Superconductive Materials

Part 7

Pinning

Critical current in a SC

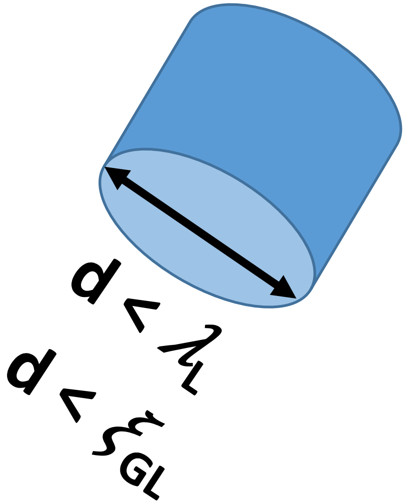
Superconductor can carry only a **limited electric current**

The strong correlation of the Cooper pairs leads to the existence of a **critical velocity** and, hence, of a **critical current density j_c**

At $j > j_c$ Cooper pairs will be broken up

At $j > j_c$ voltage appears and the material reverts back to normal state

Critical current in a SC



Calculation of the **maximum supercurrent density**
under the most favorable conditions

1. **homogeneous** SC wire extending along the *x*-direction
2. **diameter** of the wire is assumed to be **smaller than λ_L and ξ_{GL}**

Cooper pair density can be taken as **constant** over the cross-section of the wire

Ginzburg–Landau equations can be solved for this problem

Limit of the Supercurrent Due to Pair Breaking (1)

The wave function of the SC to simplify is:

$\Psi = \Psi_0 e^{ikx}$

Cooper pair density is spatially homogeneous, so:

$n_s = |\Psi|^2$ → ψ_0 *indepedent of location*

We use the 2nd GL equation to connect Ψ with the supercurrent density j

$$j_s = \frac{q\hbar}{2mi}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) - \frac{q^2}{m}|\Psi|^2 \mathrm{A}$$

in x, $\Psi = \Psi_0 e^{ikx}$ →

$$j_{s,x} = \frac{q\hbar k}{m}\Psi_0^2 - \frac{q^2}{m}\Psi_0^2 A_x$$

$$j_{s,x} = q\Psi_0^2 \frac{\hbar k - qA_x}{m}$$

Cooper pair velocity

$p_{can} = \hbar k = mv + qA$ ←

$$\boldsymbol{j_{s,x} = q\Psi_0^2 v_x}$$

Correspond exactly to current density in NC: $j = -qnv$

Limit of the Supercurrent Due to Pair Breaking (2)

From the 1st GL equation we can derive the dependence of n_s from v_x:

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathrm{A}\right)^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0 \xrightarrow[\Psi = \Psi_0 e^{ikx}]{\text{in } x} \frac{1}{2m}(\hbar k - qA_x)^2\Psi + \alpha\Psi + \beta\Psi_0^2\Psi = 0$$

$\cdot \frac{1}{\Psi}$, $v_x = \frac{\hbar k - qA}{m}$

$$\frac{1}{2}mv_x^2 + \alpha + \beta\Psi_0^2 = 0 \xrightarrow{\alpha < 0} \Psi_0^2 = -\frac{\alpha}{\beta}\left(1 - \frac{mv_x^2}{2|\alpha|}\right) \xrightarrow{\Psi_\infty^2 = \frac{\alpha}{\beta}} \boldsymbol{\Psi_0^2 = \Psi_\infty^2\left(1 - \frac{mv_x^2}{2|\alpha|}\right)}$$

Cooper pair density decreases with increasing kinetic energy
It vanishes if this energy is equal to $|\boldsymbol{\alpha}|$

$$\boldsymbol{j_{s,x} = q\Psi_0^2 v_x}$$

$j_{s,x}$ increase linearly with the velocity of the charge carriers
However, this happens only at small velocities
The larger v_x, the more pronounced is the decrease of the Cooper pair density
At $\Psi_0^2 = 0 \rightarrow \boldsymbol{j_{s,x}} = 0$

Limit of the Supercurrent Due to Pair Breaking (3)

$$\Psi_0^2 = \Psi_\infty^2\left(1 - \frac{m v_x^2}{2|\alpha|}\right)$$

$$j_{s,x} = q\Psi_0^2 v_x \xrightarrow{\Psi_0^2} j_{s,x} = -q\Psi_\infty^2 v_x + \frac{mq\Psi_\infty^2}{2|\alpha|}v_x^3$$

Between $v_x = 0$ and this limiting value, j_x has a maximum as a function of v_x

$$j_{c,x} = \frac{\partial j_{s,x}}{\partial v_x} = 0 \;\Rightarrow\; \frac{\partial j_{s,x}}{\partial v_x} = -q\Psi_\infty^2 + \frac{3}{2}\frac{mq\Psi_\infty^2}{|\alpha|}v_x^2 \;\Rightarrow\; v_{c,x} = \sqrt{\frac{2|\alpha|}{3m}}$$

Ψ_0^2

$$\Psi_0^2 = \Psi_\infty^2\left(1 - \frac{m}{2|\alpha|}\frac{2|\alpha|}{3m}\right)$$

$j_{s,x} = q\Psi_0^2 v_x$

$$\boldsymbol{j_{c,x} = q\frac{2}{3}\sqrt{\frac{2|\alpha|}{3m}}\Psi_\infty^2}$$

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s q^2}}$$

$$\alpha = -\frac{1}{\mu_0}\frac{B_{cth}^2}{n_s}$$

$$\boldsymbol{j_{c,x} = \frac{2}{3}\sqrt{\frac{2}{3}}B_{cth}\frac{1}{\mu_0\lambda_L}}$$

Critical current in SC

30 000 A !!!

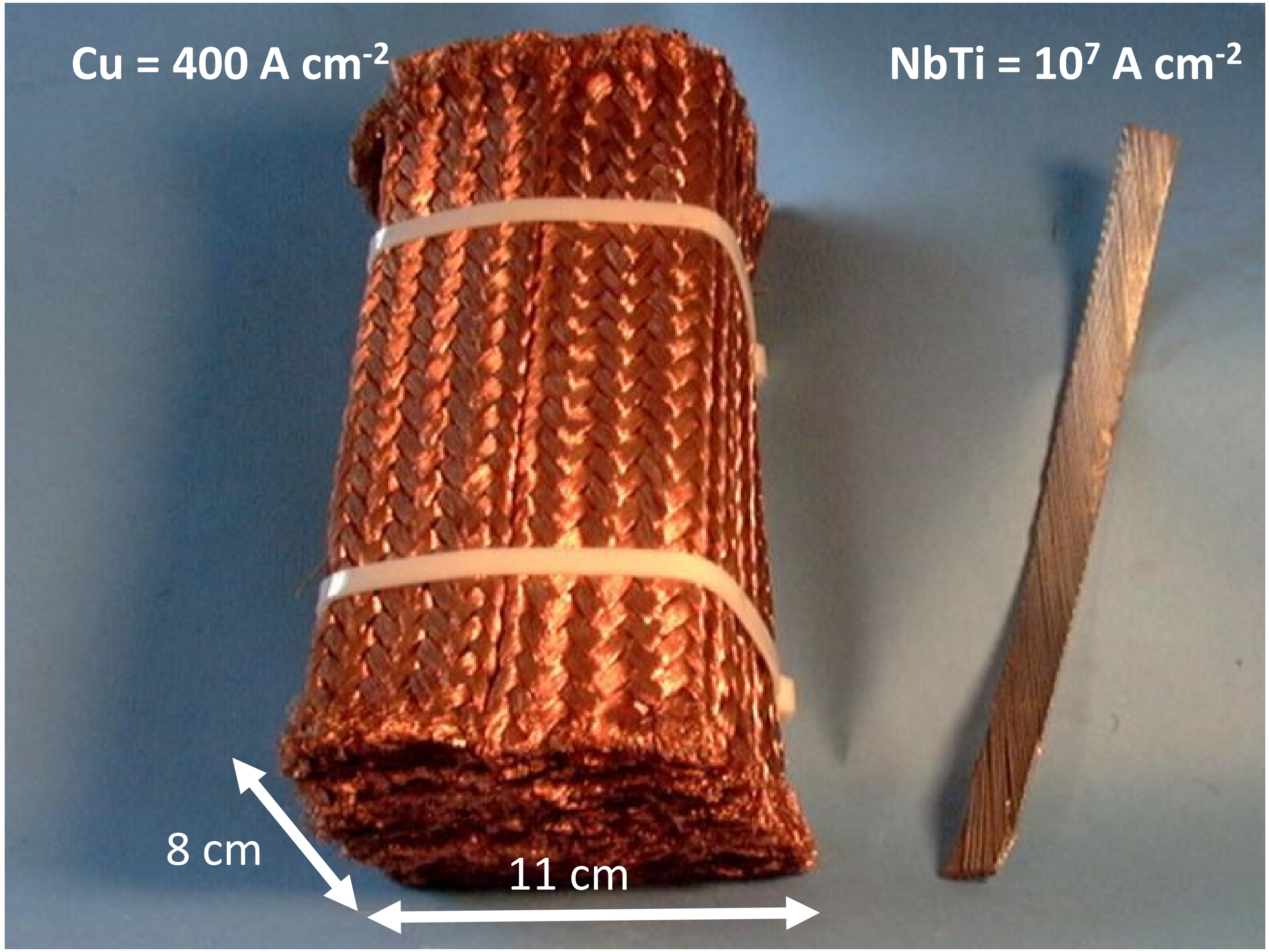
$$j_{c,x} = \frac{2}{3}\sqrt{\frac{2}{3}B_{cth}}\frac{1}{\mu_0 \lambda_L}$$

$B_{cth} = 1\text{T}$

$\lambda_L = 100\ \text{nm}$

$$j_c \sim 4.3 \times 10^8\ \text{A/cm}^2$$

$$j_{Cu} < 6 \times 10^2\ \text{A/cm}^2$$



Measure of the Critical Current

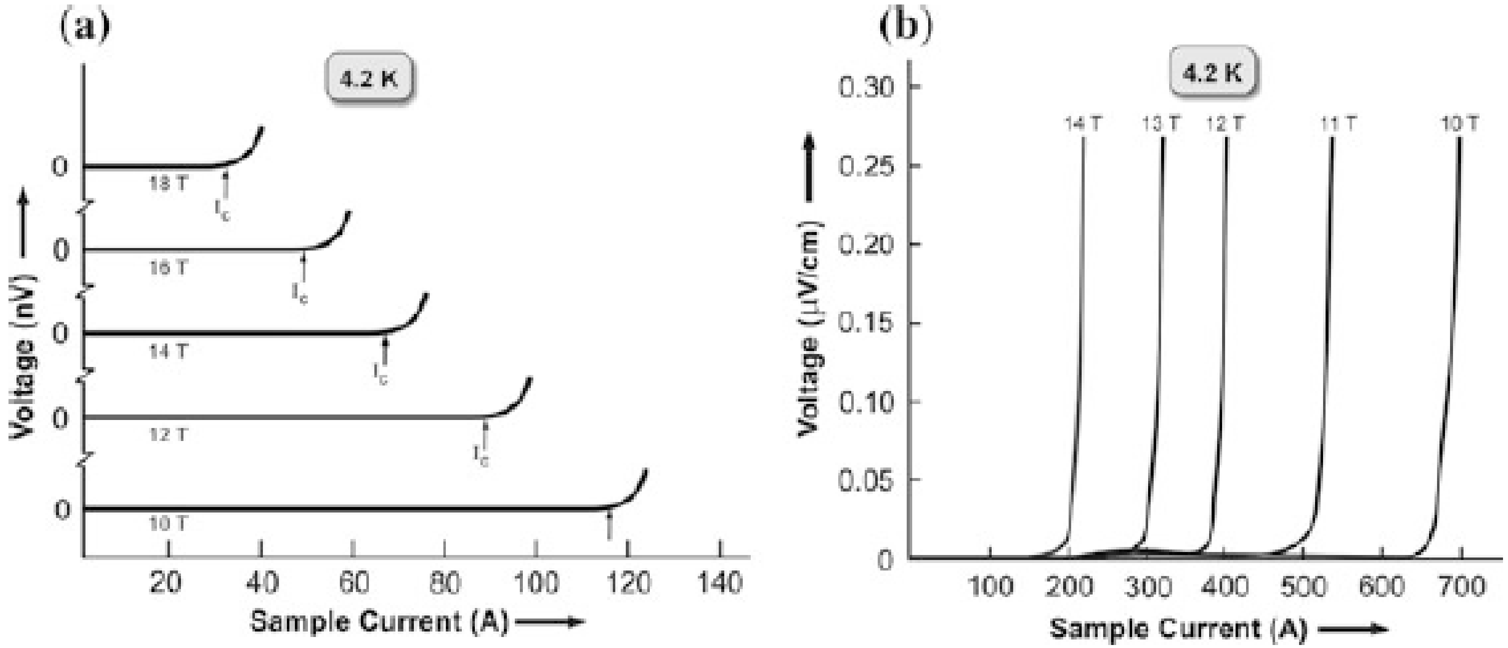
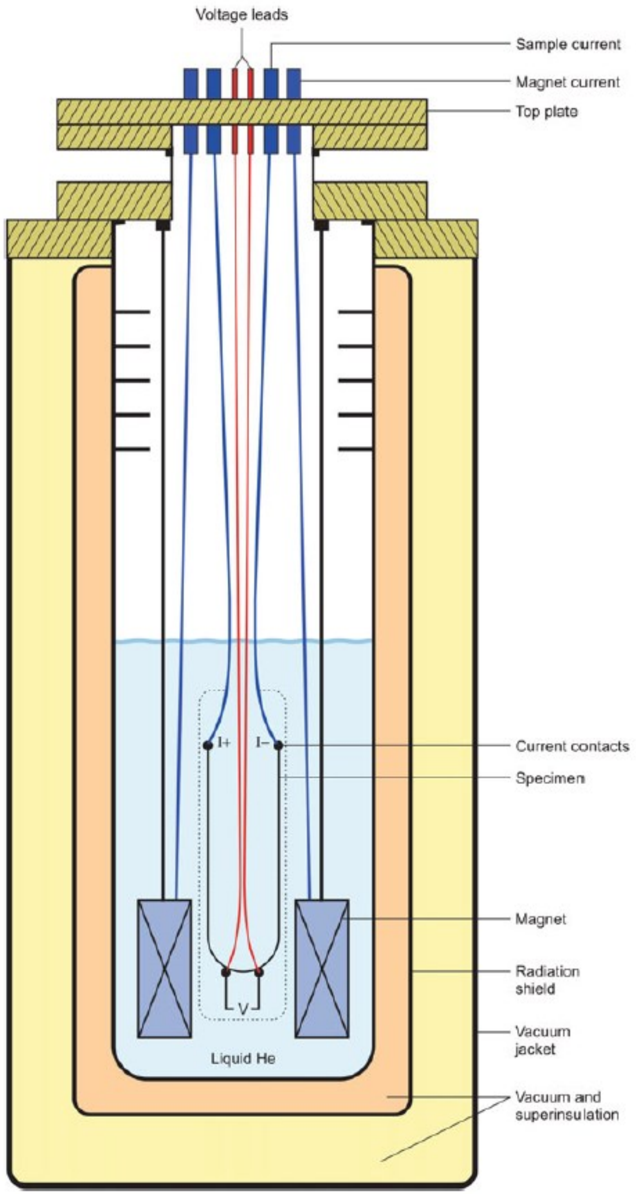


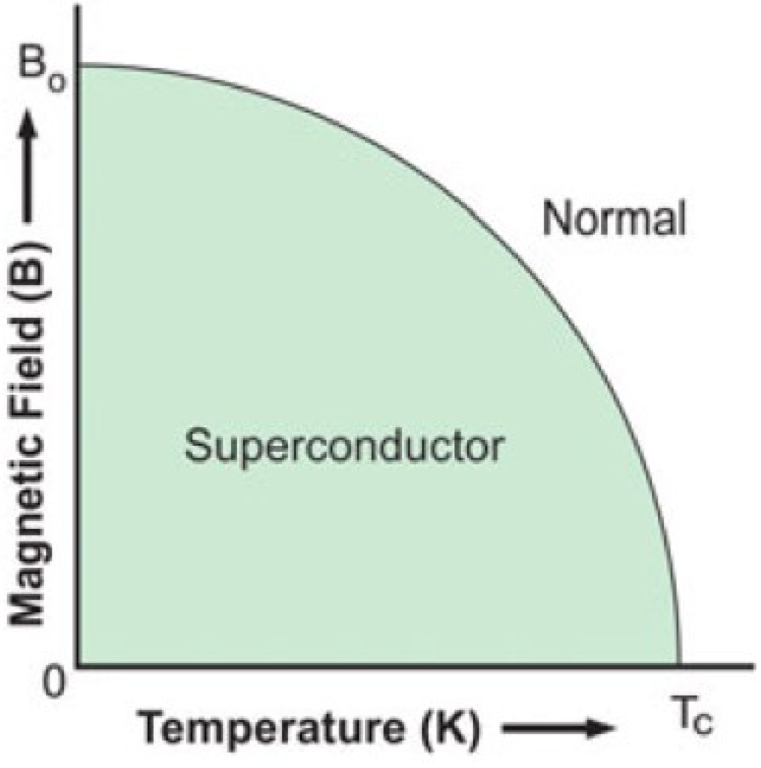
Fig. 3.7 Typical critical current, (I_c) plots at different fields. I_c decreases as the magnetic field increases



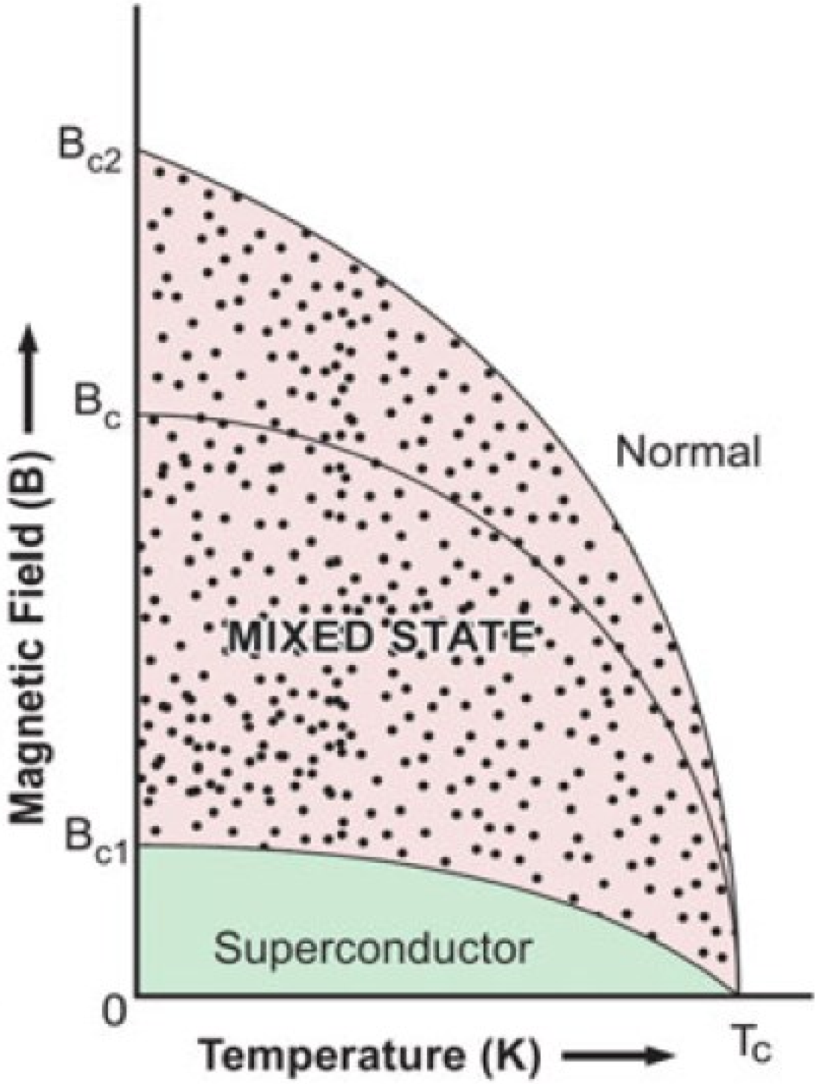
The criterion to determine I_c is usually 0.1–0.01 μV/cm voltage drop across the sample

R.G. Sharma, Superconductivity Basics and Applications to Magnets

Type I and Type II SC



TYPE I $\kappa < \frac{1}{\sqrt{2}}$

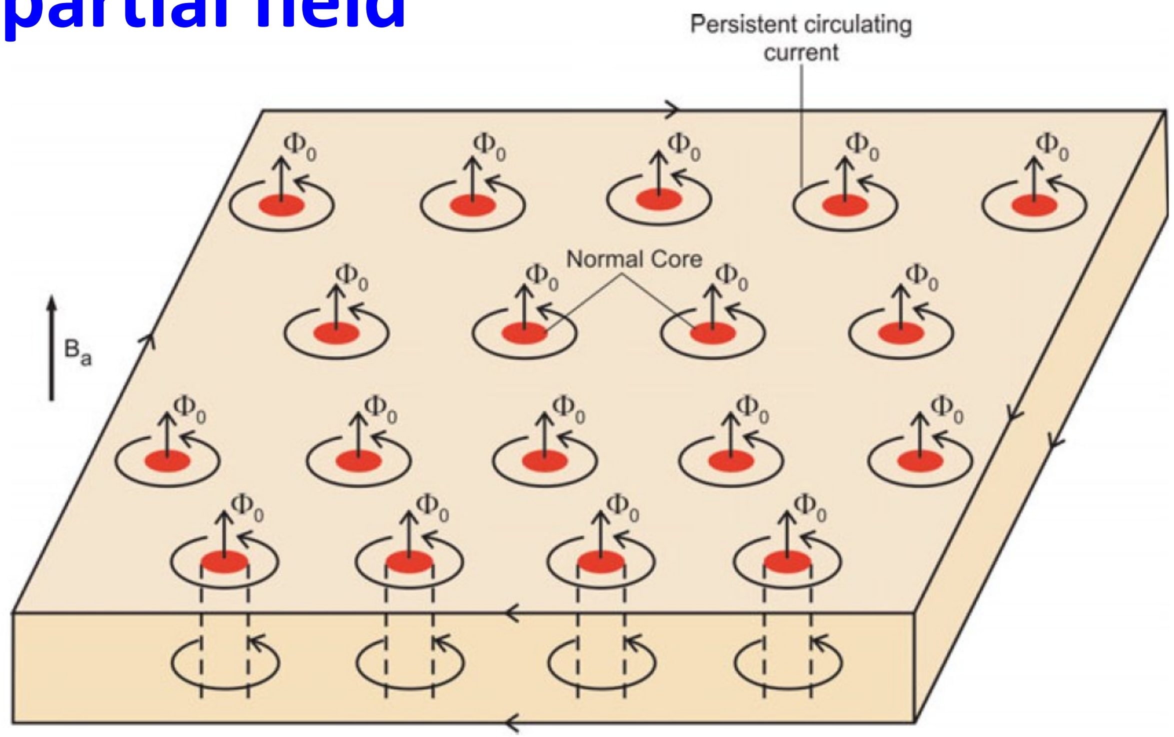


TYPE II $\kappa \geq \frac{1}{\sqrt{2}}$

Mixed state in type II SC

In type II SC the existence of a **negative surface energy** at the N–S boundary favors **partial field penetration of the material**

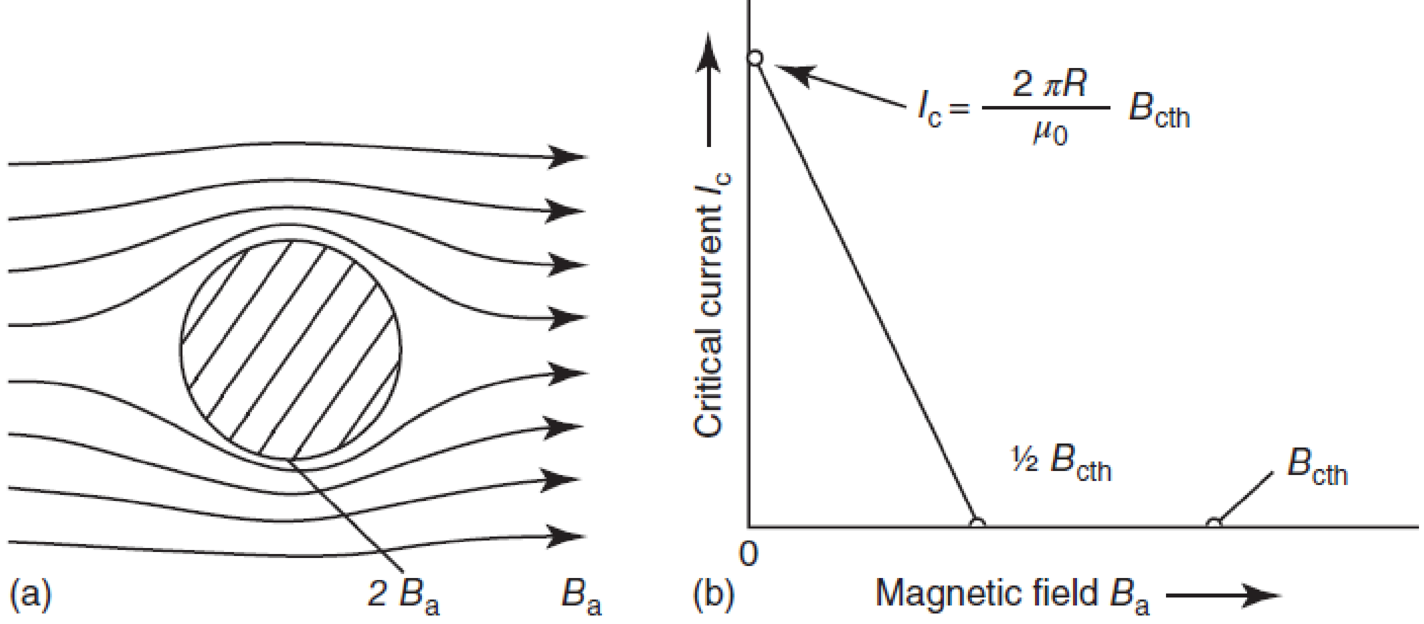
It is **energetically favorable** that the **flux lines**, each carrying a unit quantum of flux **Φ_0(=h/2e)** parallel to the applied field **penetrate the material**



The material is threaded by flux lines (Φ_0) produced by vortices of persistent current with a sense of rotation opposite to surface screening

Critical current in Type I SC

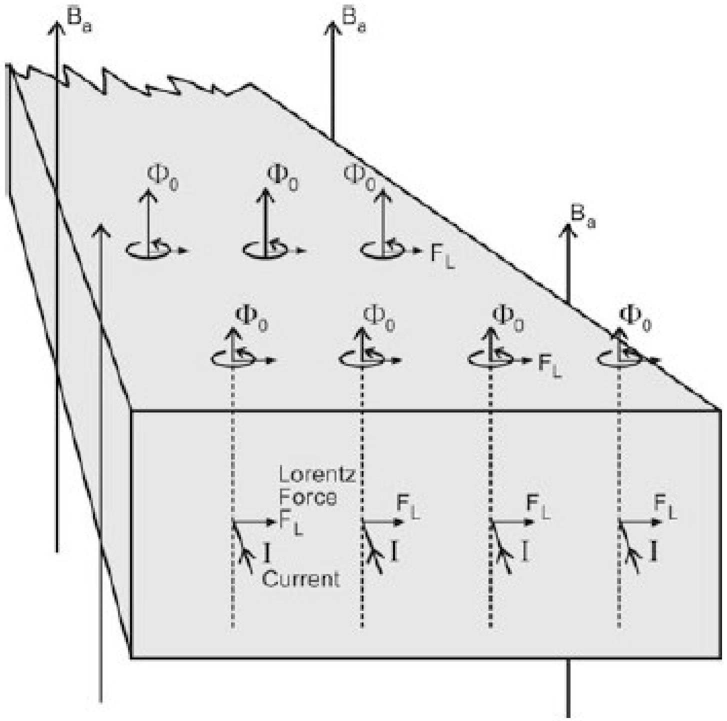
In 1916, **Silsbee** had proposed the hypothesis that in the case of “thick” SC the **critical current is reached exactly when the magnetic field of the current at the surface attains the value B_{cth}**



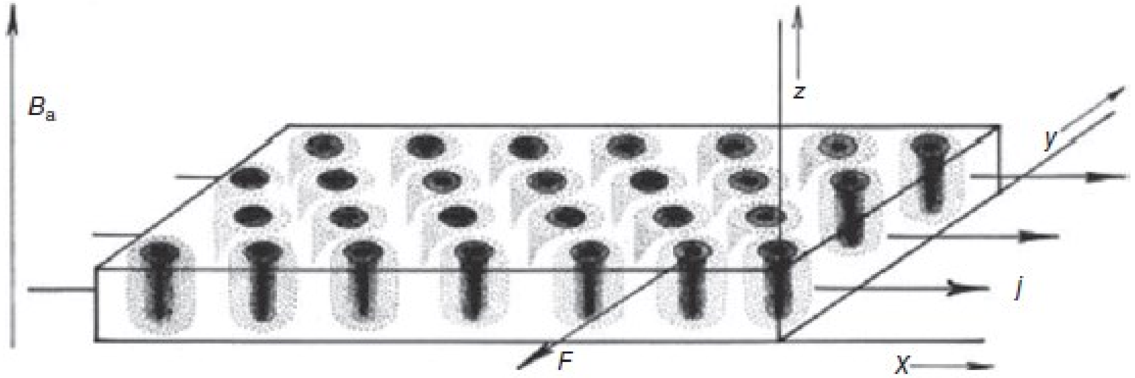
(a) Field distribution around a superconducting wire in the Meissner phase without a transport current. (b) Critical current of a wire with circular cross-section in external field B_a oriented perpendicular to the wire axis.

Current flow in a mixed state

Under this conditions the flux lines experience a **Lorentz force** under the influence of the transport current and the perpendicular transverse field which tries to move it in the third perpendicular direction

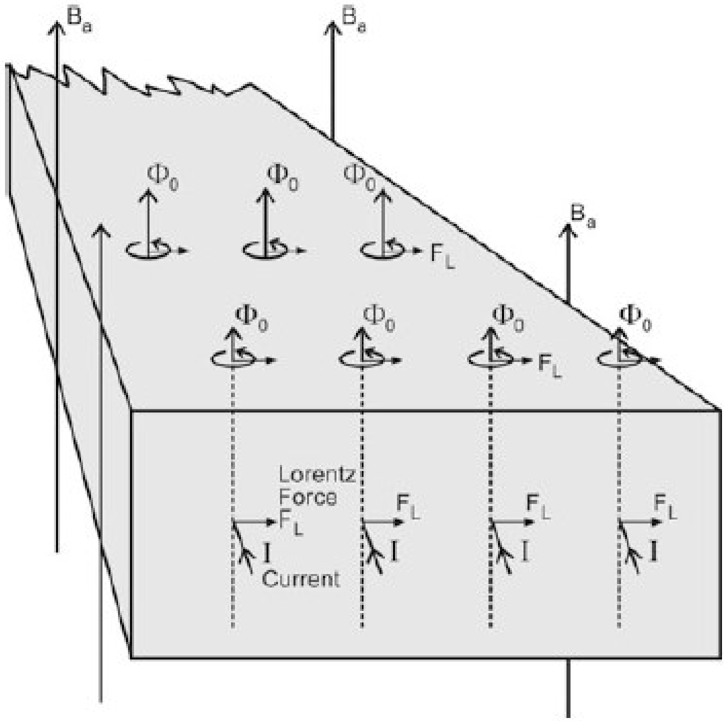


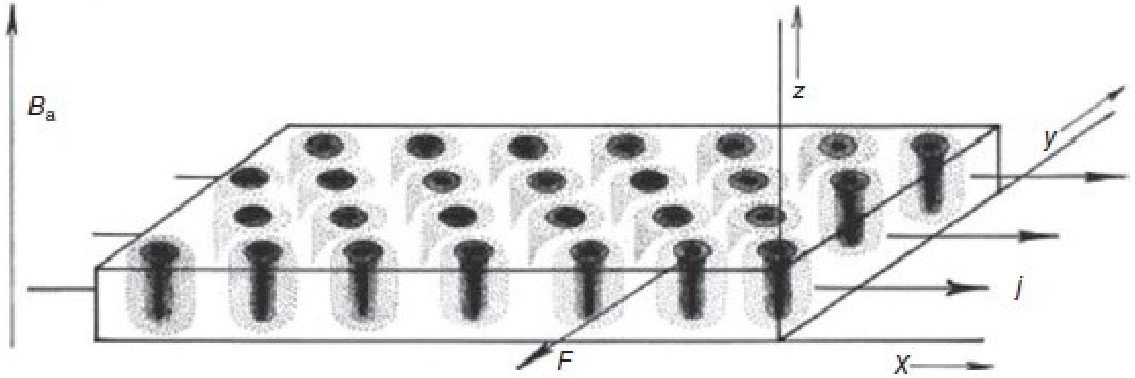
The movement of the flux lines generates **a finite voltage and a resistance appears**



Current flow in a mixed state (ideal type II SC)

In an **ideal type-II superconductor** arbitrarily small transport currents already lead to vortex motion, the **critical current** of such a superconductor in the Shubnikov phase **is zero**

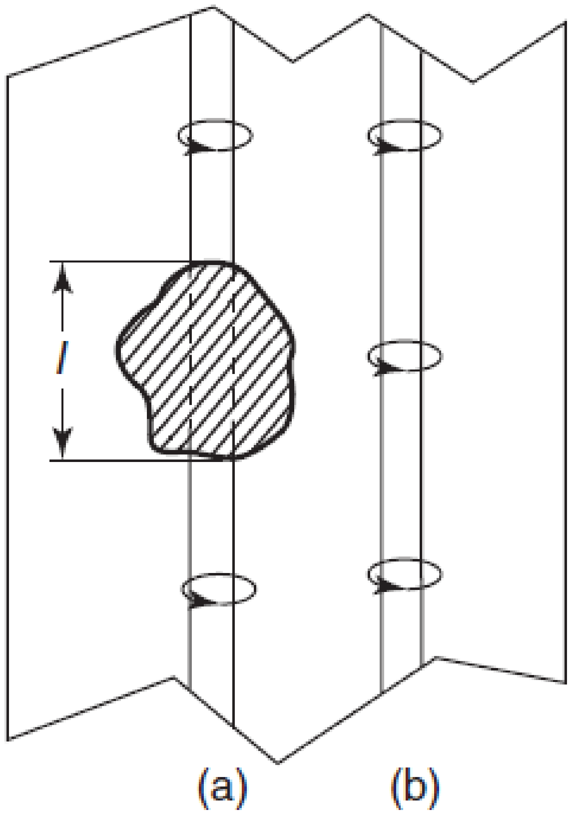




Current flow in a mixed state (real type II SC)

In an **real type-II superconductor** the presents of **defects bound “pin” the vortices in their locations** avoiding motion and dissipation

The flux lines do not move until the time **Lorentz force becomes equal to the pinning force**



Giaever's experiment (1966)

1. A thin film of a SC (A) is driven into the Shubnikov phase (B_{ext} applied)
2. A transport current is applied in A
3. Motion of the flux lines occur in A
4. Magnetic flux lines of film A are coupled to flux lines of film B (electrically insulated from film A)
5. Vortices in film B are dragged along because of the coupling
6. An electrical voltage appears in film B

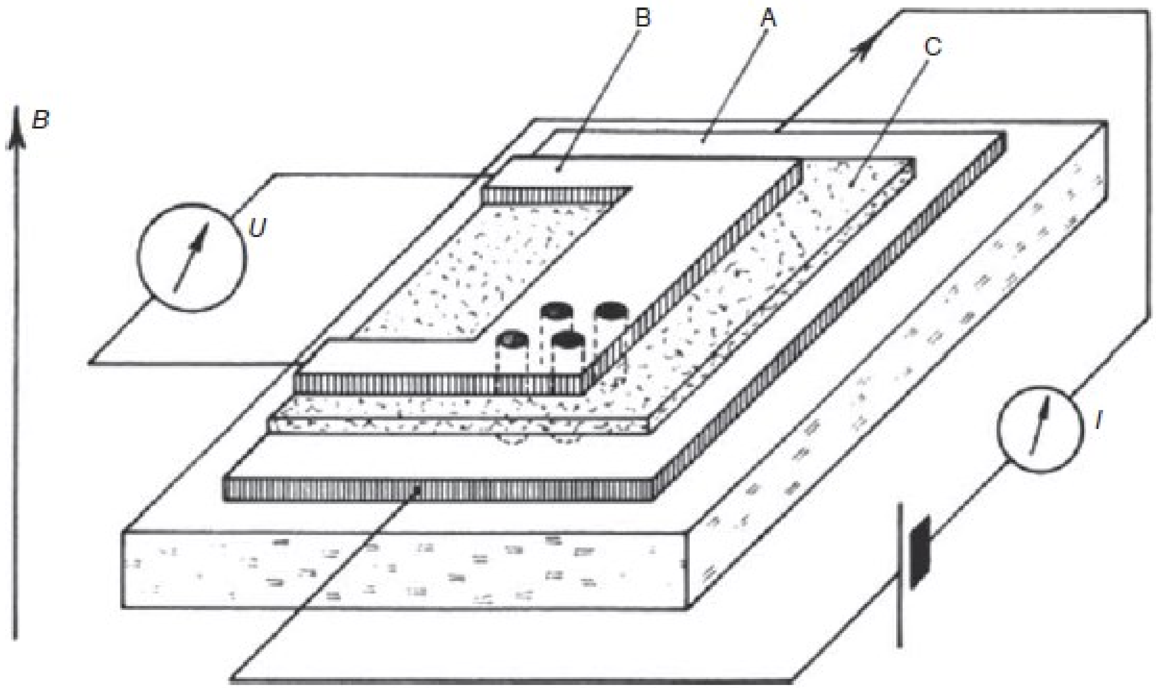
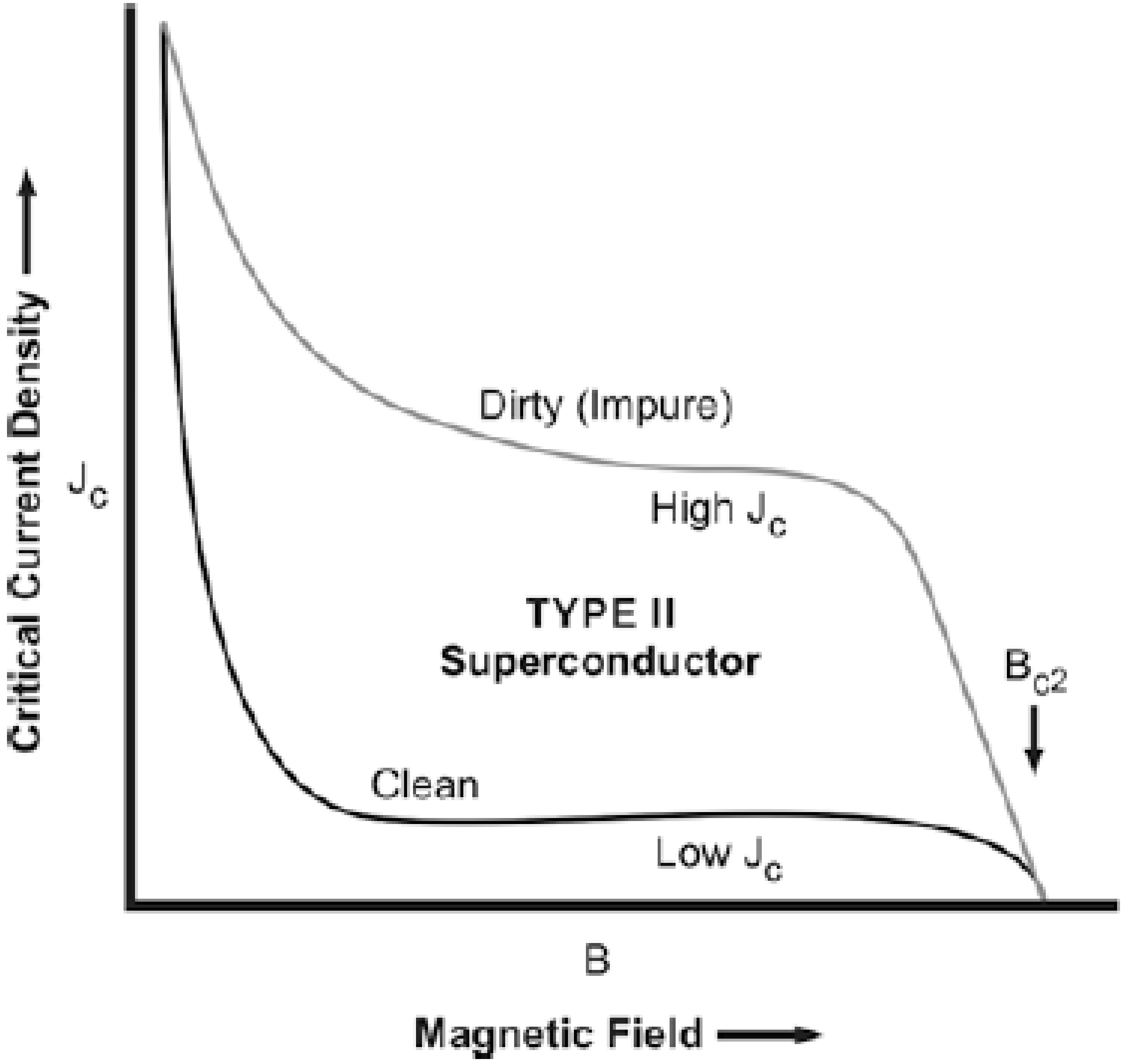


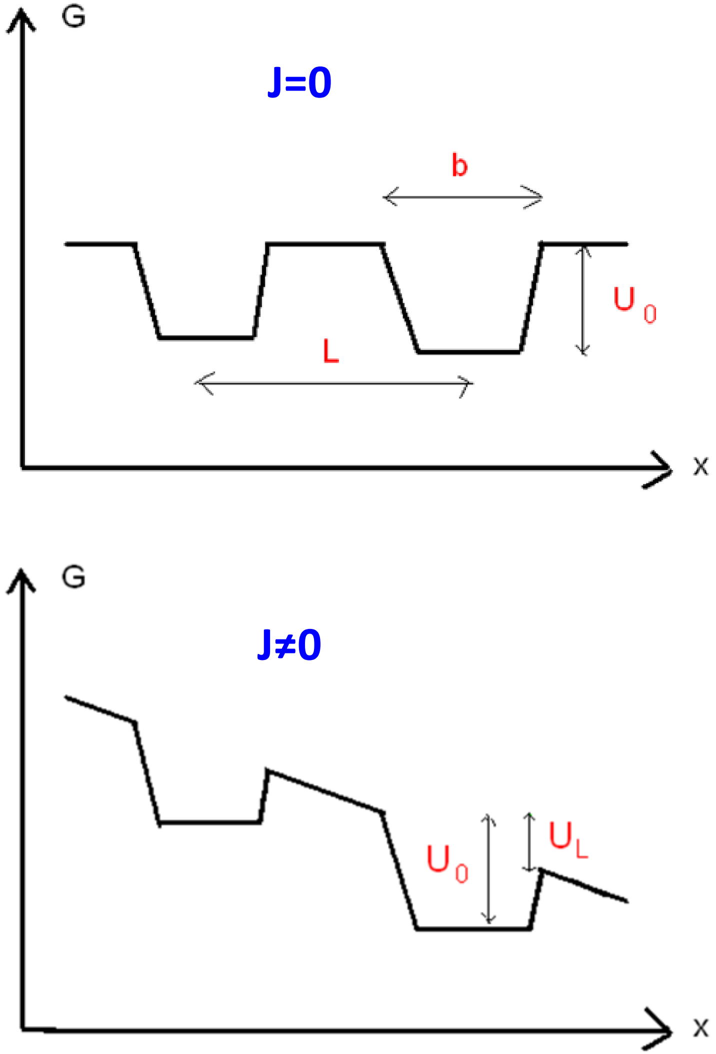
Figure 5.8 Generation of an electrical voltage *U* during vortex motion. A and B are superconductors, C is an insulating layer. All layer thicknesses are strongly exaggerated. (From [12].)

Hard superconductors

The dirty superconductors, normally referred to as “hard superconductors” with pinning sites (defects) on the other hand carries large useful current at high magnetic field. **Defects are substituted intelligently in a superconductor to create pinning sites** which increase the pinning force many fold



Interpretation of Pinning force



- Pinning center are **depression of Gibbs Energy**
- **Equilibrium position for the vortex**
- At T≠0 (J=0→ F_L=0) exist a probability of climbing out of the hole equal to:

$$\mathbf{P} \propto e^{-\frac{U_0}{K_B T}}$$

- At J≠0 Gibbs energy change and Lorenz force appears

$$\mathbf{P} \propto e^{-\frac{(U_0 - U_L)}{K_B T}}$$

Vortex configuration in a hard SC

Vortex energy is reduced passing through a pinning center

Vortex can also bend in order to reach the minimum value of the total energy

Every **inhomogeneity** of the material that is **less favorable for superconductivity acts as a pinning center** (e.g. inclusions, lattice defects, etc.)

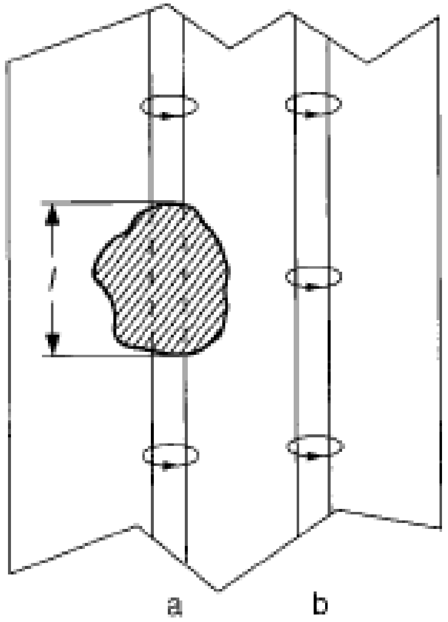


Fig. 5.9 Pinning effect of normal conducting precipitates. In location "a" the effective length of the vortex is shorter compared to that in location "b", since there are no circulating currents in the normal conducting region.

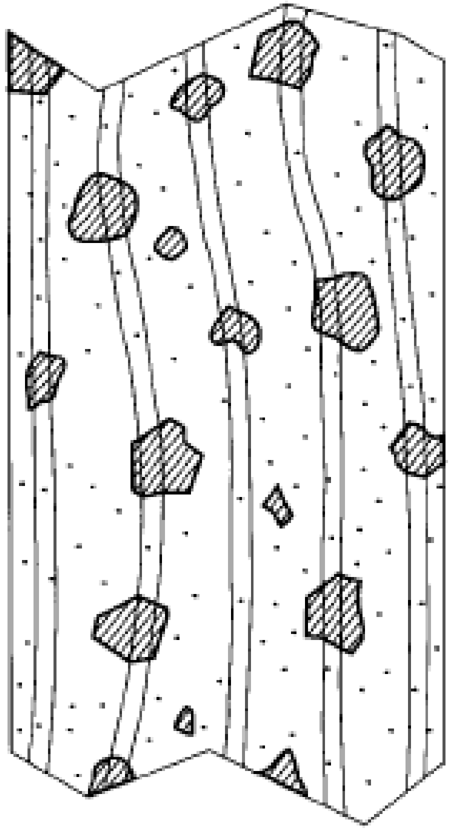


Fig. 5.10 Vortex configuration in a hard superconductor. The hatched regions represent pinning centers. The dots indicate atomic defects.

Magnetization in ideal Type II SC

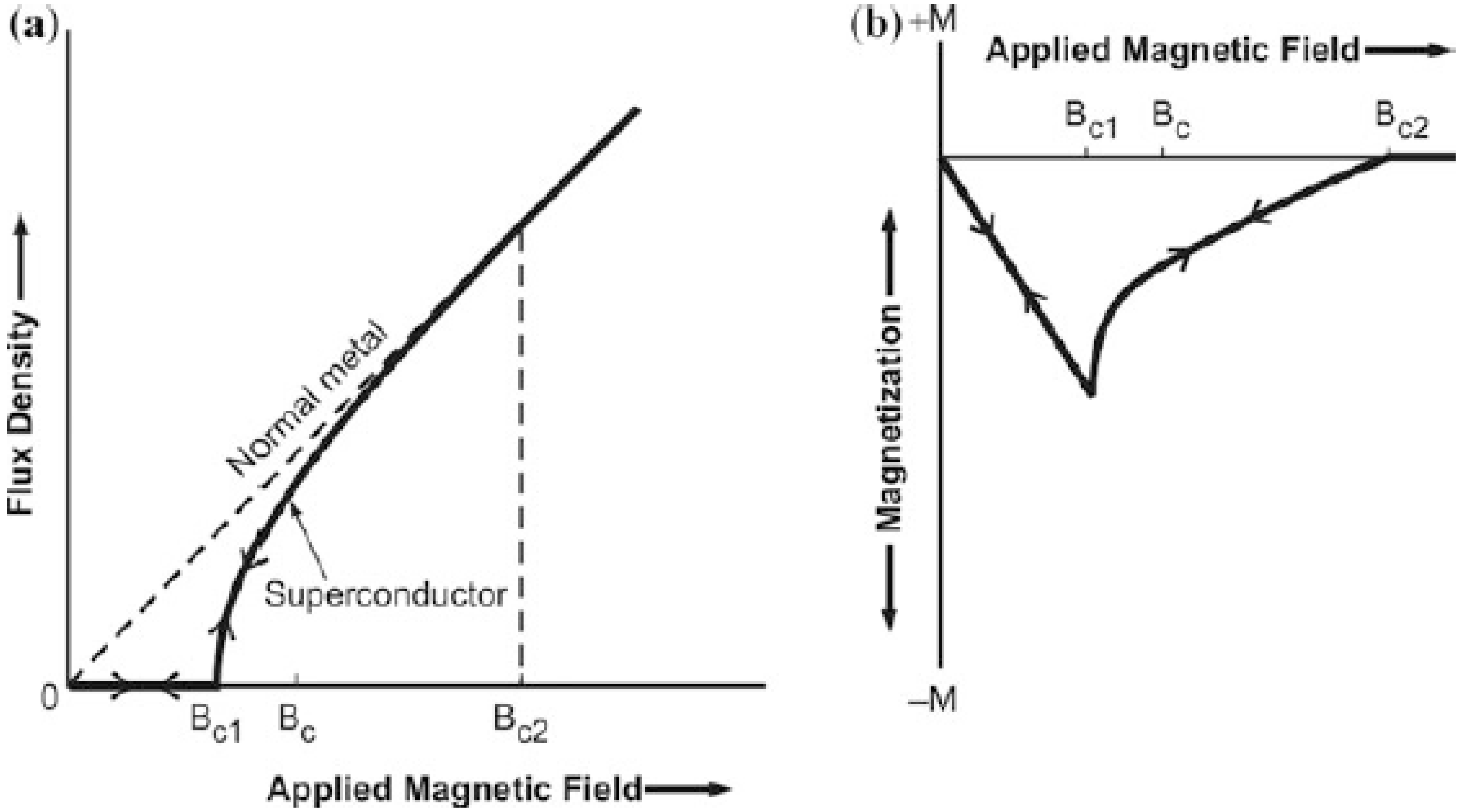


Fig. 3.8 **a** The magnetic flux density inside a type II superconductor is zero up to B_{c1} and increases in higher field. At B_{c2} flux penetrates the whole of the material. The process is reversible in ideal pure material. **b** Negative magnetization increases with magnetic field, peaks at B_{c1} and then decreases to zero at B_{c2}. This process too is reversible for pure ideal material

Magnetization of Hard Superconductors

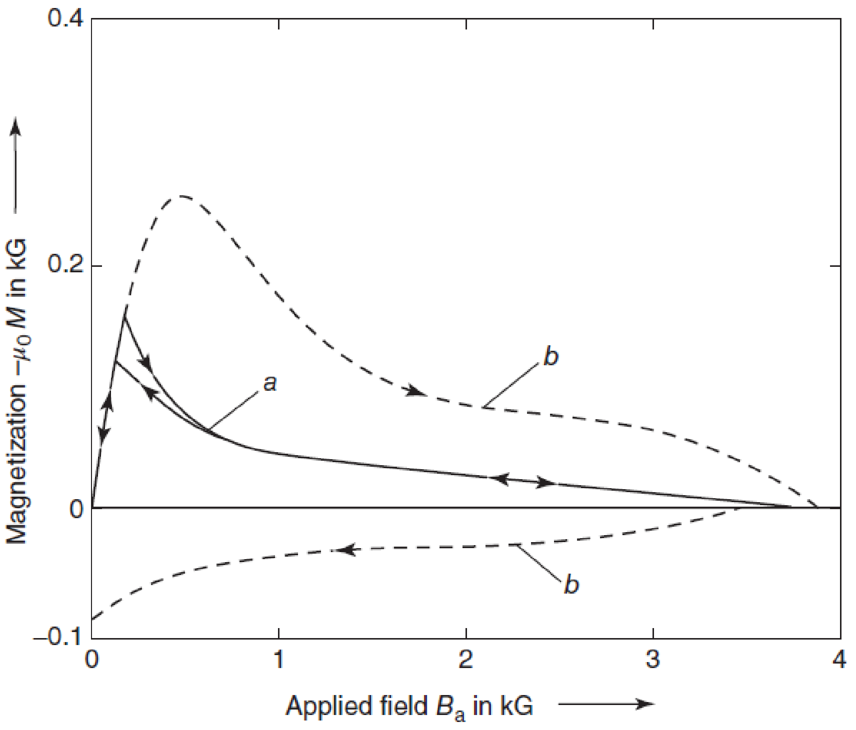


Figure 5.11 Magnetization curves of a $Nb_{55}Ta_{45}$ alloy: curve *a*, well annealed; curve *b*, with many lattice defects (1 kG = 0.1 T).(from [23])

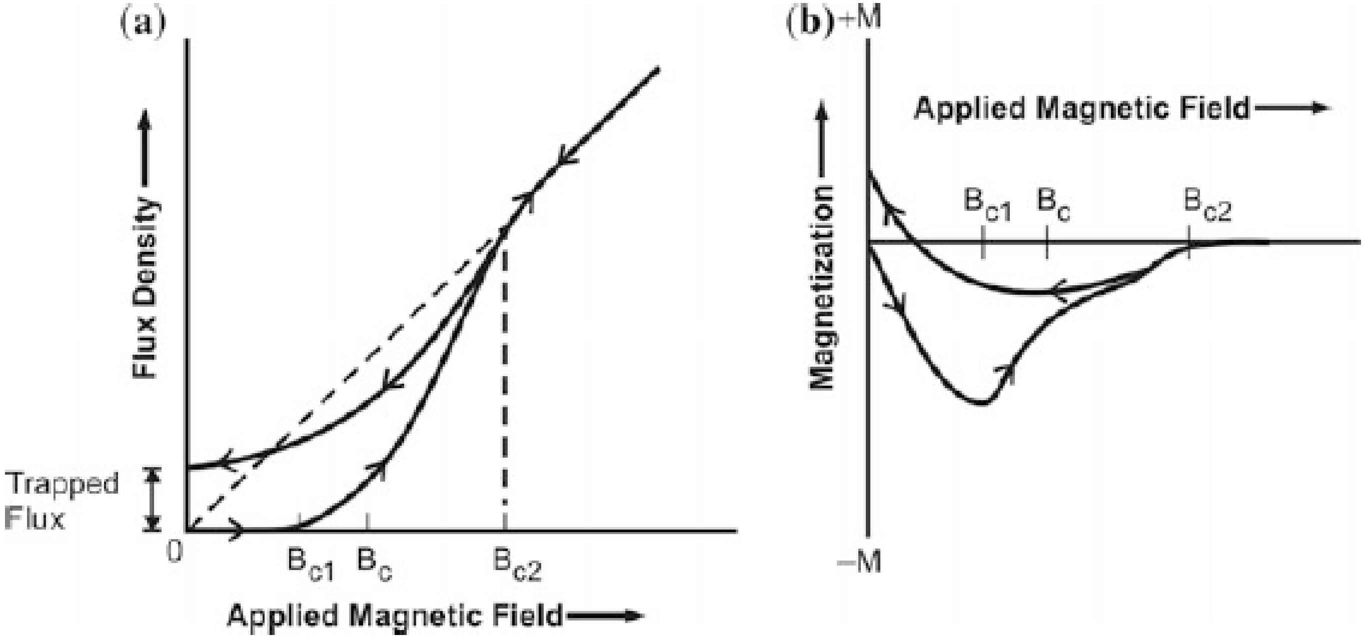
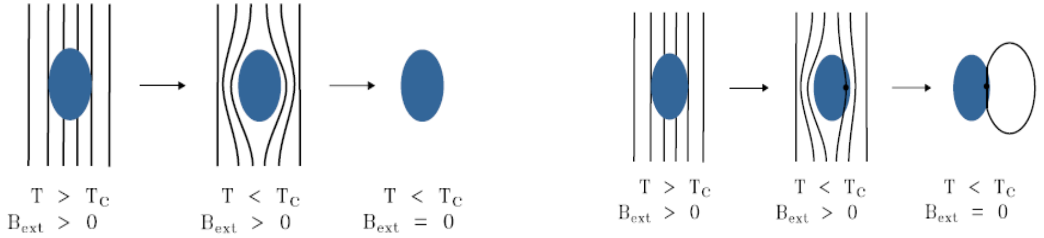


Fig. 3.9 A real type II superconductor has imperfection and show magnetic irreversibility in flux density (a) and in magnetization (b). Material can retain finite flux and magnetization even after the field is reduced to zero



(a) Sketch of the perfect Meissner effect

(b) Sketch of flux trapping

Magnetization of Hard Superconductors

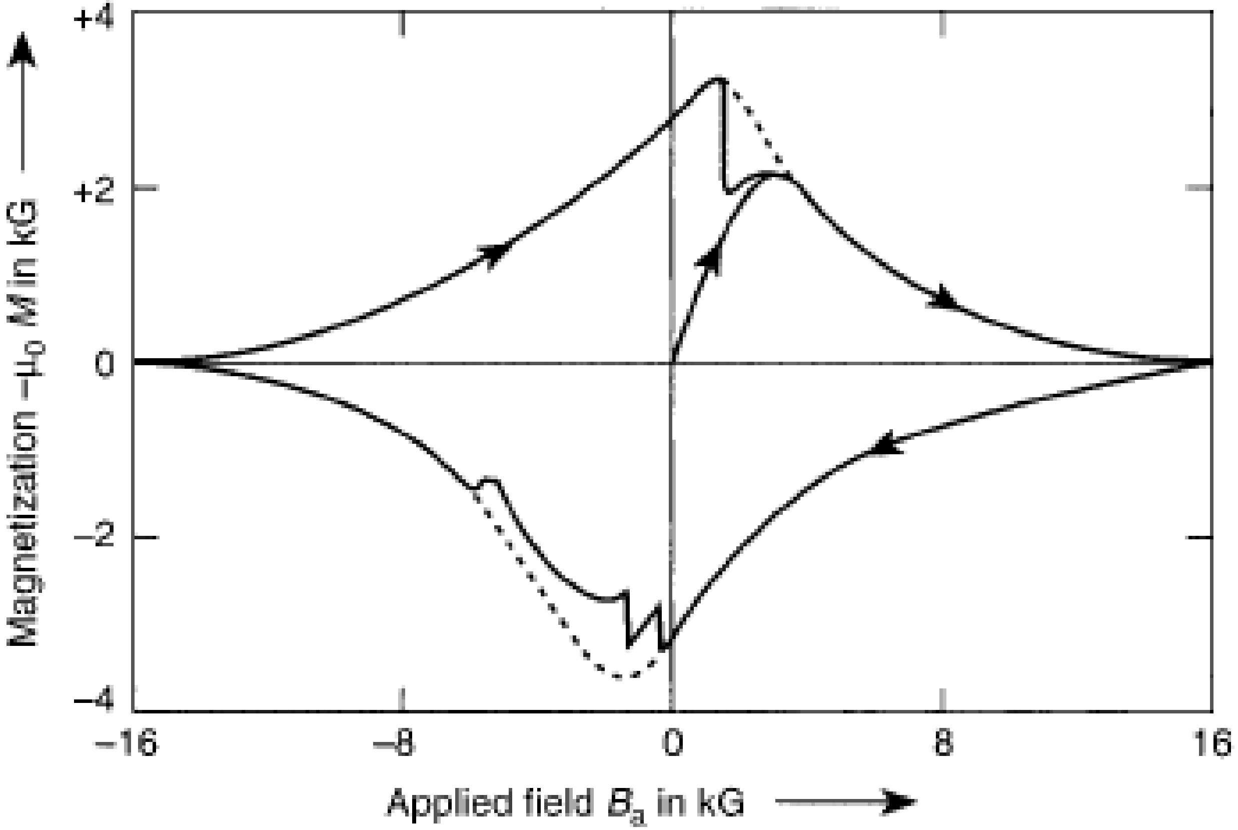
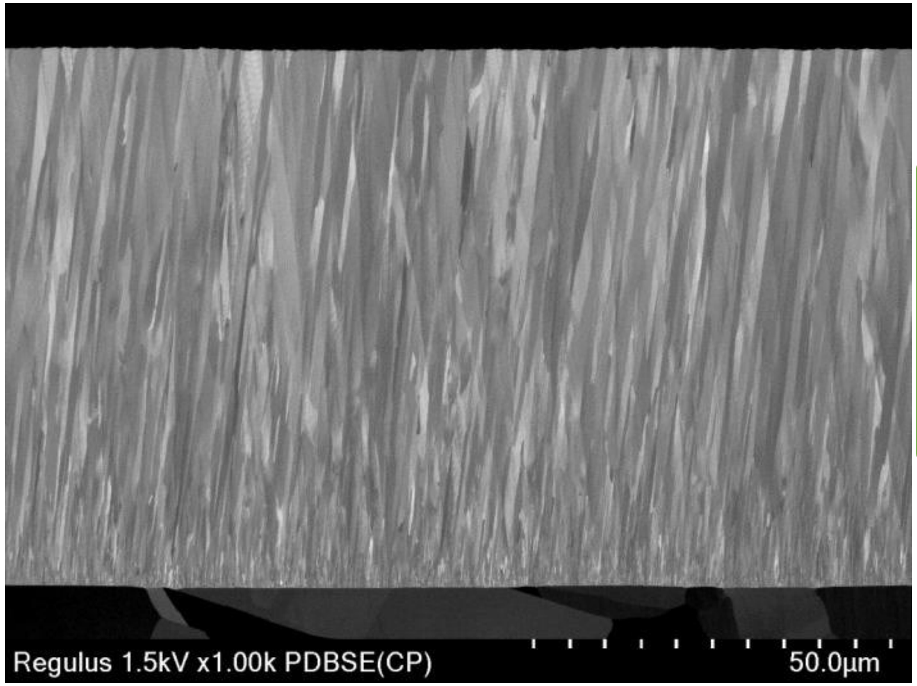


Fig. 5.12 Complete magnetization cycle of a Pb-Bi alloy (53 at.% Bi). The dashed curve is expected if there are no flux jumps (from [22]) (1 kG = 0.1 T).

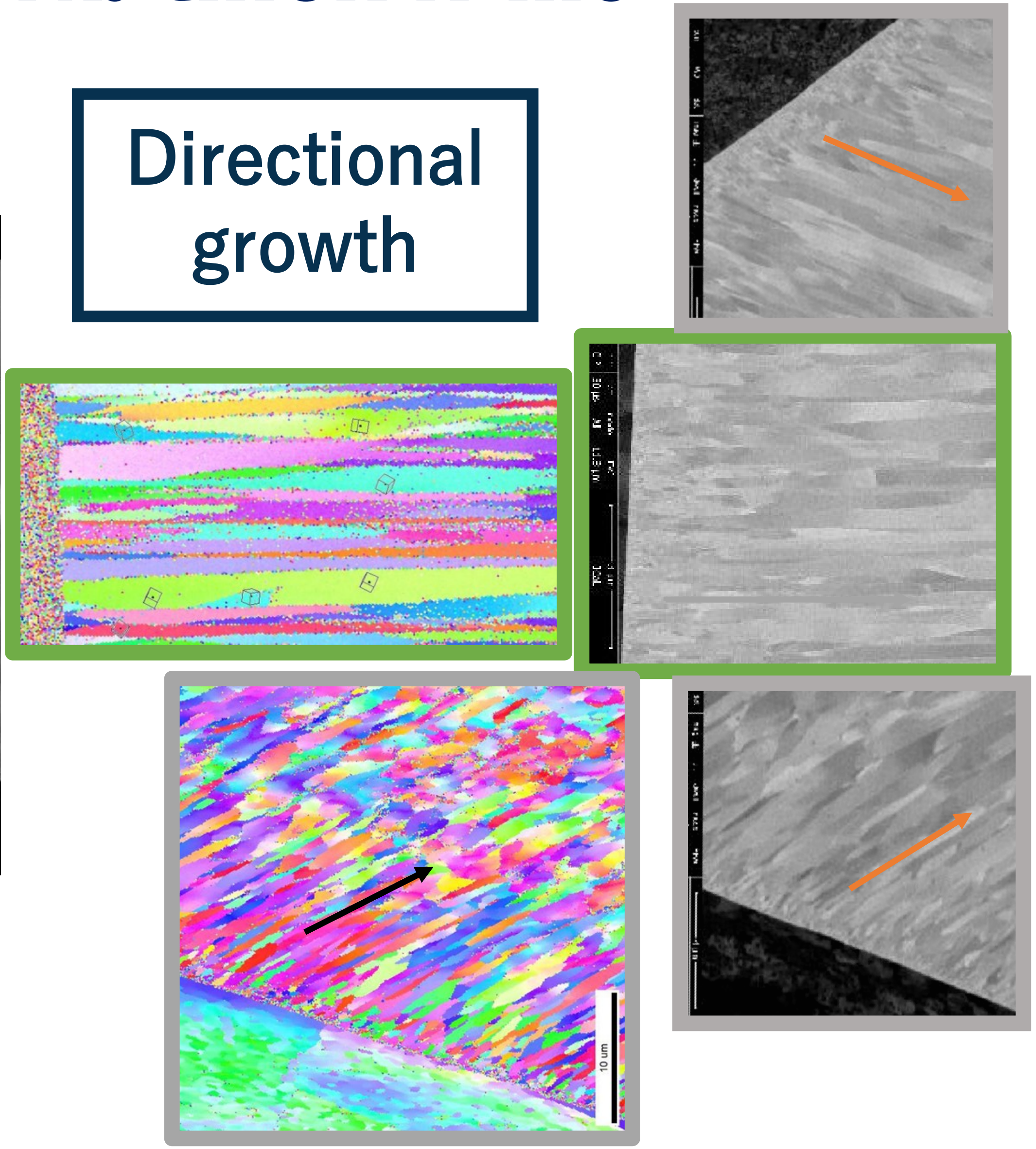
Magnetometry of Nb thick films

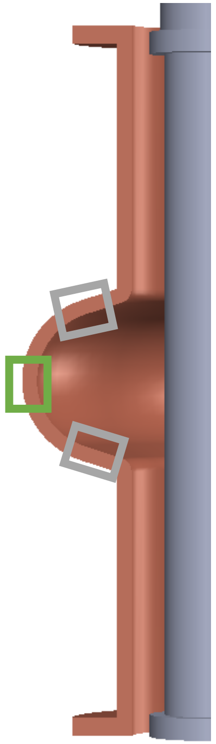
Courtesy of Reza Valizadeh (STFC)



Cavity 21.
Single layer thickness 500nm

Directional growth

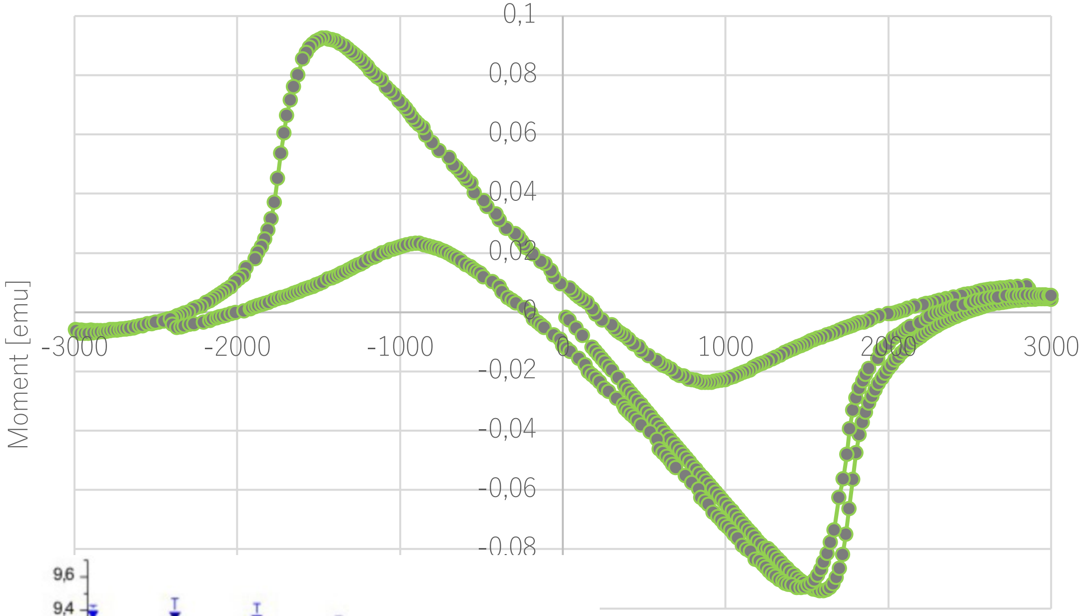


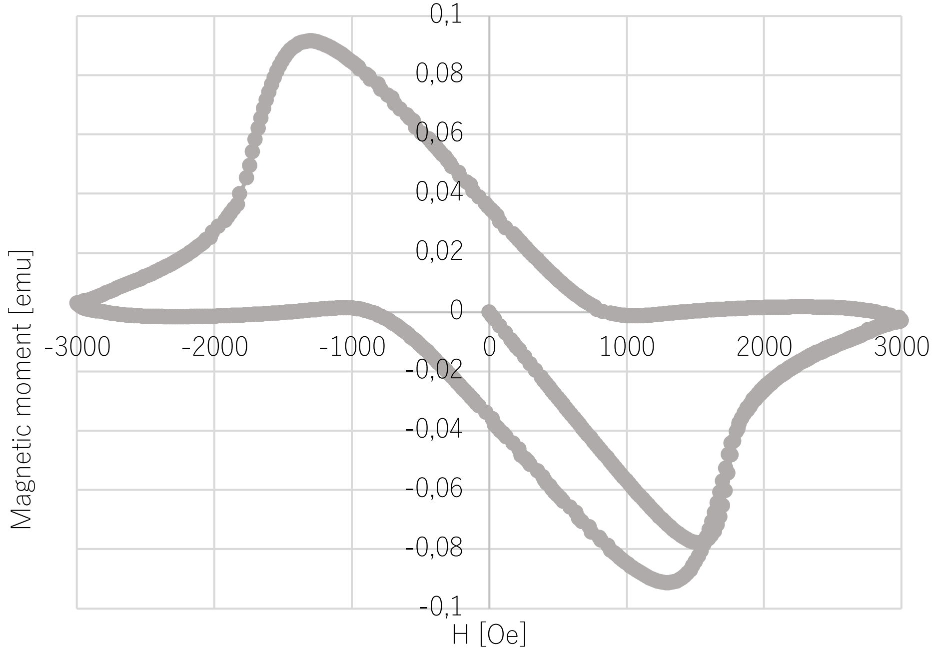


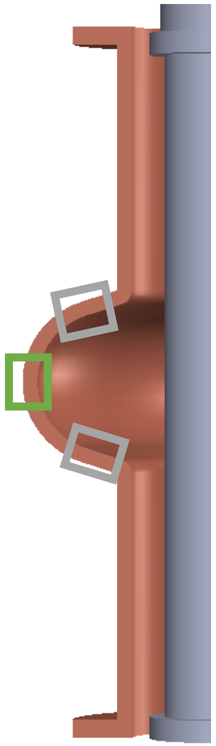
6 GHz cavity and Nb target section view

Magnetometry of Nb thick films

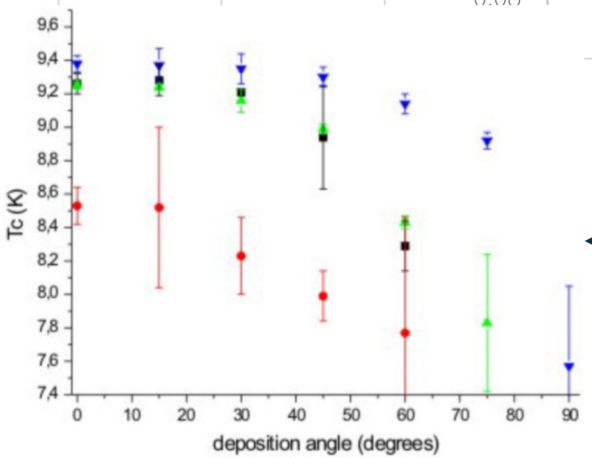
Courtesy of Reza Valizadeh (STFC)







6 GHz cavity and Nb target section view



Tonini et al 2011. Proceedings of the 11th Workshop on RF Superconductivity, Lübeck/Travemünder, Germany

Tc dependance respect to the relative angle of sputtering

Nb Thick film:
H_{fp} ~ 145 mT
H_{c2} ~ 330 mT

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  5 Critical Currents in Type-I and Type-II Superconductors

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  Chapter 3 Type II Superconductors

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