1} = inc.\, C_{n+2} + dec.\, C_n \end{cases} \quad \overset{?}{\sim} \quad C = inc.\,(C \mid \underbrace{dec.0}_{d})$$

$$\begin{tikzcd}
C_0 \arrow[d, "inc"] \\
C_1 \arrow[u, bend left, "dec"] \arrow[d, "inc"] \\
C_2 \arrow[u, bend left, "dec"] \arrow[d, "inc"] \\
\vdots \arrow[u, bend left]
\end{tikzcd}
\qquad
\begin{tikzcd}
(c|0)|0 \arrow[r, phantom, "\sim"] & c \arrow[d, "inc"] \arrow[r, phantom, "\sim"] & c|0 \\
(c|0)|d \arrow[u, "dec"] \arrow[r, phantom, "\sim"] & c|d \arrow[d, "inc"] \arrow[ur, "dec"] \arrow[r, phantom, "\sim"] & (c|d)|0 \\
& (c|\underline{d})|\underline{d} \arrow[ul] \arrow[ur] \arrow[d, phantom, "\vdots"] & \\
& {} &
\end{tikzcd}$$

$$C_0 \overset{?}{\sim} C$$

intuition $\qquad C_n \sim C \mid d^n \qquad$ where $\quad d^n = \underbrace{d(d \mid d(\ldots}_{n \text{ times}})$

$$\left(\begin{array}{l} d^0 = 0 \\ d^{n+1} = d(d^n \end{array} \right)$$

consider $\qquad \mathcal{R} = \{(C_n, C \mid d^n) \mid n \in \mathbb{N}\}$

bisimulation ?

take $\quad C_{m+1} \;\mathcal{R}\; c \mid d^{m+1}$

note $\quad C_{m+1} \xrightarrow{dec} C_m \quad \rightsquigarrow \quad c \mid d^{m+1} \xrightarrow{dec} c \mid \underbrace{d(d|..\ (0|d..}_{}\ )$

n components d
one 0

$\wr$

$\mathcal{R} \quad c \mid d^n$

$\mathcal{R}$ is a bisimulation up to bisimilarity

WEAK BISIMULATION

- equivalence
- compositional
- (largest) bisimulation
- expected equivalence $(P)Q \sim Q|P$, $\quad P|(S|T) \sim (P|S)|(T)$

special action τ (internal synchronisation)

$$b.0 \quad \approx \quad b.\tau.0$$

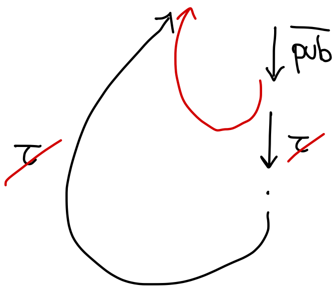
Example

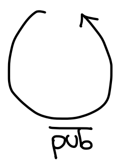
$CS = \overline{pub}.\ \overline{coin}.\ coffee.\ CS$

$CM = coin \cdot \overline{coffee}.\ CM$

$Spec = \overline{pub}.\ Spec$

$(CS \mid CM) \setminus \{coin, coffee\} \qquad \approx \qquad Spec$





$CMB = coin.\overline{coffee}.CMB + coin.CMB$

$$\begin{tikzcd}
(CS \mid CMB) \setminus \{coin, coffee\} \arrow[d, "\overline{pub}"] & \\
\cdot \arrow[d, "\tau"] \arrow[dr, "\tau"] & \\
\cdot \arrow[uu, bend left, "\tau"] & (\overline{coffee}.CS \mid CMB) \setminus \{coin, coffee\}
\end{tikzcd}$$

traces

$$\xrightarrow{\tau} \xrightarrow{\tau} \cdots \overset{a_1}{\Longrightarrow} \xrightarrow{\tau} \cdots \xrightarrow{\tau} \overset{a_2}{\Longrightarrow} \cdots \xrightarrow{\tau} \xrightarrow{\tau} \overset{a_n}{\Longrightarrow} \cdots \xrightarrow{\tau}$$

|| same as

$$\xrightarrow{a_1} \xrightarrow{a_2} \cdots \xrightarrow{a_n}$$

Weak transition

P, Q processes $\qquad P \overset{\alpha}{\Longrightarrow} Q$ when

(i) $\alpha \neq \tau \qquad P \xrightarrow{\tau}{}^{*} P' \xrightarrow{\alpha} Q' \xrightarrow{\tau}{}^{*} Q$

(ii) $\alpha = \tau \qquad P \xrightarrow{\tau}{}^{*} Q$

possibly empty sequence ($P = Q$)

Example:

$P = b.\tau.0$

$$\begin{tikzcd}
b.\tau.0 \arrow[r, Rightarrow, "b"] \arrow[rr, Rightarrow, bend left, "b"] \arrow[loop below, Rightarrow, "\tau"] & \tau.0 \arrow[r, Rightarrow, "\tau"] \arrow[loop below, Rightarrow, "\tau"] & 0 \arrow[loop below, Rightarrow, "\tau"]
\end{tikzcd}$$

Weak Bisimilarity

a relation $R \subseteq Proc \times Proc$ is a <u>weak bisimulation</u> if when $P \; R \; Q$

(i) if $P \xrightarrow{\alpha} P'$ then $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \; R \; Q'$

(ii) if $Q \xrightarrow{\alpha} Q'$ then $P \overset{\alpha}{\Rightarrow} P'$ and $P' \; R \; Q'$

We say that P, Q are weakly bisimilar $P \approx Q$ if there is R weak bisimulation st. $P \; R \; Q$

$$\approx \; = \; \bigcup \{ R \mid R \text{ weak bisimulation} \}$$

EXERCISE (exam) :

a relation $R \subseteq Proc \times Proc$ is a <u>weak strong bisimulation</u> if when $P \, R \, Q$

(i) if $P \overset{\alpha}{\Rightarrow} P'$ then $Q \overset{\alpha}{\Rightarrow} Q'$ and $P' \; R \; Q'$

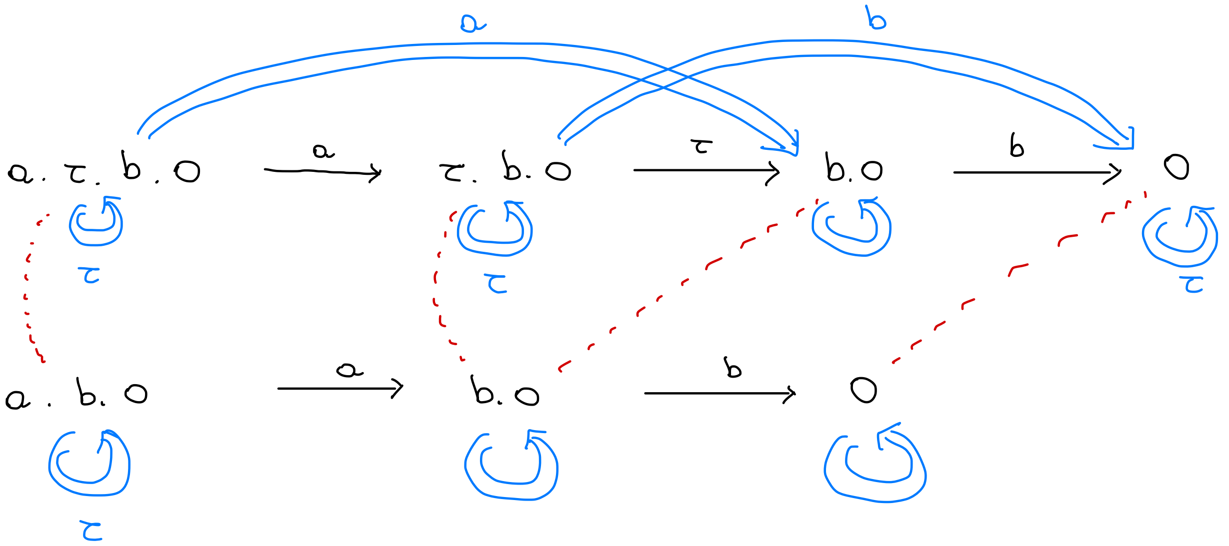
(ii) if $Q \overset{\alpha}{\Rightarrow} Q'$ then $P \overset{\alpha}{\Rightarrow} P'$ and $P' \; R \; Q'$

$\leadsto$ relation $\approx_{strong}$ weak strong bisimilarity

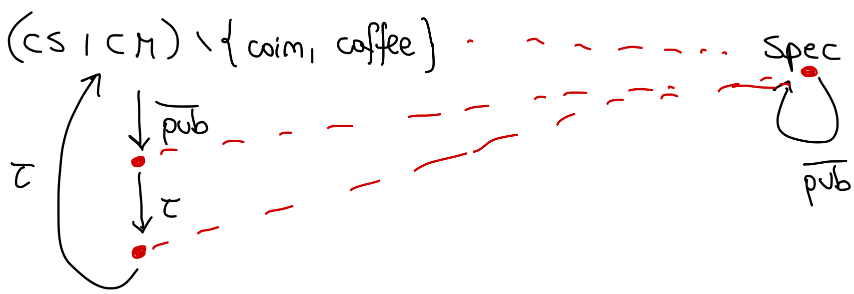
Prove $\approx \; = \; \approx_{strong}$

Example

$P = a.\tau.b.0 \qquad \approx \qquad Q = a.b.0$



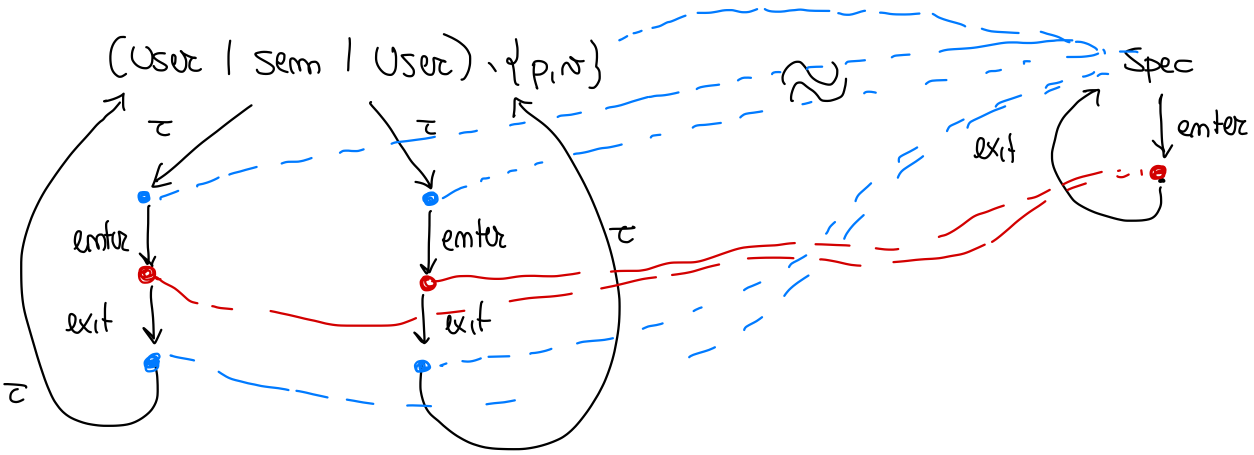
Example



Example :

$\mathrm{Sem} = p.v.\,\mathrm{Sem}$

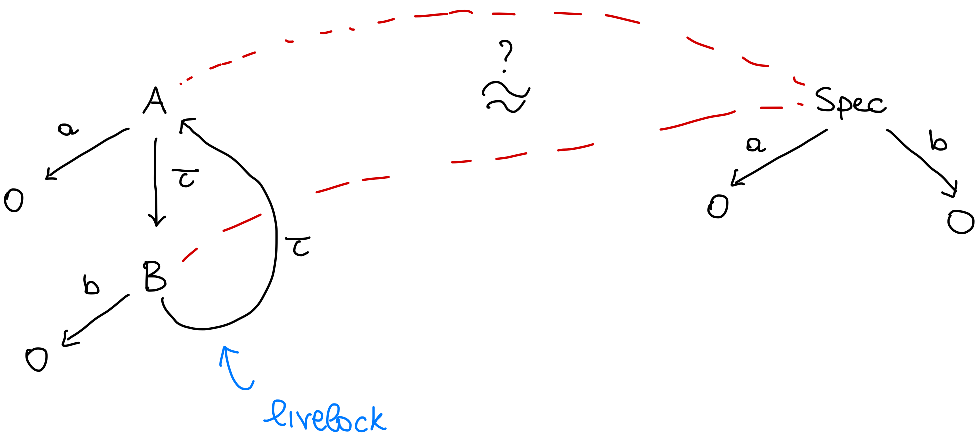
$\mathrm{User} = \bar{p}.\,\mathrm{enter}.\,\mathrm{exit}.\,\bar{v}.\,\mathrm{User}$ $\qquad$ $\mathrm{Spec} = \mathrm{enter}.\mathrm{exit}.\,\mathrm{Spec}$



* Fair abstraction from divergence

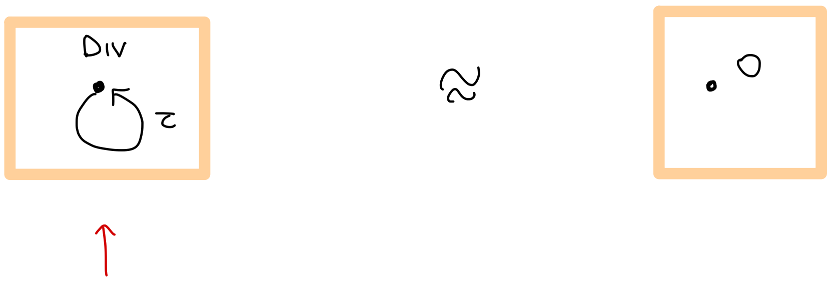
$A = a.0 + \tau.B$

$B = b.0 + \tau.A$ $\qquad \stackrel{?}{\approx} \qquad$ $\mathrm{Spec} = a.0 + b.0$



* Divergence is not observable

$$\mathrm{Div} = \tau . \mathrm{Div}$$



it could be :

$$(P_1 \mid P_2 \mid \ldots \mid P_n) \setminus L$$

e.g.

$$(A_1 \mid A_2) \setminus a$$

with

$$A_1 = a . A_1$$

$$A_2 = \bar{a} . A_2$$