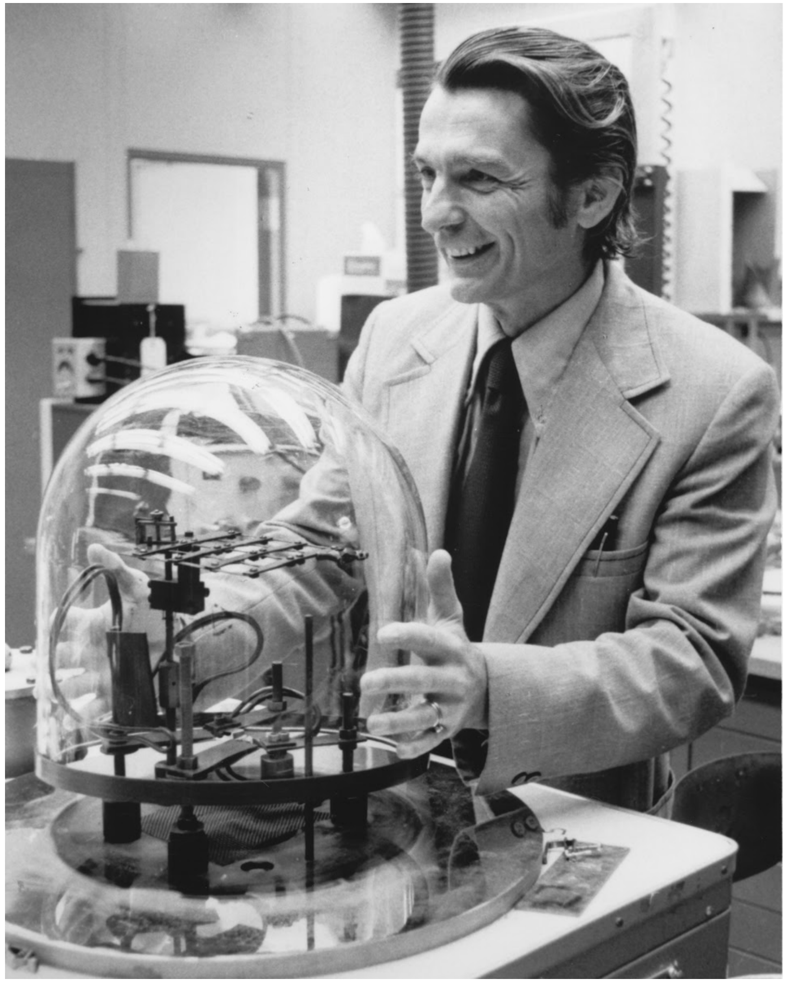
Superconductive Materials

Part 6

Josephson Effect

Consequences of BCS: energy gap measures



Ivar Giaever at General Electric Laboratories in the US made the **first superconducting tunnel junction in 1960**

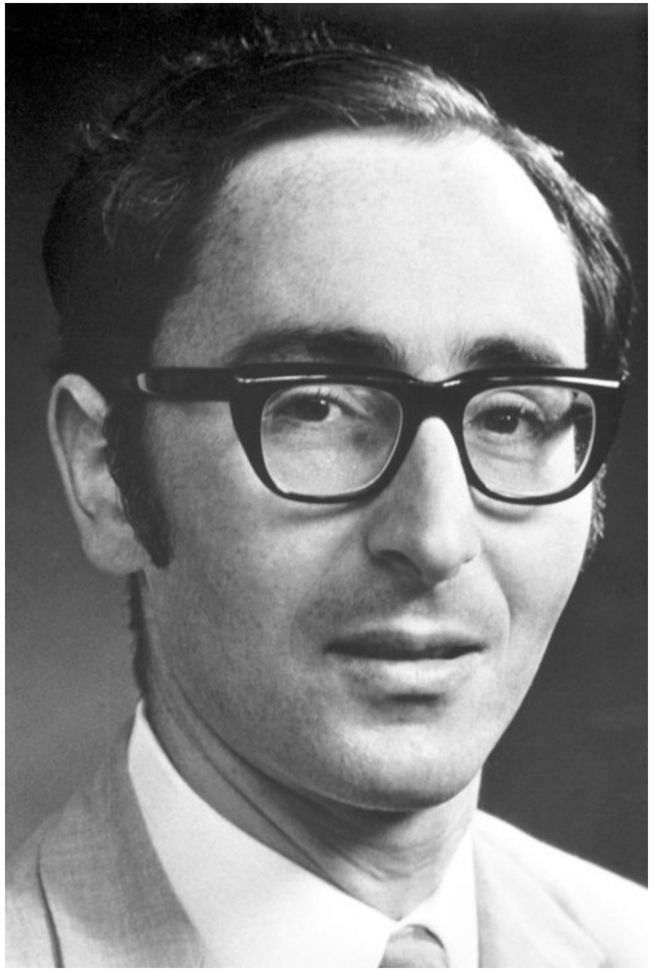
In 1961 Giaever pointed out the possibility of **determining the energy gap by means of tunneling experiments**

Dr. Ivar Giaever, 44, works in his laboratory at General Electric Company here after learning (October 23, 1973), that he shared the 1973 Nobel Prize for Physics with Dr. Leo Esaki, 48, of International Business Machines Company of Yorktown Heights, New York, and Dr. Brian D. Josephson, 33, of Cambridge, England. Dr. Giaever has been with GE for 15 years. Born in Norway, he has lived in the U.S. for 20 years and became a citizen 10 years ago. He received the prize for his work in "marrying tunneling to superconductivity."

Brian Josephson

Brian Josephson was a **doctoral student** at Cambridge University's Cavendish Laboratory in the early 1960s, working under the supervision of **Brian Pippard**

During the first year of his doctorate, he had taken some lectures from **Philip Anderson** who was at that time spending part of every year in Cambridge.
Anderson lectured on broken symmetry as a central principle underlying solid state physics and Josephson was captivated by these ideas

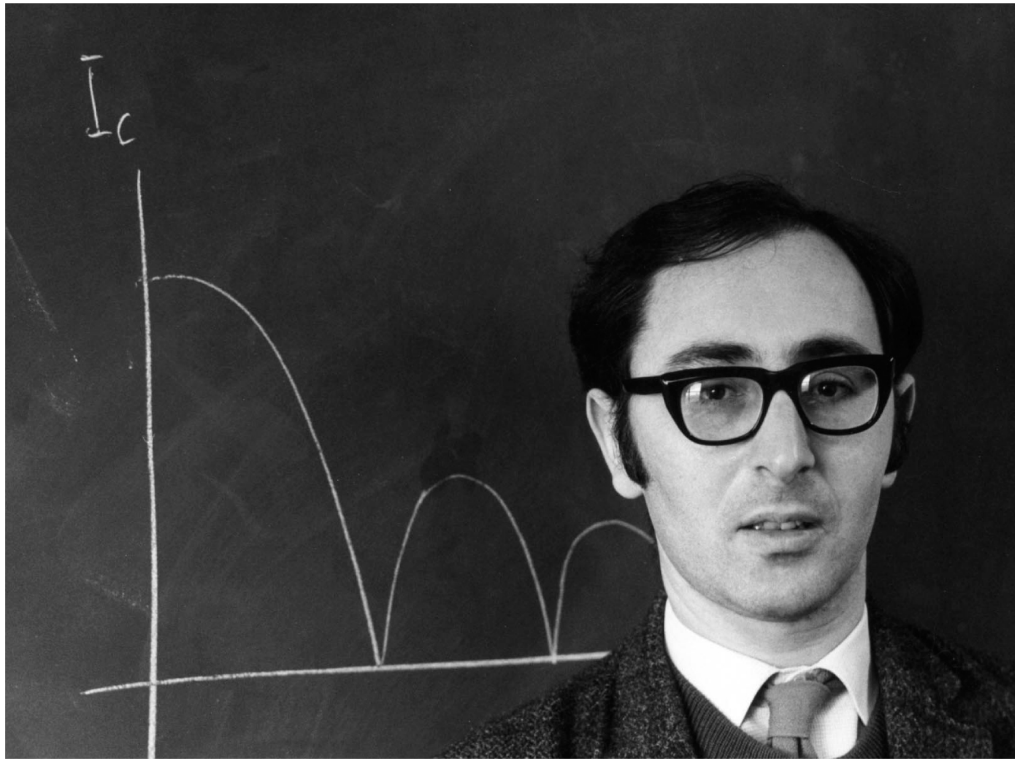


Brian Josephson

What a prediction!

Josephson realized that though **the phase** of the wavefunction inside a superconductor was fixed and uniform inside it, the phase of the wavefunction inside a second superconductor would also be fixed and uniform, but **would be fixed at a different value from the first**

If these two superconductors were brought in close proximity to one another (SIS junction) the **phase difference between them would have observable consequences**



Brian Josephson

Not enough for a PhD

Josephson performed a calculation of the quantum-mechanical tunneling current between the two superconductors and found that **a spontaneous net current would flow from one to the other which was directly related to the difference in the values of phase taken by the two superconductors**

Pippard (Josephson's PhD supervisor) was not convinced that this remarkable prediction **was of sufficient worth to win him his doctorate**

Josephson therefore spent the second year of his doctorate trying to provide an experimental confirmation of his prediction, a task that neither suited his own skills nor the facilities of his laboratory.

Sir Alfred Brian Pippard

The Nobel Laureate Versus the Graduate Student

Bardeen publicly dismissed young Josephson's **tunneling-supercurrent assertion** in a "Note added in proof" to a 1962 article in *Physical Review Letters*:

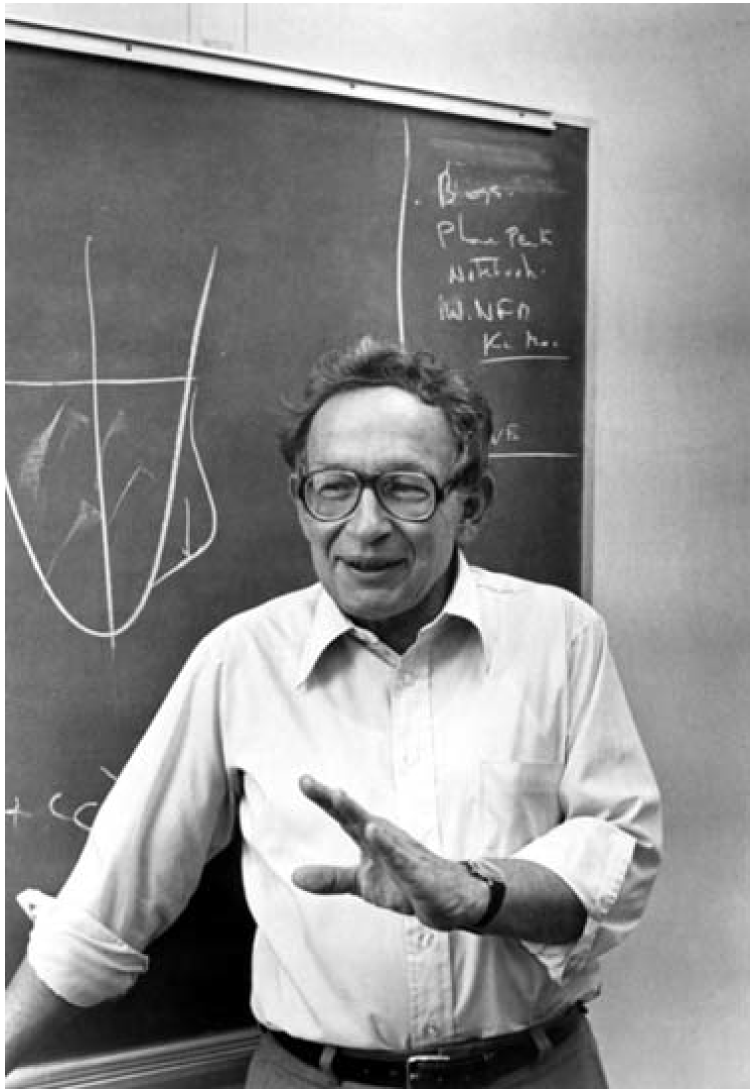
"In a recent note, Josephson uses a somewhat similar formulation to discuss the possibility of superfluid flow across the tunneling region, in which no quasi-particles are created. However, as pointed out by the author [Bardeen, in a previous publication], pairing does not extend into the barrier, so that there can be no such superfluid flow"

https://physicstoday.scitation.org/doi/full/10.1063/1.1397394

The Nobel Laureate Versus the Graduate Student
John Bardeen, the leading condensed matter theorist of his day, was quite wrong when he dismissed a startling prediction by the unknown Brian Josephson.

Experimental demonstration

It was not a trivial matter to construct what is now known as a Josephson junction, **two superconductors connected through a weak link**, and far more difficult than a conventional tunnel junction such as was made by Giaever, which has a more insulating barrier. **Philip Anderson**, who had been closely involved with the development of Josephson's thinking, and who had agreed to give Josephson a year to produce experimental justification before competing with him, eventually **constructed a working Josephson junction himself at Bell Labs in collaboration with John Rowell in 1963**



Philip Anderson

A good reward

Josephson received his PhD, the 1973 Nobel Prize (shared with **Ivar Giaever and Leo Esaki**), and a chair at Cambridge, all fitting rewards for a brilliant piece of insight which has had far-reaching consequences. He has spent most of the rest of his career devoting himself to his ‘mind-matter unification project’ which aims to find a physical basis for extrasensory perception, telepathy, and various other paranormal phenomena. It is perhaps unsurprising that his activities in this area have not won him the universal admiration of his scientific colleagues

S. Blundell
Superconductivity, a very short introduction

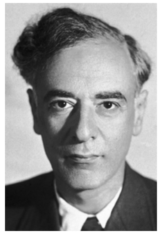
Brian Josephson's Sketch of Science

Milestones in Superconductivity

- 1908 – Helium liquefaction (Kamerligh Onnes)
- 1911 – Discovery of superconductivity (Kamerligh Onnes)
- 1933 – Meissner Effect
- 1935 – Phenomenological Theory (H. & F. London)
- 1936 – Discovery of Type II SC (Shubnikov)
- 1950 – Phenomenological (Thermodynamic) Theory (Ginzburg - Landau)
- 1951 – Vortex in type II SC (Abrikosov)
- 1957 – Microscopic Theory - BCS (Bardeen, Cooper, Schrieffer)
- 1960 – Measure of Energy Gap in SC (Giaever)
- 1961 – Quantization of Magnetic Flux (Doll & Nabauer/Deaver & Fairbank)
- 1962 – Josephson Effect
- 1986 – High TC Superconductors (Bernodz, Muller)

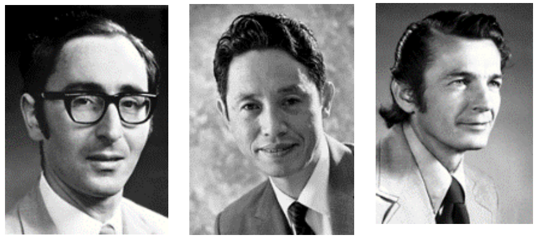
Nobel for Superconductivity

1913 Kamerlingh Onnes

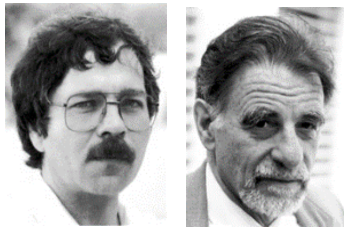


1962 Landau

1972 Bardeen, Cooper, Schrieffer



1973 Josephson, Esaki, Giaever



1987 Bednorz and Muller

2003 Abrikosov, Ginzburg, Leggett

2016 Thouless, Haldane, Kosterlitz

Tunnel effect (unpaired electrons)

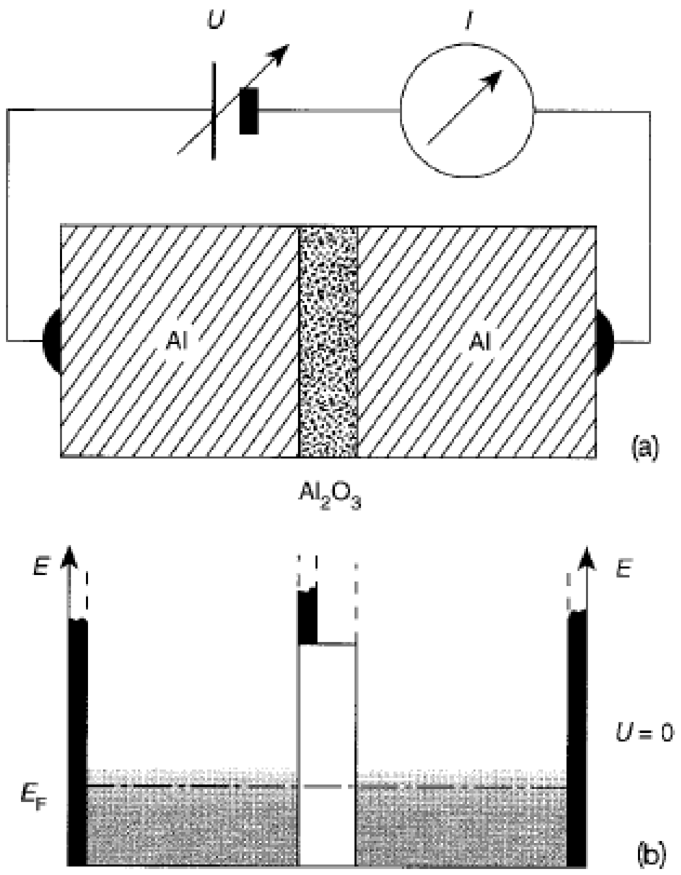
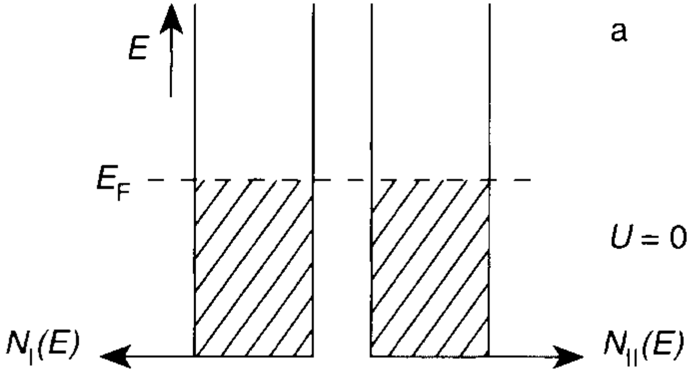


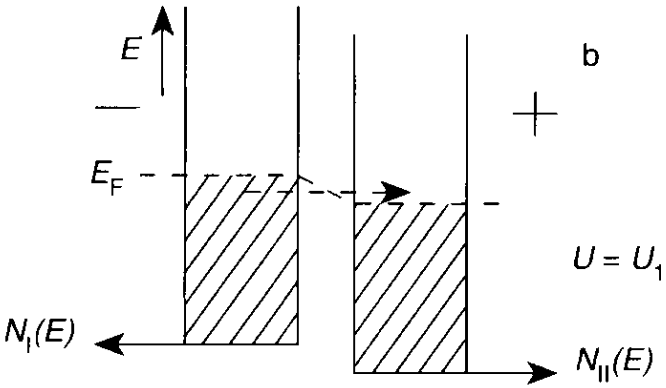
Fig. 3.11 (a) The arrangement for measuring a tunneling current. (b) The allowed energy values (black areas) and their occupation (gray shaded areas).

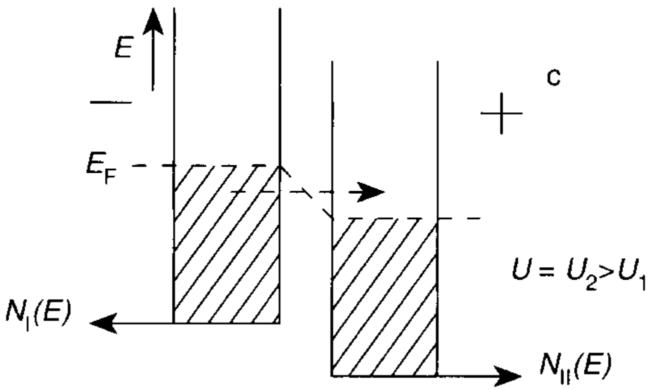
The number of particles passing across the barrier depends on the following 3 quantities:

1. **n. of electrons reaching the barrier**
2. **tunneling probability across the barrier**
3. **n. of unoccupied E levels on the other side**

Tunnel effect between NC metals

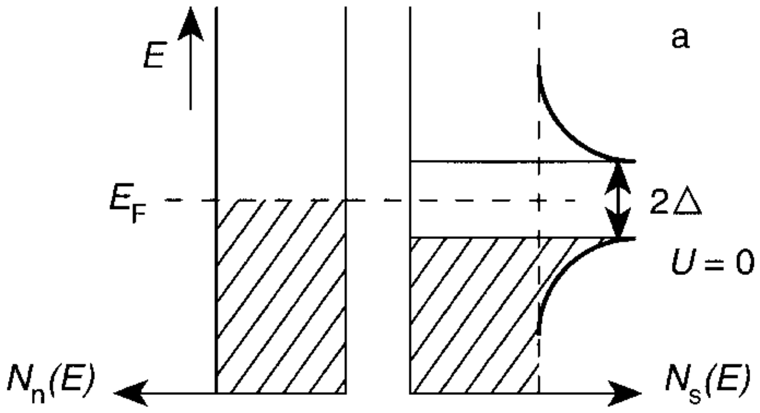


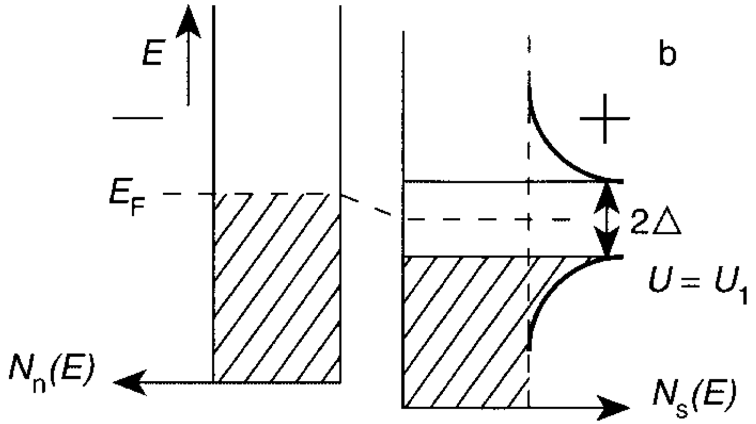


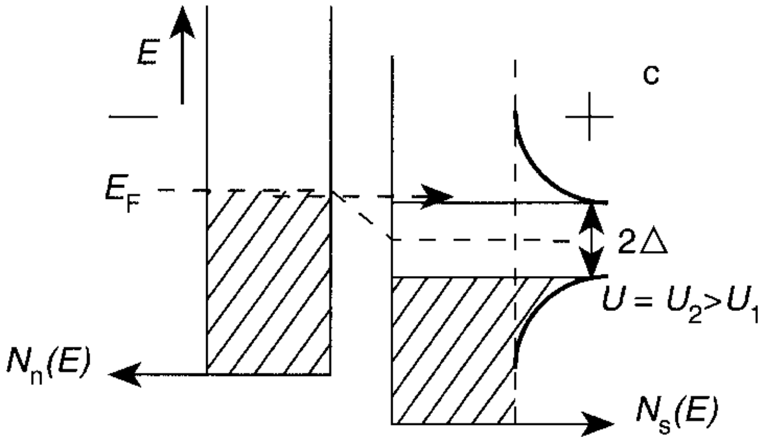


$U = 0$ $U = U_1$ $U = U_2 > U_1$

Tunnel effect between NC and SC metals

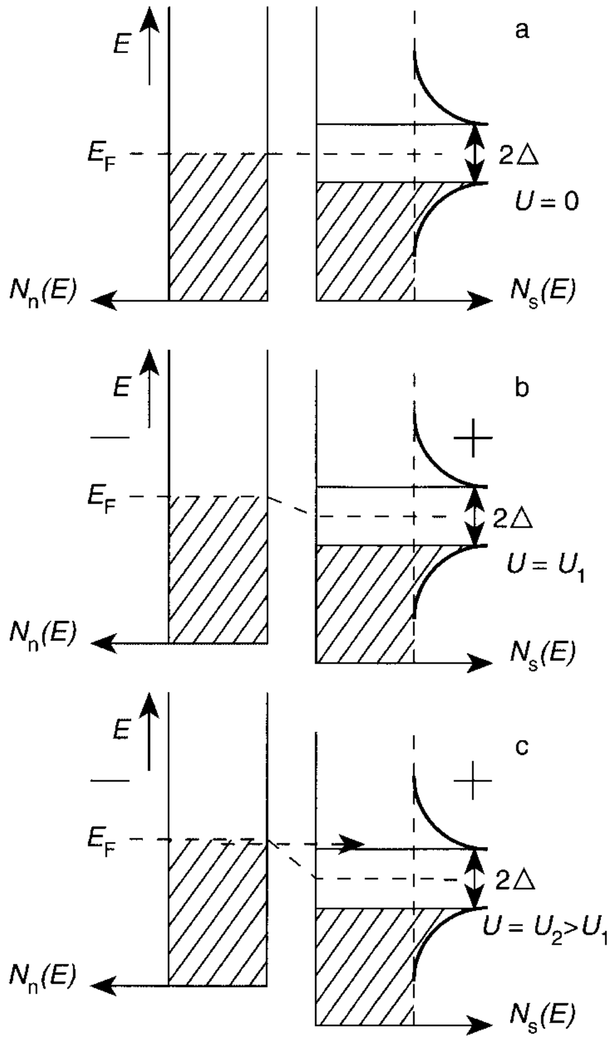






$U = 0$ $\quad$ $U = U_1$ $\quad$ $U = U_2 > U_1$

Tunnel effect between NC and SC metals



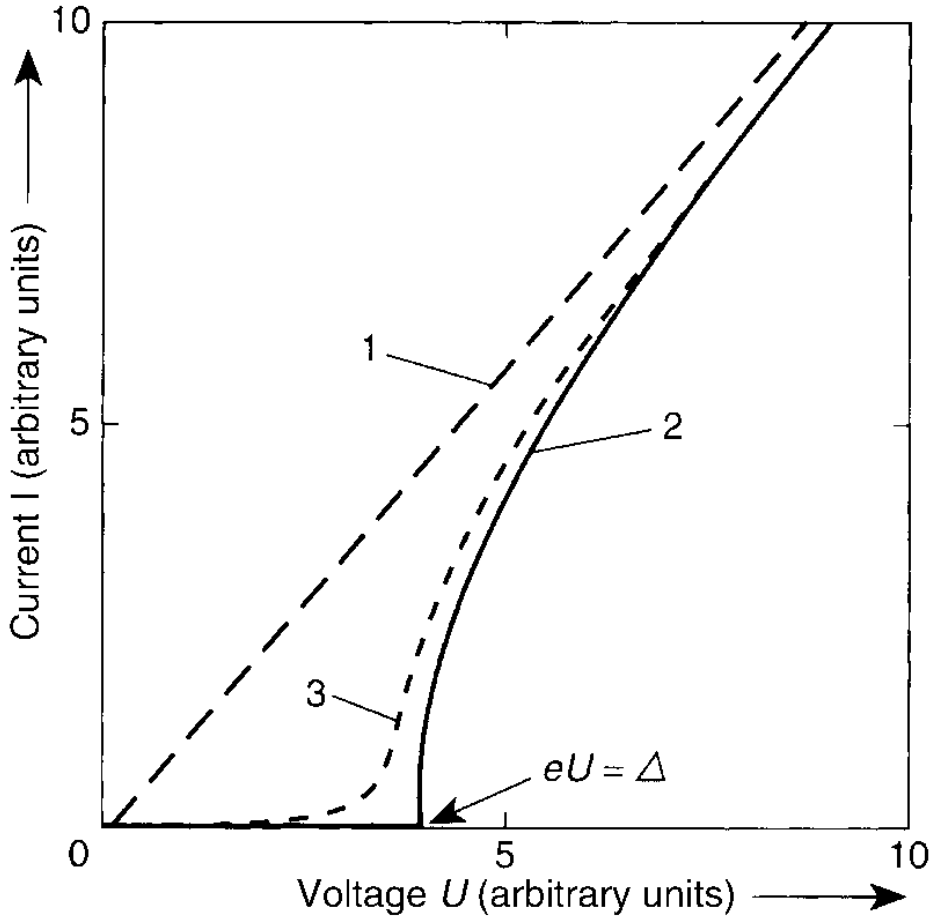
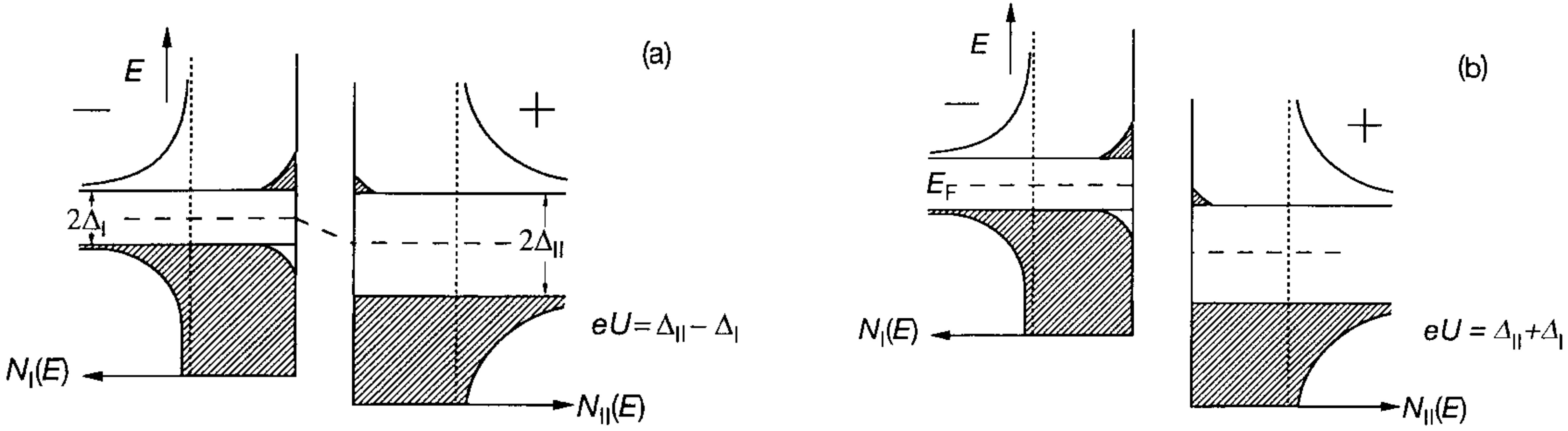
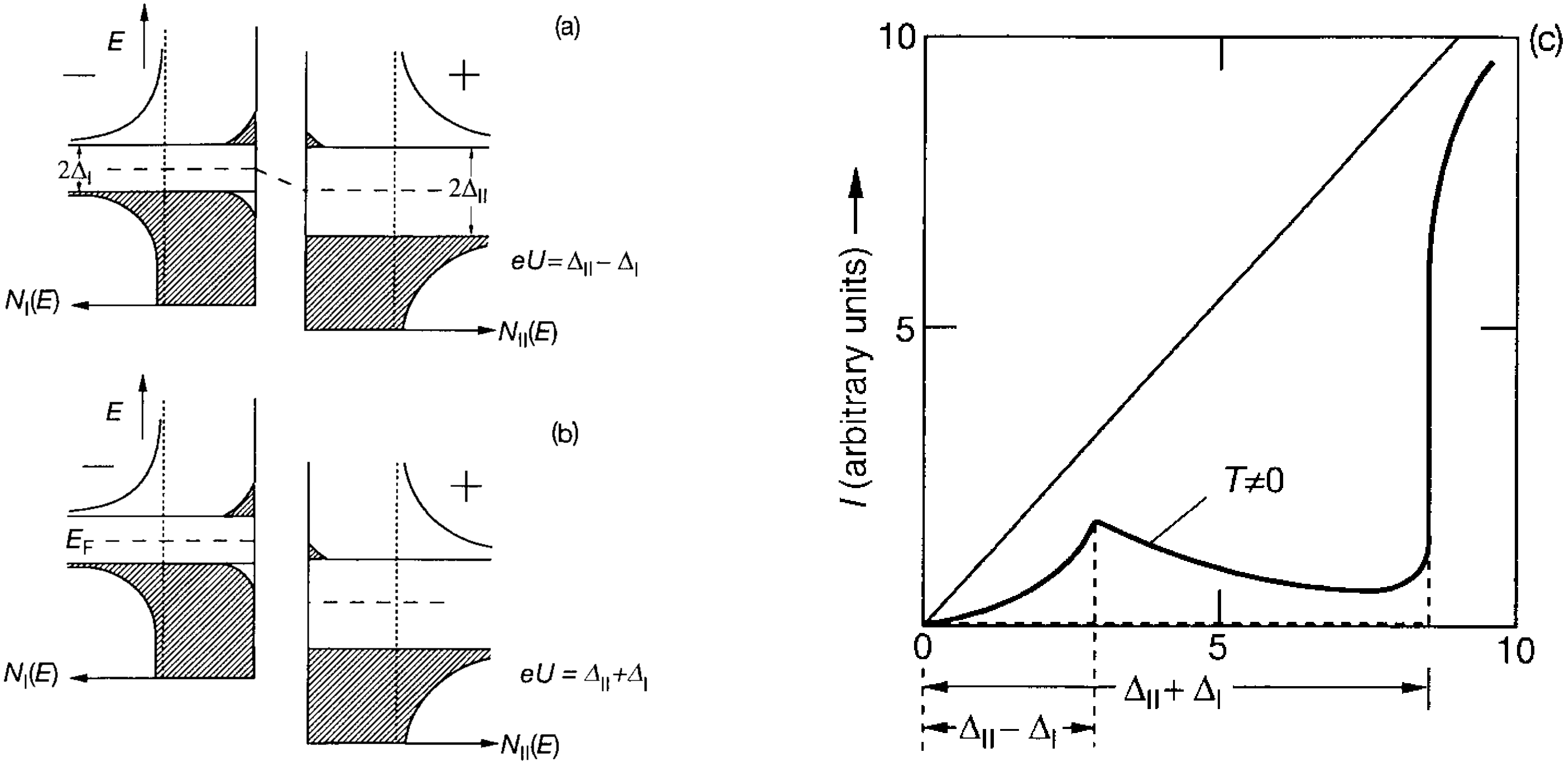


Fig. 3.14 Current-voltage characteristic of tunnel junctions: curve 1, normal conductor/normal conductor (Fig. 3.12); curve 2, normal conductor/superconductor, $T = 0$ K (Fig. 3.13); curve 3, normal conductor/superconductor, $0 < T < T_c$.

Tunnel effect between 2 SC



Tunnel effect between 2 SC



Josephson currents

In a **junction** between **2 SC separated by a thin (2-10 Å) insulator** (weak coupling) a current appear, as predicted by Josephson

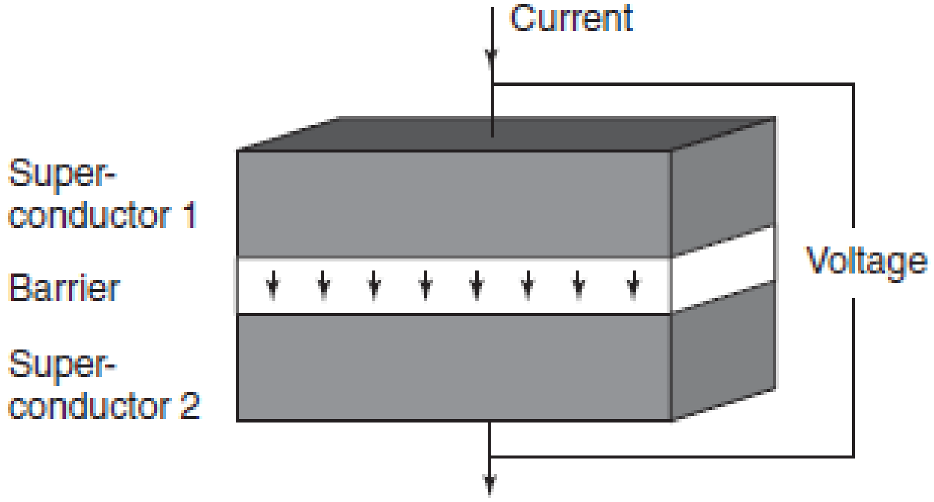
Due to the tunneling electrons or Cooper pairs, the two superconductors are coupled to each other, and a weak supercurrent (the Josephson current) can flow across the barrier at U=0

$$\boldsymbol{I_S = I_0 sin\Delta\varphi}$$

1st Josephson equation

$$\boldsymbol{\frac{\partial\varphi}{\partial t} = \frac{2e}{\hbar}U}$$

2nd Josephson equation



Josephson DC currents

$$I_S = I_0 sin(\Delta\varphi)$$

Phase difference $\varphi_1 - \varphi_2$ of the macroscopic wave function of the two superconductors

Critical current of the J-J
≠ Critical Current of SC

Typical I_0 ~ 10^{-3} – 10^{-6} A

I_0 / contact area = J_0 J-J ~ 10^2 – 10^4 A/m²

J_c SC ~ 10^9 – 10^{11} A/m²

constant

$$\frac{\partial(\Delta\varphi)}{\partial t} = \frac{2e}{\hbar} U \Rightarrow U = 0$$

If the voltage **U_0** across the junction **is zero** there is a **dc Cooper-pair current** which can assume any value in the range:

$$-I_0 < I < I_0$$

Josephson AC currents

$$I_S = I_0 sin(\Delta\varphi)$$

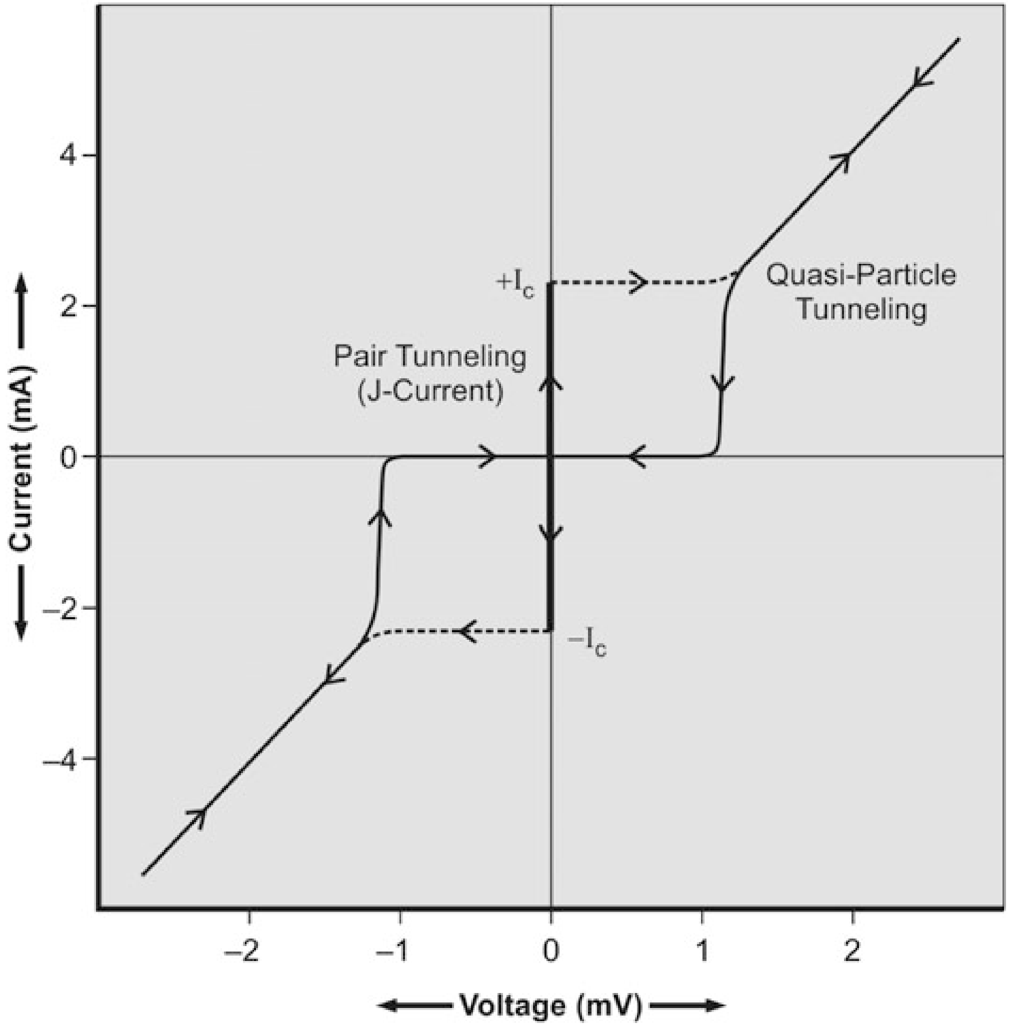
$$\frac{\partial(\Delta\varphi)}{\partial t} = \frac{2e}{\hbar}U$$

Increasing the voltage of the power supply eventually leads to **a non-vanishing voltage across the junction** and then a new phenomenon arises besides a dc current which however is now carried by single electrons there is an **alternating Cooper-pair current**

$$\boldsymbol{I(t) = I_0 sin(\omega t)}$$

$$\boldsymbol{\omega} = \frac{2e}{\hbar}U = \frac{2\pi}{\Phi_0}U$$

Typical I-V behavior in J-J



Weakly coupling

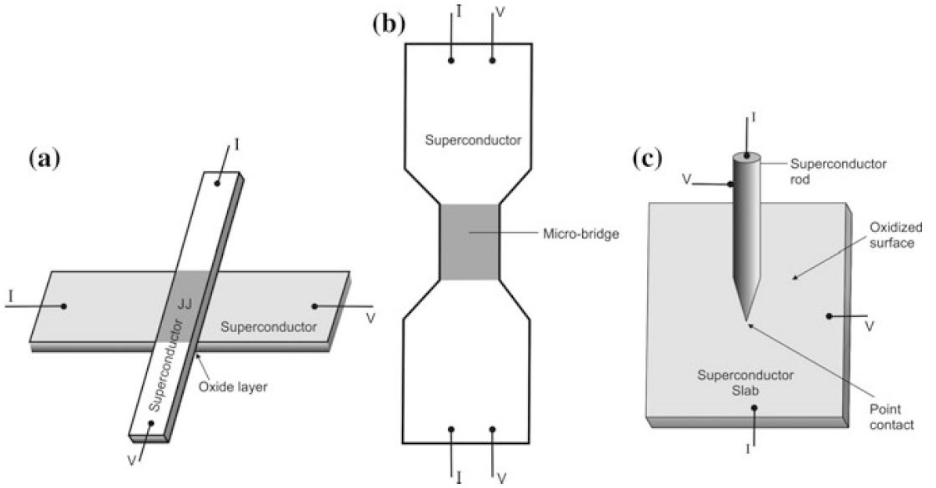


Fig. 2.30 Three different configurations of the J-J, **a** a "crossed film Junction", **b** a "weak link" or a "microbridge", **c** a "point contact" [22] (With permission of AIP)

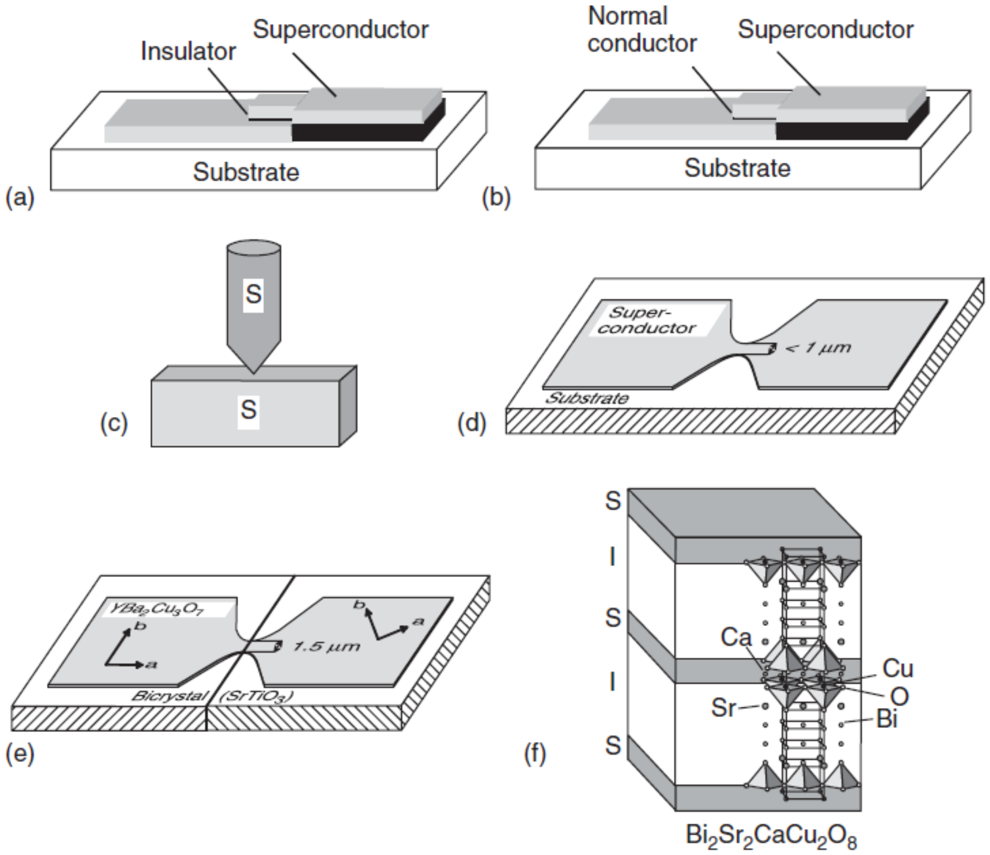


Fig. 1.21 Schematics of the different possibilities for producing a weak coupling between two superconductors: (a) SIS junction with an oxide layer as a barrier; (b) SNS junction with a normal conducting barrier; (c) point contact; (d) microbridge; (e) $YBa_2Cu_3O_7$ grain boundary junction; (f) intrinsic Josephson junction in $Bi_2Sr_2CaCu_2O_8$.

Derivation of Josephson Equations

A possible derivation of the two Josephson equation comes from Feynman

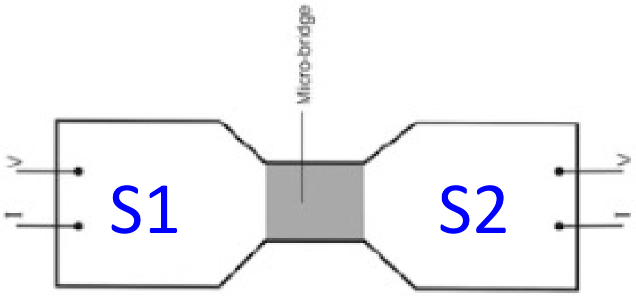
One considers **two weakly coupled quantum mechanical systems** and **solves the Schrodinger equation** for this problem by means of an approximation

The magnetic field is neglected at this stage

The two *separate* systems will be described by the 2 wave functions ψ_1 and ψ_2

Derivation of Josephson Equations (2)

$$i\hbar\frac{\partial\psi_1}{\partial t} = E_1\psi_1$$

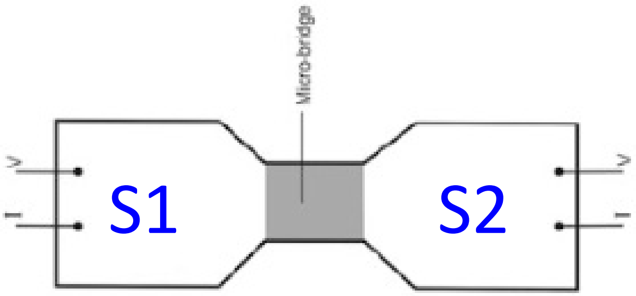


$$i\hbar\frac{\partial\psi_2}{\partial t} = E_2\psi_2$$

If there is weak coupling between the systems, the temporal change of ψ_1 will also be affected by ψ_2 and vice versa

$$i\hbar\frac{\partial\psi_1}{\partial t} = E_1\psi_1 + K\psi_2$$

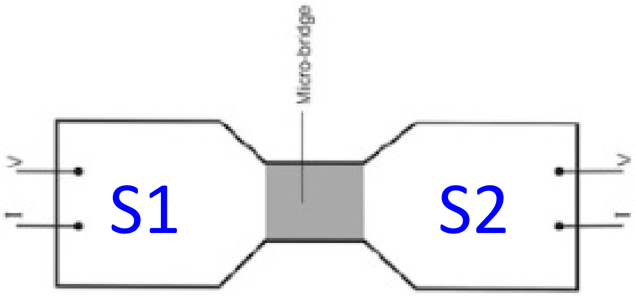
coupling parameter



$$i\hbar\frac{\partial\psi_2}{\partial t} = E_2\psi_2 + K\psi_1$$

Derivation of Josephson Equations (3)

$$i\hbar\frac{\partial\psi_1}{\partial t} = E_1\psi_1 + K\psi_2$$



$$i\hbar\frac{\partial\psi_2}{\partial t} = E_2\psi_2 + K\psi_1$$

$$\psi_1 = \sqrt{n_1}e^{i\varphi_1}$$ → Phase factor

↓ Cooper pair density

$$\psi_2 = \sqrt{n_2}e^{i\varphi_2}$$

$$\frac{\partial\varphi_1}{\partial t} = \dot{\varphi}_1$$

$$\frac{\partial n_1}{\partial t} = \dot{n}_1$$

$$\left(\frac{\dot{n}_1}{2\sqrt{n_1}} + i\sqrt{n_1}\dot{\varphi}_1\right)e^{i\varphi_1} = -\frac{i}{\hbar}\left(E_1\sqrt{n_1}e^{i\varphi_1} + K\sqrt{n_2}e^{i\varphi_2}\right)$$

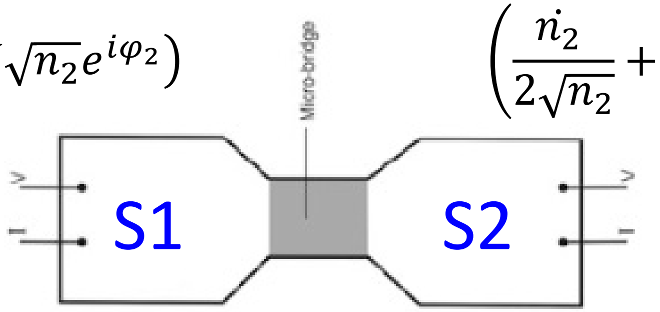
$$\left(\frac{\dot{n}_2}{2\sqrt{n_2}} + i\sqrt{n_2}\dot{\varphi}_2\right)e^{i\varphi_2} = -\frac{i}{\hbar}\left(E_2\sqrt{n_2}e^{i\varphi_2} + K\sqrt{n_1}e^{i\varphi_1}\right)$$

Derivation of Josephson Equations (4)

$$\left(\frac{\dot{n}_1}{2\sqrt{n_1}} + i\sqrt{n_1}\dot{\varphi}_1\right)e^{i\varphi_1} = -\frac{i}{\hbar}\left(E_1\sqrt{n_1}e^{i\varphi_1} + K\sqrt{n_2}e^{i\varphi_2}\right)$$

$$\left(\frac{\dot{n}_2}{2\sqrt{n_2}} + i\sqrt{n_2}\dot{\varphi}_2\right)e^{i\varphi_2} = -\frac{i}{\hbar}\left(E_2\sqrt{n_2}e^{i\varphi_2} + K\sqrt{n_1}e^{i\varphi_1}\right)$$

$\cdot\, e^{-i\varphi_1}$ $\cdot\, e^{-i\varphi_2}$



Real part

$$\dot{n}_1 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_2 - \varphi_1)$$

$$\dot{n}_2 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_1 - \varphi_2) \Rightarrow -\dot{n}_1$$

Imaginary part

$$\dot{\varphi}_1 = -\frac{1}{\hbar}\left[E_1 + K\sqrt{\frac{n_2}{n_1}}\cos(\varphi_2 - \varphi_1)\right]$$

$$\dot{\varphi}_2 = -\frac{1}{\hbar}\left[E_2 + K\sqrt{\frac{n_1}{n_2}}\cos(\varphi_1 - \varphi_2)\right]$$

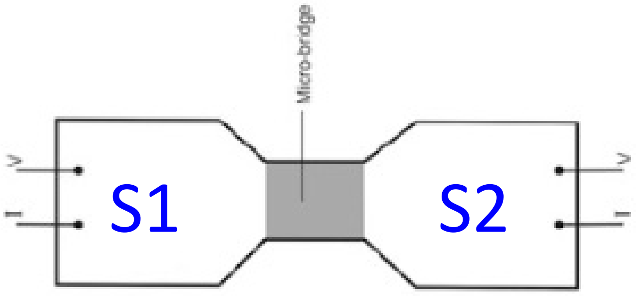
In the simple case in which S1=S2 → $n_1 = n_2$

$$\dot{n}_1 = \frac{2K}{\hbar}n_1 \sin(\varphi_2 - \varphi_1) = -\dot{n}_2$$

Derivation of Josephson Equations (5)

$$\dot{n}_1 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_2 - \varphi_1)$$

$$\dot{\varphi}_1 = -\frac{1}{\hbar}\left[E_1 + K\sqrt{\frac{n_2}{n_1}}\cos(\varphi_2 - \varphi_1)\right]$$



$$\dot{n}_2 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_1 - \varphi_2)$$

$$\dot{\varphi}_2 = -\frac{1}{\hbar}\left[E_2 + K\sqrt{\frac{n_1}{n_2}}\cos(\varphi_1 - \varphi_2)\right]$$

$$\dot{n}_1 = \frac{2K}{\hbar}n_1 \sin(\varphi_2 - \varphi_1) = -\dot{n}_2$$

Now we can calculate the current that cross the junction $\Rightarrow$ $I(t) = \frac{\partial n}{\partial t}Vq$

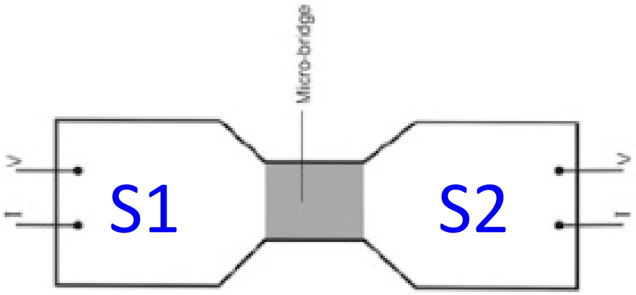
$$I = \dot{n}_1 V2e \Rightarrow I = \frac{2K \cdot 2e}{\hbar}Vn_s \sin(\varphi_2 - \varphi_1) \Rightarrow I = I_0 \sin(\varphi_2 - \varphi_1)$$

$$I_0 = \frac{2K \cdot 2e}{\hbar}Vn_s$$

Derivation of Josephson Equations (6)

$$\dot{n}_1 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_2 - \varphi_1)$$

$$\dot{\varphi}_1 = -\frac{1}{\hbar}\left[E_1 + K\sqrt{\frac{n_2}{n_1}}\cos(\varphi_2 - \varphi_1)\right]$$



$$\dot{n}_2 = \frac{2K}{\hbar}\sqrt{n_1 n_2}\sin(\varphi_1 - \varphi_2)$$

$$\dot{\varphi}_2 = -\frac{1}{\hbar}\left[E_2 + K\sqrt{\frac{n_1}{n_2}}\cos(\varphi_1 - \varphi_2)\right]$$

$$\dot{n}_1 = \frac{2K}{\hbar}n_1 \sin(\varphi_2 - \varphi_1) = -\dot{n}_2$$

$$\boldsymbol{I(t) = I_0\, sin(\varphi_2 - \varphi_1)}$$

$$\boldsymbol{I_0 = \frac{2K \cdot 2e}{\hbar} V n_s}$$

Derivation of $(\boldsymbol{\varphi_2 - \varphi_1})$ ➡ $\frac{d}{dt}(\varphi_2 - \varphi_1) = -\frac{1}{\hbar}(E_2 - E_1)$

The Cooper-pair energies E_1 and E_2 differ by the energy gained upon crossing the voltage U ➡ $E_2 = E_1 - 2eU$

$$\frac{d}{dt}(\varphi_2 - \varphi_1) = -\frac{1}{\hbar}(E_2 - E_1) = \frac{2eU}{\hbar}$$ ➡ $$\varphi_2(t) - \varphi_1(t) = \frac{2eU}{\hbar} \cdot t + \varphi_0$$

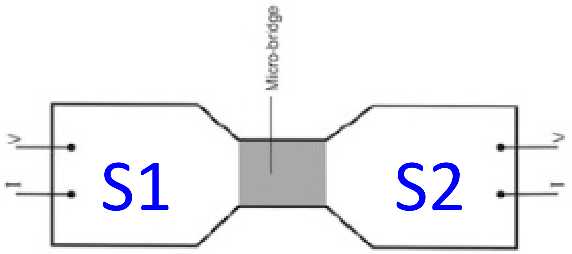
Derivation of Josephson Equations (7)

$$\boldsymbol{I(t) = I_0\, sin(\varphi_2 - \varphi_1)}$$

$$\boldsymbol{I_0} = \frac{2K \cdot 2e}{\hbar} V n_s$$

$$\boldsymbol{\varphi_2(t) - \varphi_1(t)} = \frac{2eU}{\hbar} \cdot t + \varphi_0$$

$$\boldsymbol{I(t) = I_0\, sin}\left(\frac{2eV}{\hbar} t + \varphi_0\right)$$



CASE 1 ➡ **V=0** ➡ **dc current** $\boldsymbol{I = I_0\, sin}\, \varphi_0$

CASE 2 ➡ **V≠0** ➡ **ac current** $\boldsymbol{I(t) = I_0 sin(\omega t)}$

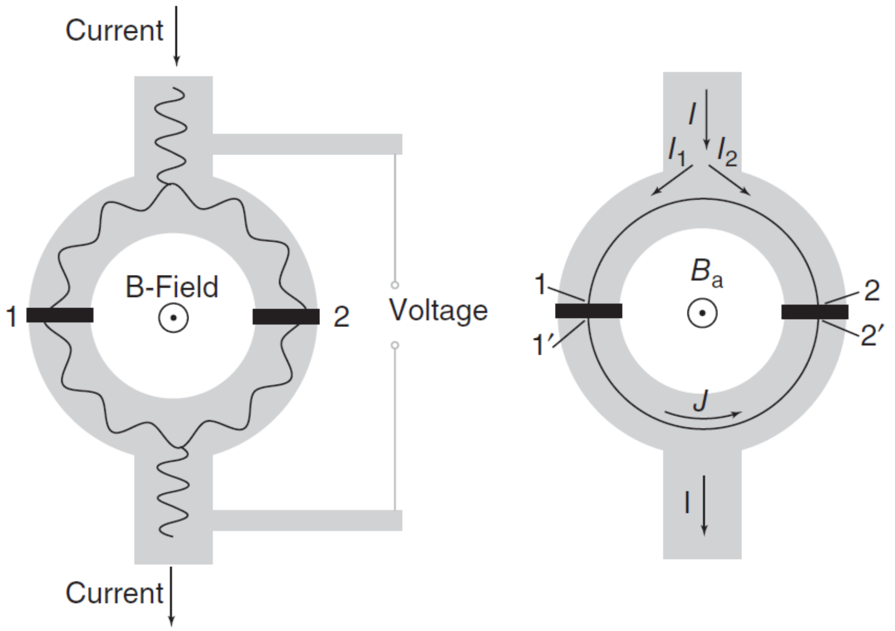
$\boldsymbol{\omega = \frac{f_J}{2\pi}}$ ➡ $\boldsymbol{f_J} = \frac{2e}{\hbar} U$ ➡ **For U=1mV → f_J = 483.6 GHz**

Josephson frequency

Standard Volt Definition

SQUID

The **superconducting quantum interference device (SQUID)** consists of two superconductors separated by thin insulating layers to form **two parallel Josephson junctions**. The device may be configured as a magnetometer to detect **incredibly small magnetic fields.** Small enough to measure the magnetic fields in living organisms. Squids have been used to measure the magnetic fields in mouse brains to test whether there might be enough magnetism to attribute their navigational ability to an internal compass.



Generation of spatial interferences of the superconducting wave function in a ring structure

Electromagnetic field phase change

In quantum mechanics in presence of an electromagnetic field:

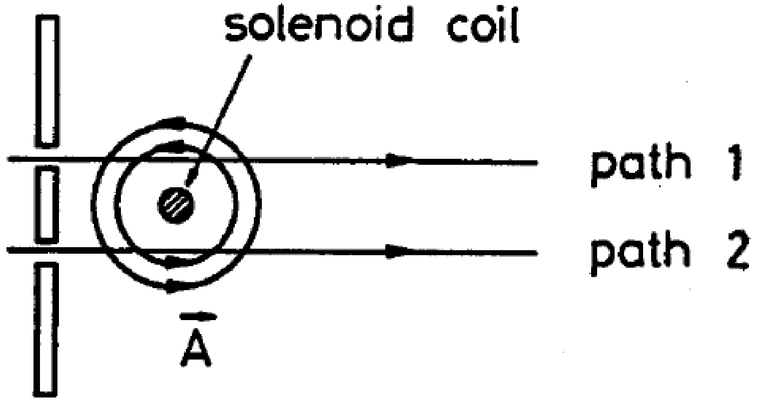
$$\mathbf{p} = m\mathbf{v} + q\mathbf{A}$$ *Canonical momentum*

Travelling along x (Δx) a phase variation (Δφ) occurs

$$\Delta\varphi = \frac{2\pi}{\lambda}\Delta x$$

In an electromagnetic field there is an additional phase change

$$\Delta\varphi' = -\frac{q}{\hbar}\boldsymbol{A\Delta x}$$ *Aharonov-Bohm effect*



Schematic arrangement for observing the phase shift due to a vector potential

$$\delta\varphi = \delta\varphi_0 + \frac{q}{\hbar}\oint \boldsymbol{A}\cdot\boldsymbol{ds} = \delta\varphi_0 + \frac{q}{\hbar}\Phi_{mag}$$

SQUID

$$I = 2I_0 \sin\delta \cos\left(\pi \frac{2e}{\hbar}\Phi_{mag}\right) \Rightarrow I = 2I_0 \sin\delta \cos\left(\pi \frac{\Phi_{mag}}{\Phi_0}\right)$$

$$I_{s,max} = 2I_0 \sin\left|\cos\left(\pi \frac{\Phi_{mag}}{\Phi_0}\right)\right|$$

The quantity $I_{s,max}$ reaches a maximum if the flux corresponds to an integer multiple of a flux quantum

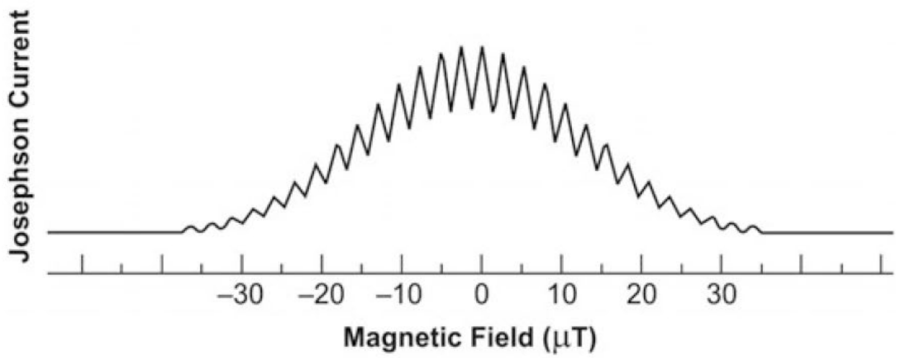


Fig. 2.32 A typical Josephson current versus magnetic field pattern in a dc SQUID

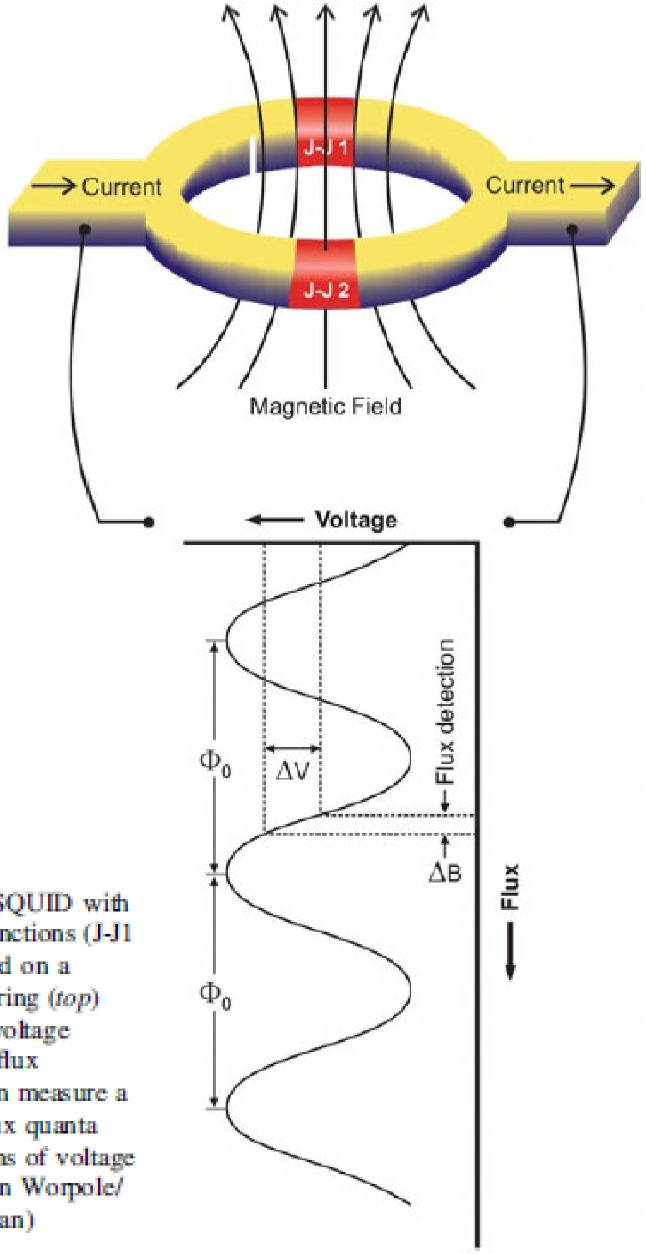


Fig. 2.31 A dc SQUID with two Josephson-junctions (J-J1 and J-J2) mounted on a superconducting ring (*top*) and the SQUID voltage oscillations with flux (*bottom*). One can measure a fraction of the flux quanta accurately in terms of voltage [23] (*Courtesy* Ian Worpole/ Scientific American)

SQUID

The **effect** is **similar** to the one observed in **optics** when a coherent light beam from a **laser** source passes through two **parallel slits** and **interfere** with each other to produce dark and bright **fringes**

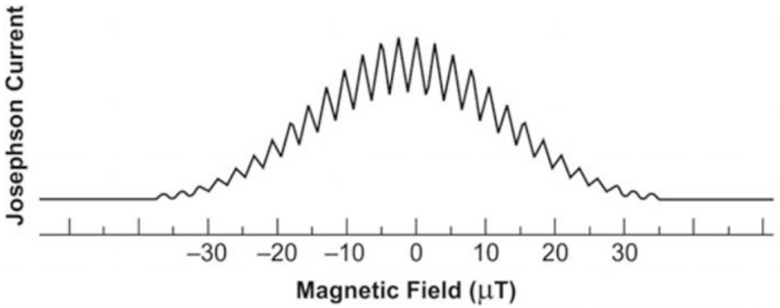


Fig. 2.32 A typical Josephson current versus magnetic field pattern in a dc SQUID

SQUIDs can detect and **measure a fraction of a flux quantum** digitally and accurately

SQUIDs can resolve changes of the magnetic flux down to about $10^{-6}\,\Phi_0$

Threshold for SQUID: ***10^{-15} T***

Magnetic field of earth: ***$20\text{-}70 \cdot 10^{-6}$ T***

Magnetic field of heart: ***$1\text{-}10 \cdot 10^{-11}$ T***

Magnetic field of brain: ***$1\text{-}100 \cdot 10^{-14}$ T***

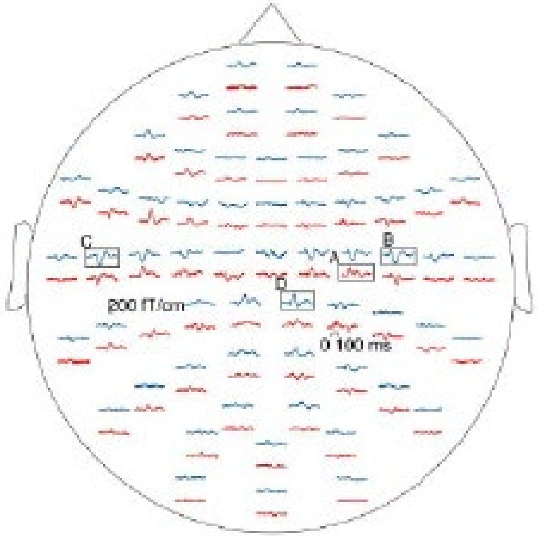
Magnetoencephalography (MEG)

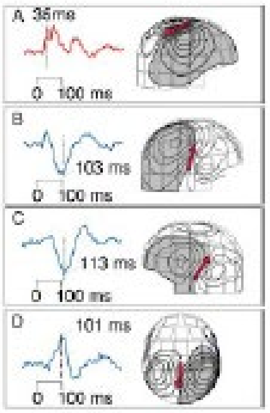
MEG = functional neuroimaging technique for mapping brain activity

Neurons are able to generate action potentials (voltage pulses)

Neuro-surgeons can pin point the source of epileptic seizure and can study real time brain activity. A combination of MEG and MRI can enable a surgeon to have detailed brain map and remove only the damaged tissues

Magnetic shielded room necessary
(2 layers of aluminium + 2 layers of mu-metal)





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  1.1.5 Josephson Currents

  3.1.3.2 The Energy Gap → Tuneling experiments

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