Superconductive Materials

Part 4

Ginzburg-Landau Theory

Ginzburg Landau Theory

Represents an important **extension of the London theory**

Incorporated two aspects:

1. **Spatial variation** of the **Gibbs energy** function of the SC state
2. Superconducting state can be **described in terms of a macroscopic wave function** with a well-defined phase

Ginzburg Landau Theory

The theory was published in 1950 (7 years before BCS)

For a long time the Ginzburg-Landau theory did not receive the proper attention (Abrikosov and Ginzburg received the Nobel Prize in physics in 2003)

Theory was generally recognized only after **Gor'kov demonstrated that for T near Tc the theory can be derived from the BCS theory**

One of the great successes of the theory was the **prediction of the vortex state by Abrikosov**

Often the theory is **now referred to as GLAG theory** after the four scientists Ginzburg, Landau, Abrikosov, and Gor'kov

Ginzburg Landau Theory starting point

In the absence of a magnetic field NC-SC is a **second-order phase transition**

In the Landau theory the so-called **order parameter *$\psi(r)$*** was defined

In the SC phase $\boldsymbol{\Psi}$ (r) should increase continuously from 0 at Tc up to the value 1 at T = 0

The quantity $|\boldsymbol{\Psi}(r)|^2$ can be interpreted as the density of the SC charge carriers

Ginzburg Landau Theory starting point

$|\boldsymbol{\Psi}\ (\mathrm{r})|^2$ must approach zero continuously for $T \rightarrow Tc$

near *Tc* we can **expand the Gibbs function** $\boldsymbol{g_s}$ of the SC phase **in a Taylor series** of the density $|\boldsymbol{\Psi}\ (\mathrm{r})|^2$

$$\boldsymbol{g_s = g_n + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \ldots}$$

Ginzburg Landau Theory starting point

$$g_s = g_n + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \ldots$$

for $T \sim T_c$ very general statements about the sign of the coefficients **α** *and* **β** *are possible*

β > 0 otherwise a very large value of $|\Psi|$ would always lead to a value of g_s that is smaller than g_n

for $T<T_c$ → **α < 0** since for $T<T_c$ → $\mathbf{g_s < g_n}$

for $T>T_c$ → **α > 0** since for $T>T_c$ → $\mathbf{g_s > g_n}$

$\mathbf{g_s = g_n}$ when $\boldsymbol{\Psi}$**=0**

Derivation of Ginzburg Landau equations

$$g_s = g_n + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \ldots$$

For temperatures close to T_c

α and **β** can also be expanded in Taylor series of T

$$\alpha(T) = \alpha(0)\left(\frac{T}{T_c} - 1\right)$$

However **β** can be taken as constant:

$$\beta(T) = \beta = const$$

Thermodynamic critical field (B_{cth})

Ψ_∞ is the **equilibrium value** of Ψ sufficiently **far away from any interface**

$n_s = |\Psi_\infty|^2$ → **equilibrium density in zero field**

$$g_n - g_s = \frac{1}{2\mu_0} B_{cth}^2$$

Look the derivation done in part2-properties of superconductors

$$g_n - g_s = -\alpha|\Psi|^2 - \frac{1}{2}\beta|\Psi|^4 = \frac{1}{2\mu_0} B_{cth}^2$$

Thermodynamic critical field (B_{cth})

at equilibrium g_s $|\boldsymbol{\Psi}_\infty|^2$ reaches a minimum

For the equilibrium value $|\boldsymbol{\Psi}_\infty|^2$ we must have also $\dfrac{dg_s(|\Psi_\infty|^2)}{d|\Psi|^2} = 0$

$g_s = g_n + \alpha|\psi|^2 + \frac{1}{2}\beta|\psi|^4$

$$\boldsymbol{\alpha + \beta|\Psi_\infty|^2 = 0}$$

$$\boldsymbol{n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta}}$$

$-\alpha|\Psi|^2 - \frac{1}{2}\beta|\Psi|^4 = \frac{1}{2\mu_0}B_{cth}^2$

$$\boldsymbol{B_{cth}^2 = \mu_0 \frac{\alpha^2}{\beta}}$$

Thermodynamic critical field (B_{cth})

$$n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta} \qquad\qquad B_{cth}^2 = \mu_0 \frac{\alpha^2}{\beta}$$

From the temperature dependences of **α** and **β**: $\alpha(T) = \alpha(0)\left(\frac{T}{T_c} - 1\right)$ $\quad\beta(T) = \beta = const$

We see that (near T_c): n_s and B_{cth} are proportional to $(1-T/T_c)$

$$n_s(T) = n_s(0)\left(1 - \frac{T}{T_c}\right) \qquad\qquad B_{cth}(T) = B_{cth}(0)\left(1 - \frac{T}{T_c}\right)$$

for $T \rightarrow T_c$ ⇒ $n_s \rightarrow 0$ ⇒ $B_{cth} \rightarrow 0$

Second order phase transition (1)

$$B_{cth}^2 = \mu_0 \frac{\alpha^2}{\beta} \qquad n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta}$$

By solving for α and β ➡ $\alpha = -\frac{1}{\mu_0}\frac{B_{cth}^2}{n_s}$ ➡ $\beta = \frac{1}{\mu_0}\frac{B_{cth}^2}{n_s^2}$

We can now demonstrate that a **second-order phase transition** is described by:

$$g_s = g_n + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4$$

<u>REMEMBER</u>: in a second-order phase transition, ***g*** *and* ***first derivate are continuous****, but* ***not the second derivate***

Second order phase transition (2)

$$g_n - g_s = -\alpha|\Psi|^2 - \frac{1}{2}\beta|\Psi|^4 \qquad \alpha = -\frac{1}{\mu_0}\frac{B_{cth}^2}{n_s} \qquad \beta = \frac{1}{\mu_0}\frac{B_{cth}^2}{n_s^2} \qquad B_{cth}(T) = B_{cth}(0)\left(1 - \frac{T}{T_c}\right)$$

g @T_c ➡ for $T = T_c$ ➡ $|\Psi|=0$ ➡ $g_s = g_n$ — **g passes through T_c continuously** ✓

1st derivate @T_c:

$$\frac{\partial g_s}{\partial T} = \frac{\partial g_n}{\partial T} - \frac{B_{cth}}{\mu_0}\frac{\partial B_{cth}}{\partial T} = \frac{\partial g_n}{\partial T} + \frac{B_{cth}^2(0)}{\mu_0}\left(1 - \frac{T}{T_c}\right)\frac{1}{T_c}$$

$\left(1 - \frac{T}{T_c}\right)$ **=0** @ $T=T_c$

$\partial g/\partial T$ passes through T_c continuously ✓

2nd derivate @T_c:

$$\frac{\partial g_s^2}{\partial T^2} = \frac{\partial g_n^2}{\partial T^2} - \frac{B_{cth}^2(0)}{\mu_0}\frac{1}{T_c^2}$$

$\partial^2 g/\partial T^2$ jump @ T_c ✓

Derivation of Ginzburg Landau equations

The crucial extension of the phenomenological description is achieved by the assumption (*ansatz*) of the **Gibbs function of the SC in a magnetic field** under the assumption of a **possible spatial variation of $\boldsymbol{\Psi}$**

$$g_s(B) = g_n + \alpha|\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta|\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0}|\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)\boldsymbol{\Psi}\right|^2$$

Derivation of Ginzburg Landau equations

$$g_s(B) = g_n + \alpha|\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta|\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0}|\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)\boldsymbol{\Psi}\right|^2$$

Derivation of Ginzburg Landau equations

$$g_s(B) = g_n + \alpha|\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta|\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0}|\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)\boldsymbol{\Psi}\right|^2$$

↓

Energy needed to **change the magnetic field from B_a** (external applied field) **to the value B_i** (field in the SC)

↓

In the **Meissner phase** → $B_i = 0$

↓

the term yields the **total energy to be supplied for the magnetic field expulsion**

Derivation of Ginzburg Landau equations

$$g_s(B) = g_n + \alpha|\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta|\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0}|\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)\boldsymbol{\Psi}\right|^2$$

$$\nabla\Psi = \mathrm{grad}\Psi = \frac{\partial\Psi}{\partial x}\boldsymbol{e}_x + \frac{\partial\Psi}{\partial y}\boldsymbol{e}_y + \frac{\partial\Psi}{\partial z}\boldsymbol{e}_z$$

For Cooper pairs → ***$m = 2m_e$*** and ***$q = |2e|$***

takes into account a possible **spatial variation of B_i and $\boldsymbol{\Psi}$** within the SC

It includes the **supercurrents leading to a variation of the magnetic field**

it contains the energy needed to establish a **spatial variation of the Cooper pair density**

Derivation of Ginzburg Landau equations

The Gibbs function for **the total SC sample** can be obtained **integrating over the volume V of the sample**

$$G_s = \int_V g_s \cdot \mathrm{d}V$$

$$G_s = \int_V \left\{ g_n + \alpha |\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta |\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0} |\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m} \left| \left(\frac{\hbar}{i} \nabla - q\mathbf{A} \right) \boldsymbol{\Psi} \right|^2 \right\} \cdot \mathrm{d}V$$

Derivation of Ginzburg Landau equations

$$G_s = \int_V \left\{ g_n + \alpha|\boldsymbol{\Psi}|^2 + \frac{1}{2}\beta|\boldsymbol{\Psi}|^4 + \frac{1}{2\mu_0}|\boldsymbol{B_a} - \boldsymbol{B_i}|^2 + \frac{1}{2m}\left|\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)\boldsymbol{\Psi}\right|^2 \right\} \cdot \mathrm{d}V$$

This function $\mathbf{G_s}$ must be **minimized by the variation of $\boldsymbol{\Psi}$ and A**

The variation then yields the **two equations of the Ginzburg-Landau theory**:

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathrm{A}\right)^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0$$

$$j_s = \frac{q\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{q^2}{m}|\Psi|^2\mathrm{A}$$

$\boldsymbol{\Psi}$ is the complex conjugate function to $\boldsymbol{\Psi}$*

Coming back to London

$$j_s = \frac{q\hbar}{2mi}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) - \frac{q^2}{m}|\Psi|^2\mathbf{A}$$

normalizing Ψ to the value Ψ_∞ (the equilibrium value of Ψ sufficiently far away from any interface)
with $\psi = \Psi / \Psi_\infty$:

$$j_s = \frac{q\hbar|\Psi_\infty|^2}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{q^2|\Psi_\infty|^2}{m}|\psi|^2\mathbf{A}$$

Remember the definition of London penetration depth: $\lambda_L = \sqrt{\frac{m}{\mu_0 n_s q^2}}$ and: $n_s = |\Psi_\infty|^2$

$$j_s = \frac{\hbar}{2iq}\frac{1}{\mu_0\lambda_L^2}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{1}{\mu_0\lambda_L^2}|\psi|^2\mathbf{A}$$

Coming back to London (2)

$$j_s = \frac{\hbar}{2iq}\frac{1}{\mu_0\lambda_L^2}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{1}{\mu_0\lambda_L^2}|\psi|^2\mathrm{A}$$

$\psi = \psi_0 e^{i\varphi}$

$$j_s = \psi_0^2\frac{\hbar}{q}\frac{1}{\mu_0\lambda_L^2}\nabla\varphi - \frac{1}{\mu_0\lambda_L^2}|\psi|^2\mathrm{A}$$

If the function $\boldsymbol{\Psi}$ is spatially constant, $\nabla\boldsymbol{\varphi}$ vanishes, and $|\ \boldsymbol{\psi}\ |^2$ is equal to 1

$$j_s = -\frac{1}{\mu_0\lambda_L^2}\mathrm{A}$$

we recover the London theory in the case of a **spatially constant Cooper pair density n_s**

However, the 2nd GL equation can take into account the supercurrents for a **spatially varying wave function**

Coherence length in GL Theory

Starting from the 1st GL equation:

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2 \Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0 \quad \Rightarrow \quad \frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2 \Psi + \alpha\Psi - \alpha\frac{|\Psi|^2}{|\Psi_\infty|^2}\Psi = 0$$

$$n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta}$$

We normalize Ψ to the value Ψ_∞ (with $\psi = \Psi / \Psi_\infty$):

$$\frac{1}{2m}\left(\frac{\hbar}{i}\nabla - q\mathbf{A}\right)^2 \psi + \alpha\psi - \alpha|\psi|^2\psi = 0$$

$$\frac{1}{\alpha} = \frac{1}{\alpha}$$

$$\frac{\hbar^2}{2m\alpha}\left(\frac{1}{i}\nabla - \frac{q}{\hbar}\mathbf{A}\right)^2 \psi + \psi - |\psi|^2\psi = 0$$

Coherence length in GL Theory

$$\frac{\hbar^2}{2m\alpha}\left(\frac{1}{i}\nabla - \frac{q}{\hbar}\mathbf{A}\right)^2\psi + \psi - |\psi|^2\psi = 0$$

Dimension of [lenght]²

$$\xi_{GL} = \sqrt{-\frac{\hbar^2}{2m\alpha}}$$

Ginzburg Landau Coherence lenght ξ_{GL}

$$-\xi_{GL}^2\left(\frac{\nabla}{i} - \frac{q}{\hbar}\mathbf{A}\right)^2\psi + \psi - |\psi|^2\psi = 0$$

Meaning of GL Coherence length

Imagine a simple situation in which the superconductor extends in the x direction (from x = 0 up to x → ∞)

No external magnetic field applied → A = 0

For x = 0 we assume | $\boldsymbol{\psi}$ | = 0

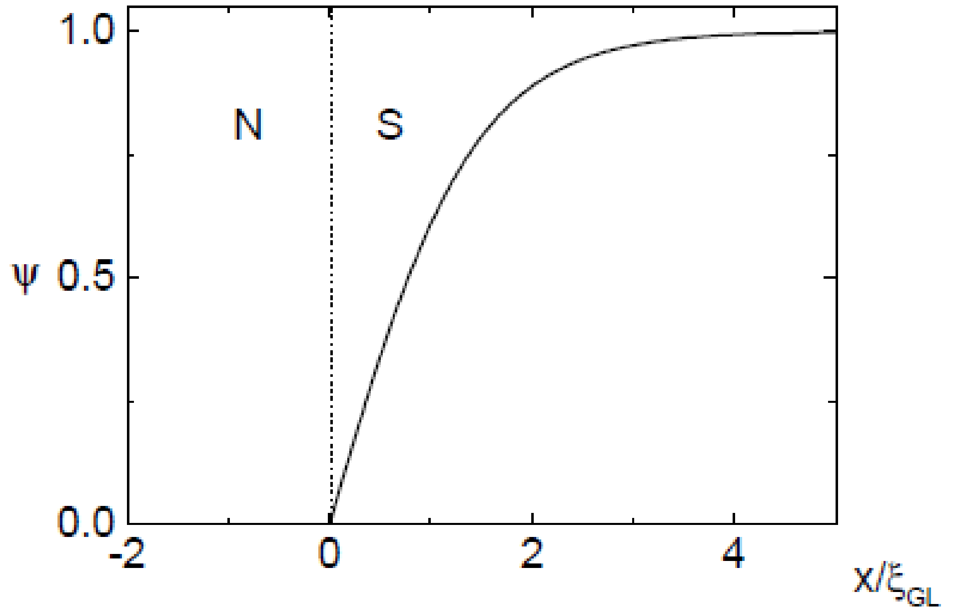
Then we can find a real solution for $\boldsymbol{\psi}$ from $-\xi_{GL}^2\left(\frac{\boldsymbol{\nabla}}{\boldsymbol{i}}-\frac{\boldsymbol{q}}{\hbar}\mathbf{A}\right)^2\boldsymbol{\psi}+\boldsymbol{\psi}-|\boldsymbol{\psi}|^2\boldsymbol{\psi}=\mathbf{0}$

Meaning of GL Coherence length (2)

$$-\xi_{GL}^2\left(\frac{\nabla}{i}-\frac{q}{\hbar}\mathbf{A}\right)^2\psi+\psi-|\psi|^2\psi=0 \quad\Longrightarrow\quad \xi_{GL}^2\frac{d^2\psi}{dx^2}+\psi-\psi^3=0$$

For x ≥ 0 this equation has the solution: $\psi(x)=tanh\left(\frac{x}{\sqrt{2\xi_{GL}}}\right)$

ξ_{GL} can be interpreted as the **characteristic length within which the order parameter ψ can change**



Characteristic Lengths of the GL Theory

London Penetration Depth λ_L

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s q^2}}$$

$$n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta}$$

Ginzburg Landau Coherence lenght ξ_{GL}

$$\xi_{GL} = \sqrt{-\frac{\hbar^2}{2m\alpha}}$$

Characteristic Lengths of the GL Theory (T)

London Penetration Depth λ_L

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s q^2}}$$

$$n_s = |\Psi_\infty|^2 = -\frac{\alpha}{\beta}$$

$$\alpha(T) = \alpha(0)\left(\frac{T}{T_c} - 1\right)$$

Ginzburg Landau Coherence lenght ξ_{GL}

$$\xi_{GL} = \sqrt{-\frac{\hbar^2}{2m\alpha}}$$

Characteristic Lengths of the GL Theory (T)

$$\lambda_L(T) = \frac{\lambda_L(0)}{\sqrt{1 - T/T_C}}$$ **London Penetration Depth λ_L**

$$\xi_{GL}(T) = \frac{\xi_{GL}(0)}{\sqrt{1 - T/T_C}}$$ **Ginzburg Landau Coherence lenght ξ_{GL}**

For T → T_c both quantities approach infinity

Remember that **Ginzburg-Landau theory is valid only in the limit of T close to T_c**

$$\kappa = \frac{\lambda_L}{\xi_{GL}}$$ **Ginzburg Landau Parameter**

κ is independent of the temperature and of the magnetic field

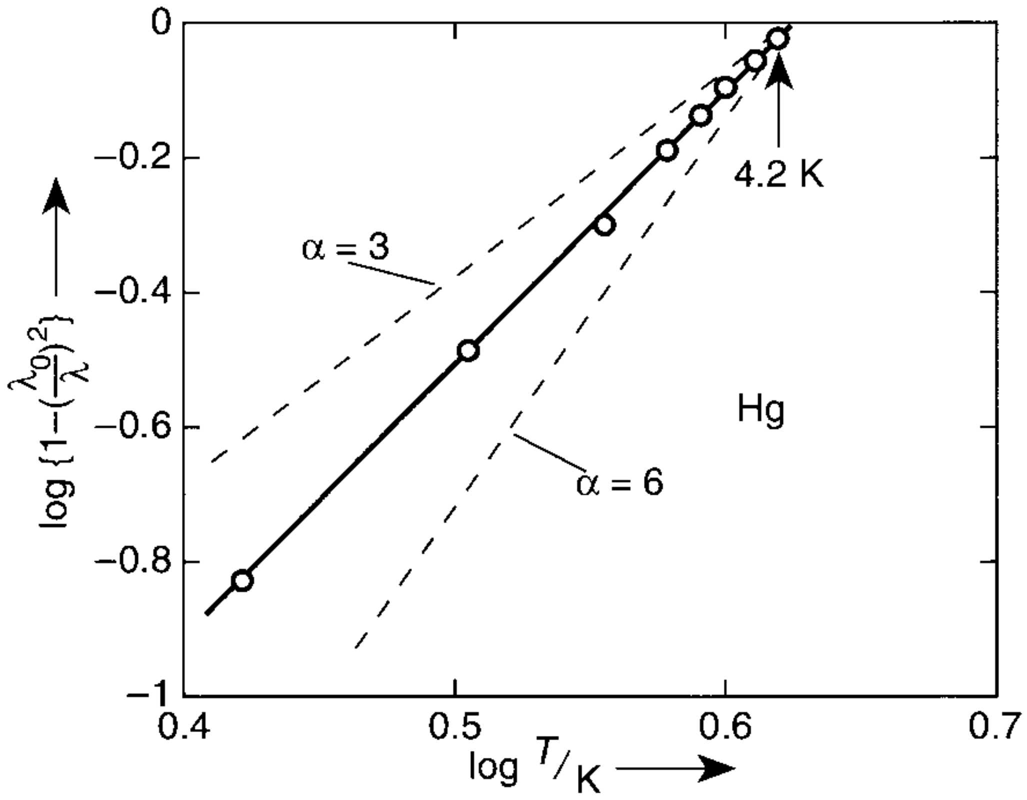
Experimental T dependence of λ_L

T dependence for conventional SC

can be approximated by the following **empirical expression**:

$$\frac{\lambda_L(T)}{\lambda_L(0)} \propto \left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2}$$

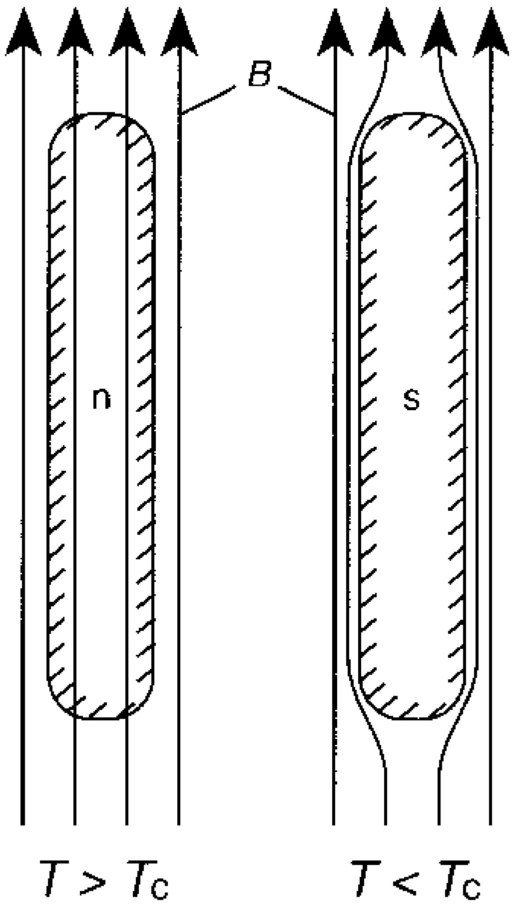
For **unconventional SC**, for which the energy gap has locations with value zero along certain crystal directions, the difference $\Delta\lambda_L = \lambda_L(T) - \lambda_L(0)$ increases from zero following a power law

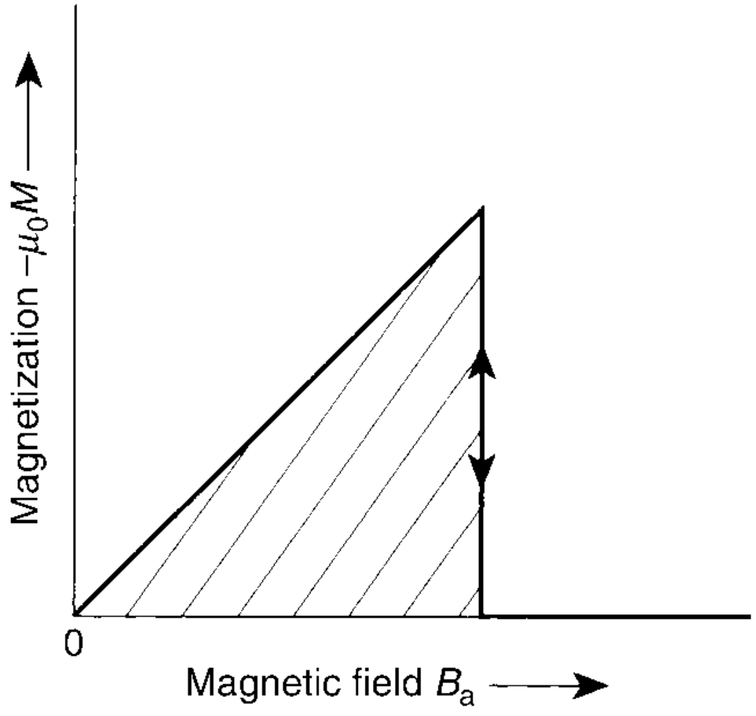


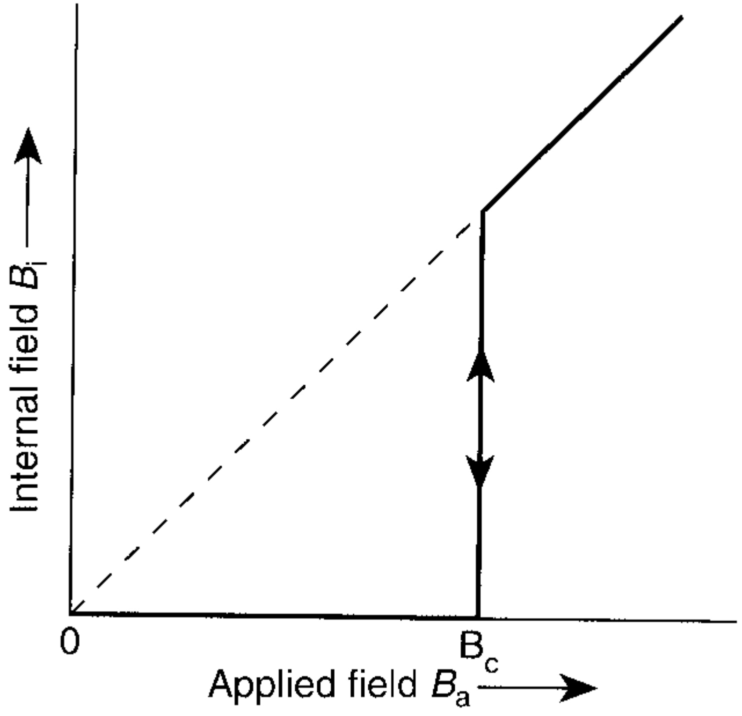
Temperature dependence of the penetration depth of Hg. The solid line corresponds to the exponent a = 4 in the bracket of the equation in this slide.
For comparison, the cases a = 3 and a = 6 are also shown by the dashed curves. (from Buckel-Kleiner book)

Type I Superconductor - geometric effects

Rod-shaped sample

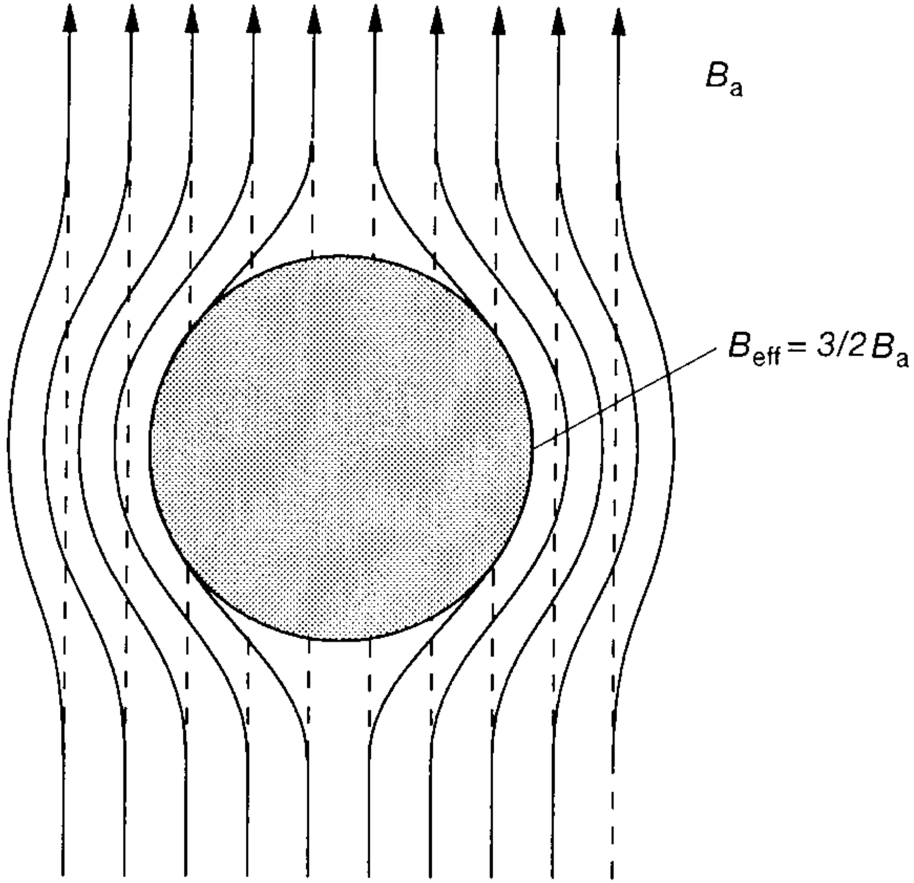


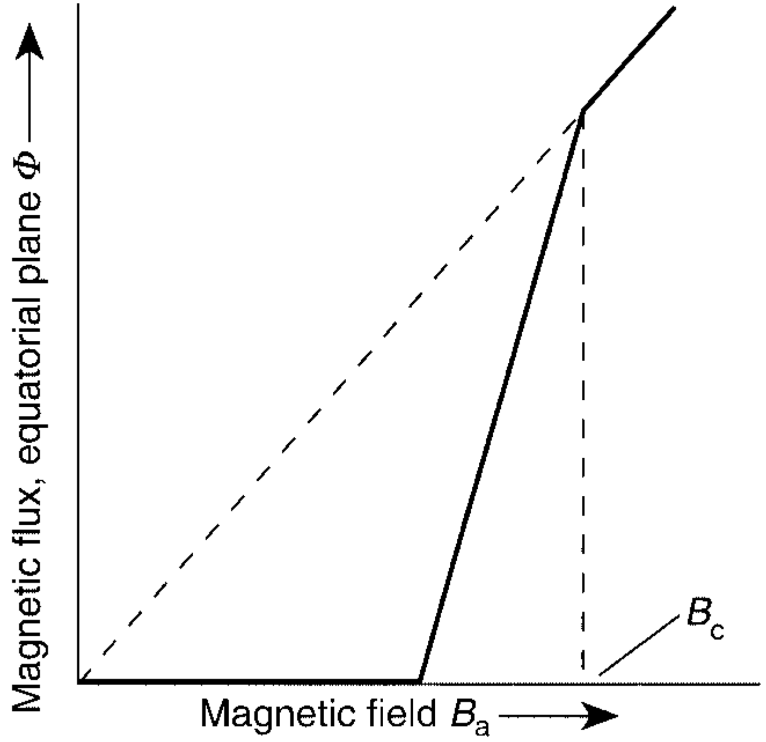




Type I Superconductor - geometric effects

Sphere sample





Intermediate state in Type I SC

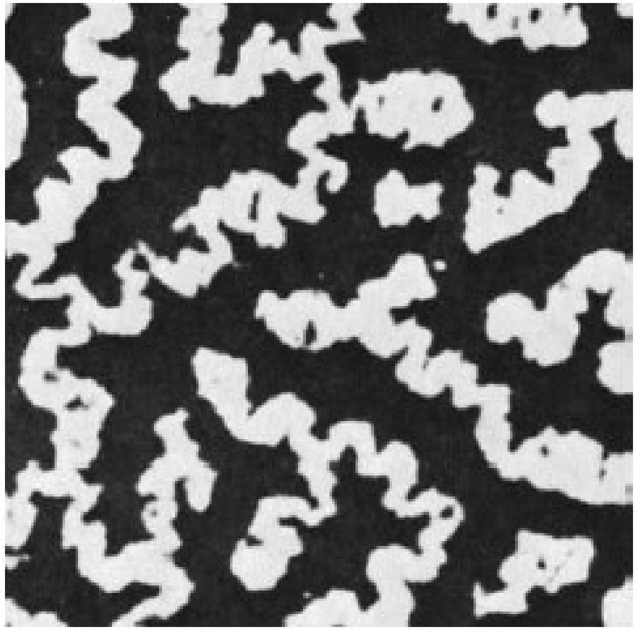


Fig. 4.16 Intermediate state structure of an indium plate. The dark regions represent the superconducting domains. In purity: 99.999 at.%; thickness: 11.7 mm; diameter: 38 mm; $B_a/B_{cth} = 0.1$; $T = 1.98$ K; T_c of indium: 3.42 K; transition N → S; magnification: 5×. Because of its high demagnetization coefficient, the plate enters the intermediate state already at $B_a/B_{cth} << 1$ [29].

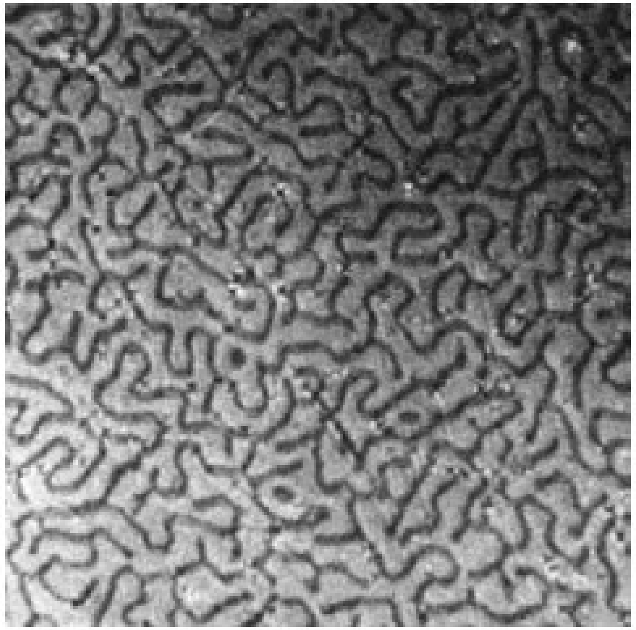


Fig. 4.17 Intermediate state structure imaged by means of the Faraday effect. Pb layer with 7 µm thickness; thickness of the magneto-optic film of EuS and EuF_2: about 100 nm; magnetic field $B_a = 0.77 B_c$ perpendicular to the layer. The dark regions represent the superconducting domains. The imaged area is about 0.5 mm × 0.5 mm [30]. (By countesy of Dr. Kirchner, Siemens, Munich).

The Wall Energy

Type I SC

a **positive wall energy** is associated to **generate a SC-NC interface**

Type II SC

generation of a SC-NC **interface** does **not require an expense of E**

The Wall Energy (2)

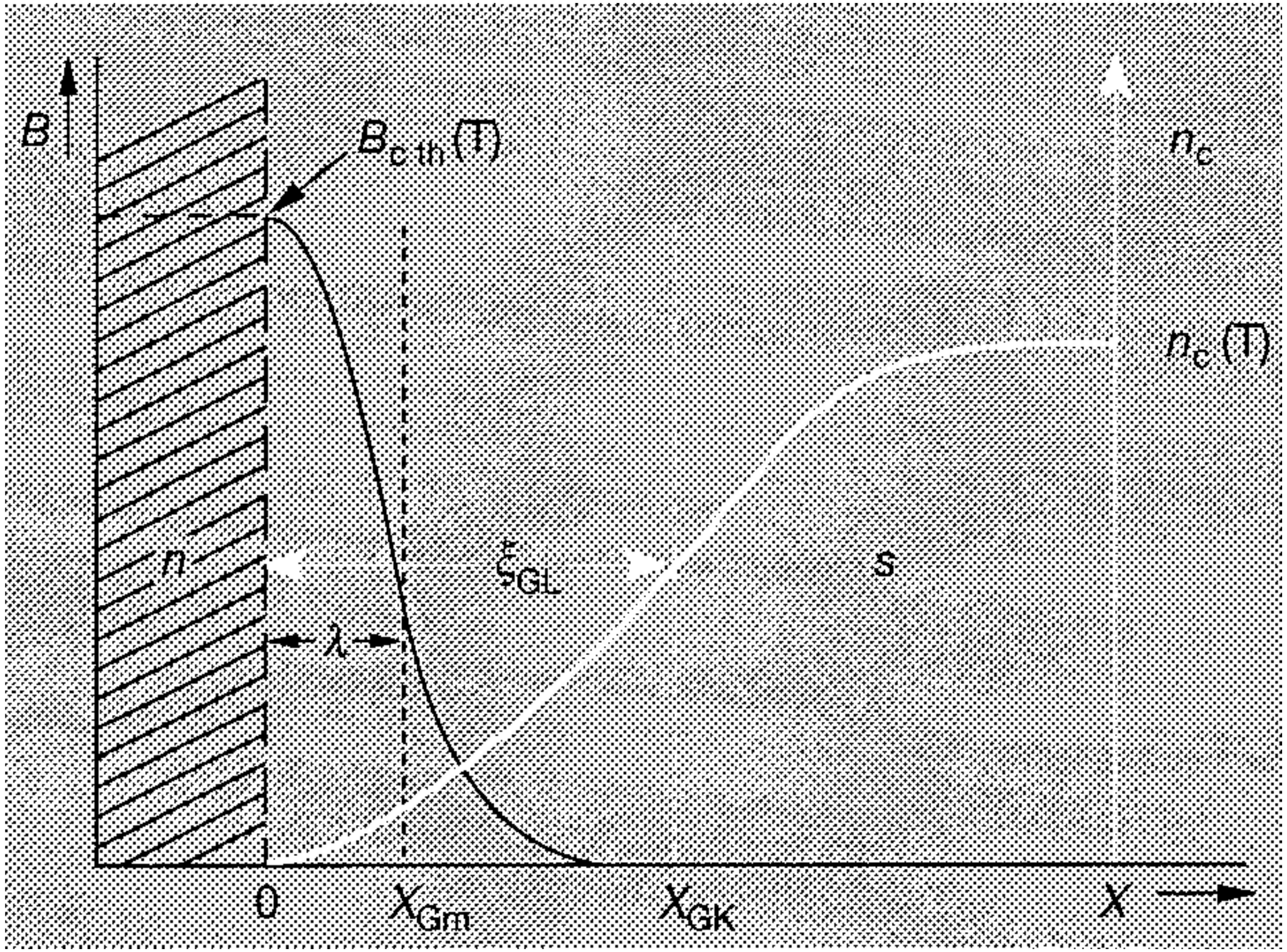
We assume:

- Homogeneous material
- SC thickness ≫ λ_L
- T costant

NC (x<0) → B > B_{cth}

SC (x>0) → B vanishes after ~ λ_L

- n_s(T) cannot drop discontinuously from n_s (T) to zero
- spatial variation of n_s (T) possible only for x > $\boldsymbol{\xi_{GL}}$



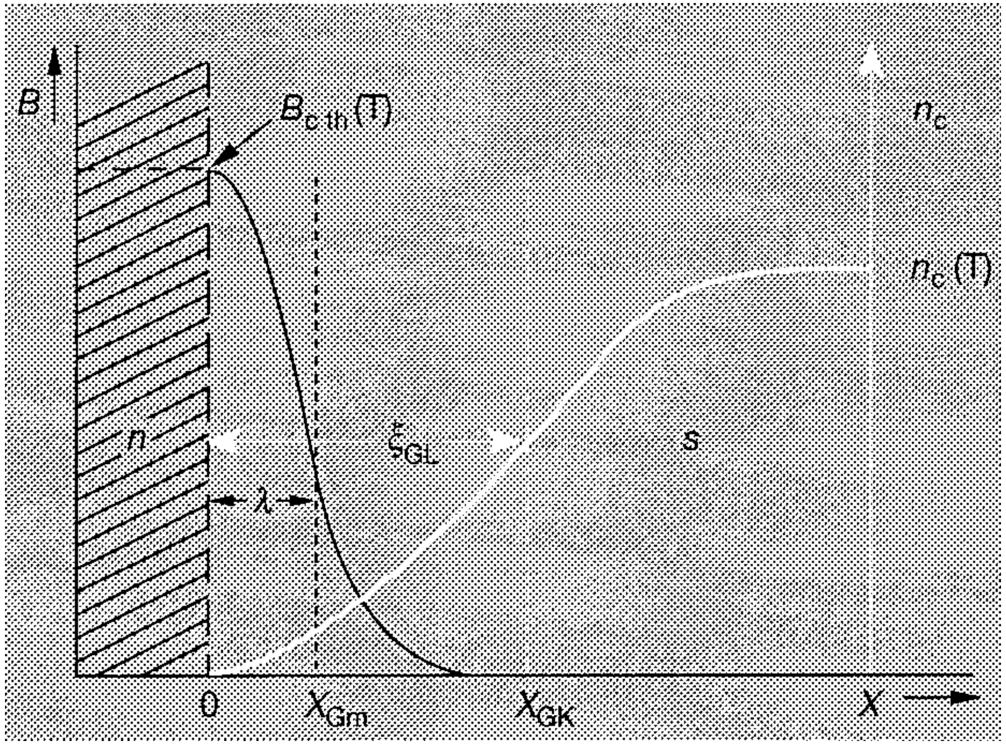
Spatial variation of *B* and n_s at an interface between a normal conducting and a superconducting domain within a homogeneous material at temperature *T*.
(Here x_{Gm} = "magnetic boundary", and x_{Gk} = "condensate boundary"). *(from Buckel-Kleiner book)*

The Wall Energy (3)

E_B = Energy associated with the expulsion of B

E_C = Energy gained because of the condensation into Cooper pairs

NC state is stabilized, since the **expulsion** of the magnetic field would **require more free enthalpy** than can be supplied by the **transition into the SC state**



The Wall Energy (4)

In the NC:

$E_B = E_C = 0$

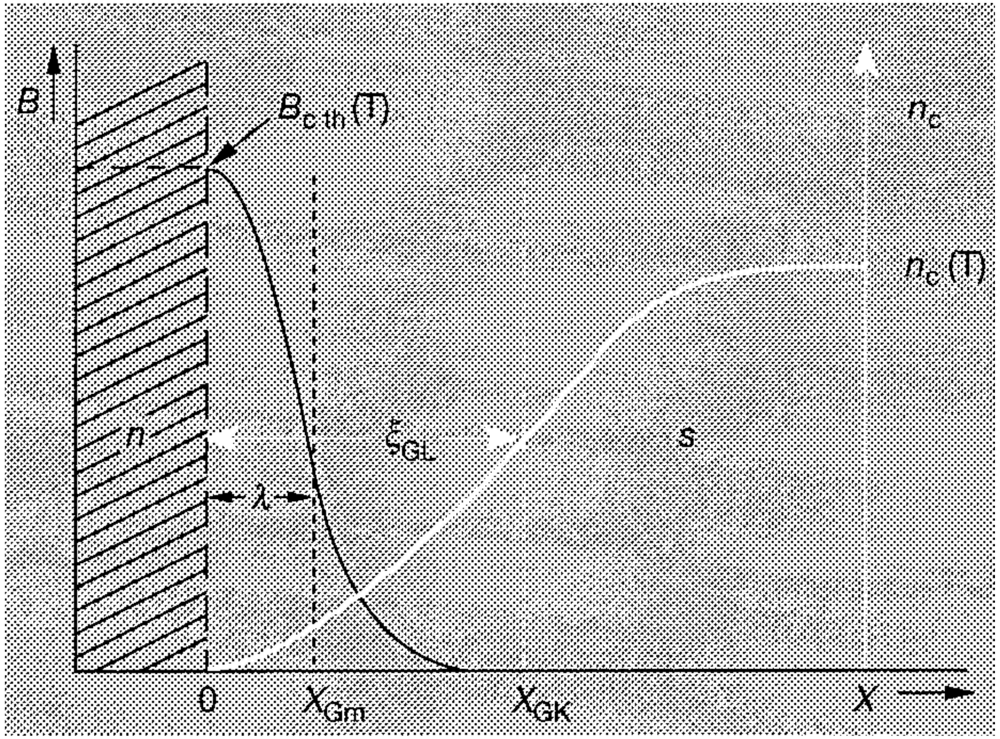
At the boundary:

$$E_B = E_C = \frac{1}{2\mu_0} B_{cth}^2 V$$

In the boundary layer:

Both E_B and E_C are reduced by the amount F (the area of the boundary layer) and the factors λ_L and ξ_{GL} respectively

$$\Delta E_B = F\lambda_L \frac{1}{2\mu_0} B_{cth}^2 \qquad \Delta E_C = F\xi_{GL} \frac{1}{2\mu_0} B_{cth}^2$$



Spatial variation of B and n_s at an interface between a normal conducting and a superconducting domain within a homogeneous material at temperature T. (Here x_{Gm} = "magnetic boundary", and x_{Gk} = "condensate boundary"). *(from Buckel-Kleiner book)*

The Wall Energy in TYPE I Superconductor

TYPE I Superconductor

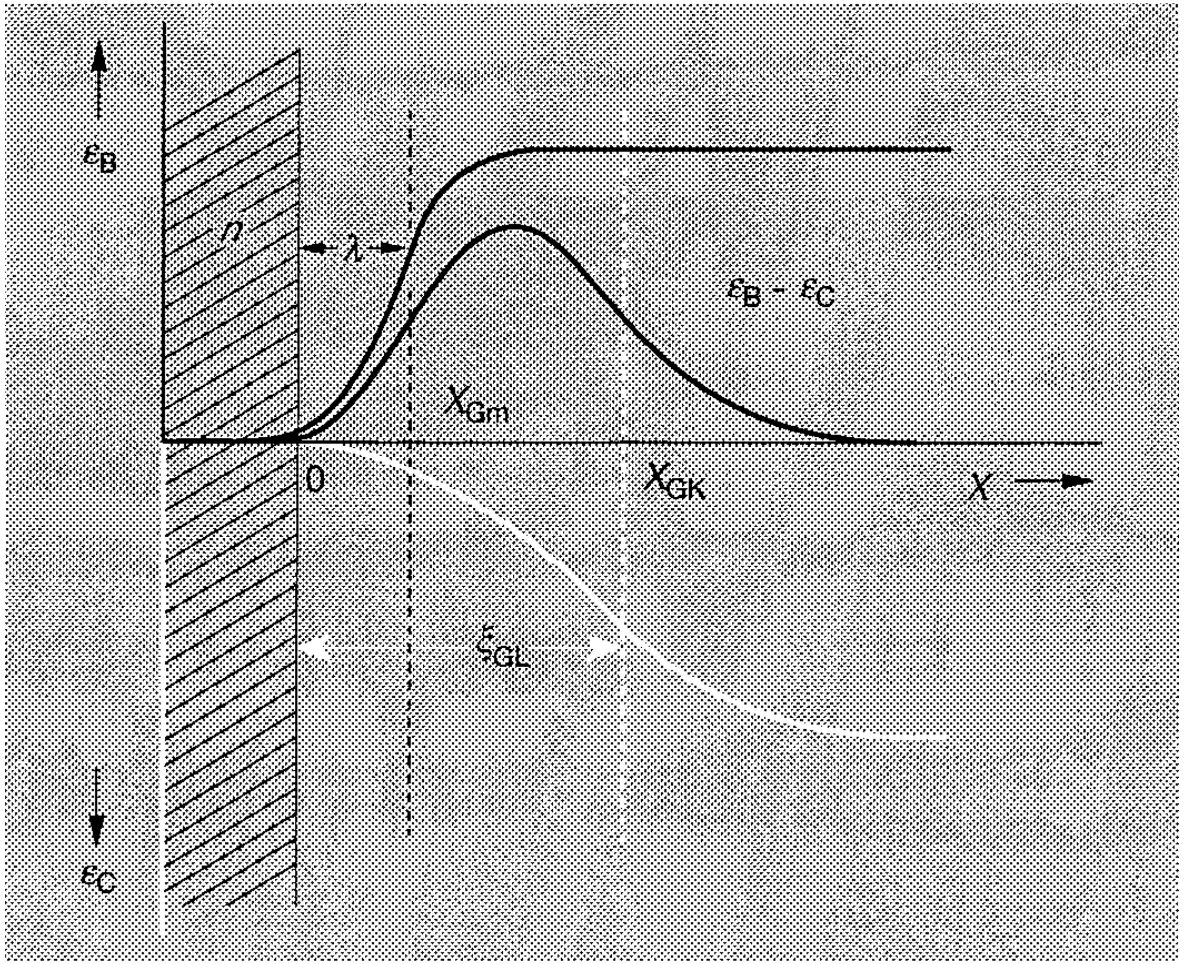
$\xi_{GL} > \lambda_L$

$$\Delta E_C - \Delta E_B = (\xi_{GL} - \lambda_L) F \frac{1}{2\mu_0} B_{cth}^2 > 0$$

Wall Energy α_W

$$\alpha_W = (\xi_{GL} - \lambda_L) F \frac{1}{2\mu_0} B_{cth}^2$$

Energy per unit area to generate a NC-SC boundary



The Wall Energy in TYPE II Superconductor

TYPE II Superconductor

$\xi_{GL} < \lambda_L$ +

2 Critical Magnetic Fields: B_{c1} and B_{c2}

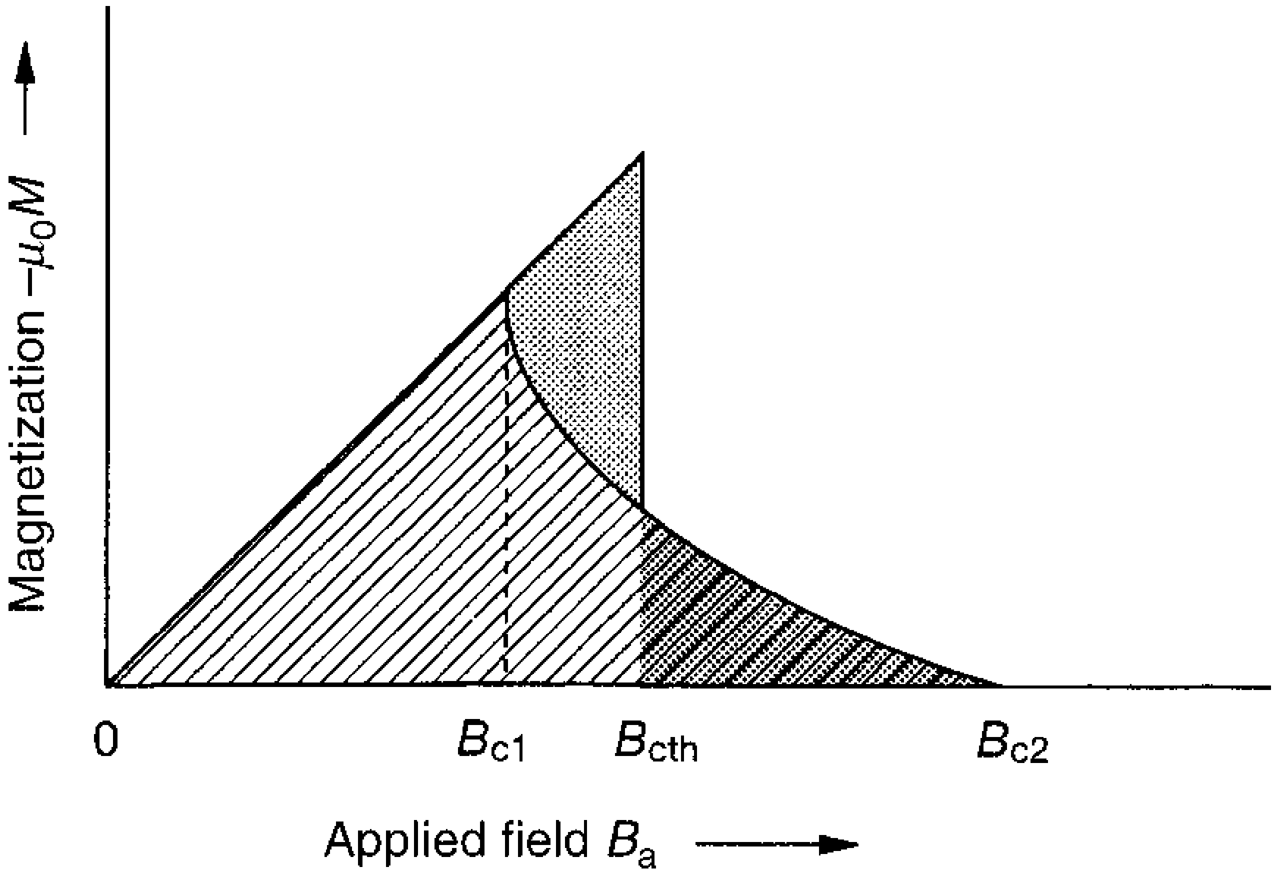
Magnetic field penetrate in the SC above B_{c1}

$$\Delta E_C - \Delta E_B = \xi_{GL} F \frac{1}{2\mu_0} B_{cth}^2 - \lambda_L F \frac{1}{2\mu_0} B^2 < 0$$

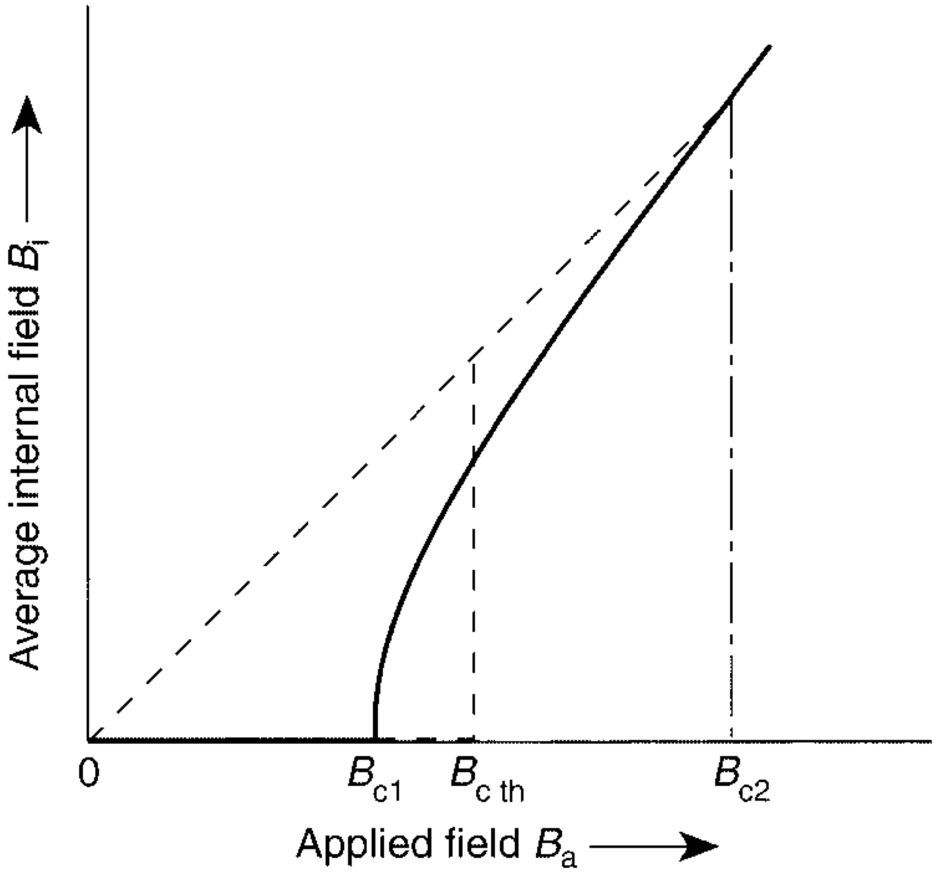
$$\xi_{GL} B_{cth}^2 < \lambda_L B^2 \Rightarrow \frac{B_{cth}^2}{B^2} < \frac{\lambda_L}{\xi_{GL}} \xrightarrow{\xi_{GL} < \lambda_L}$$

Magnetic field **can penetrate** in the SC at fields **$B < B_{CTH}$**

The Wall Energy in TYPE II Superconductor



Magnetization curve of a type-II superconductor. Rod-shaped sample with $N_M = 0$. Because of the definition of B_{cth}, the shaded areas must be equal



Average magnetic field in the interior of a type-II superconductor plotted versus the external field

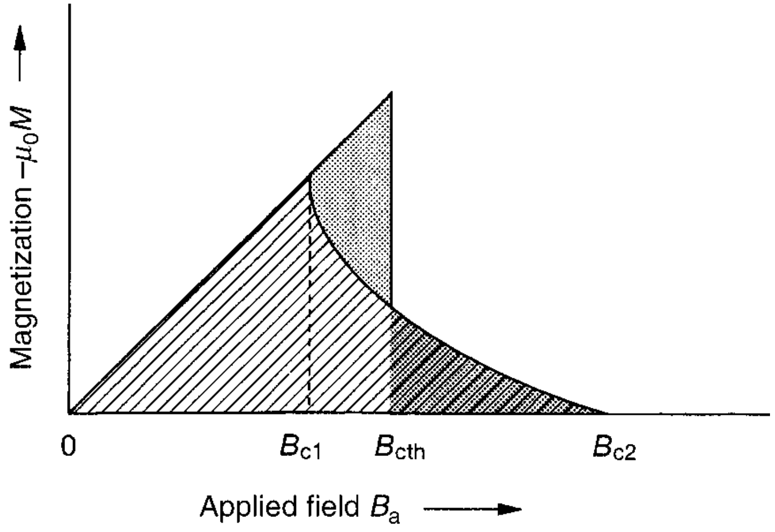
(from Buckel-Kleiner book)

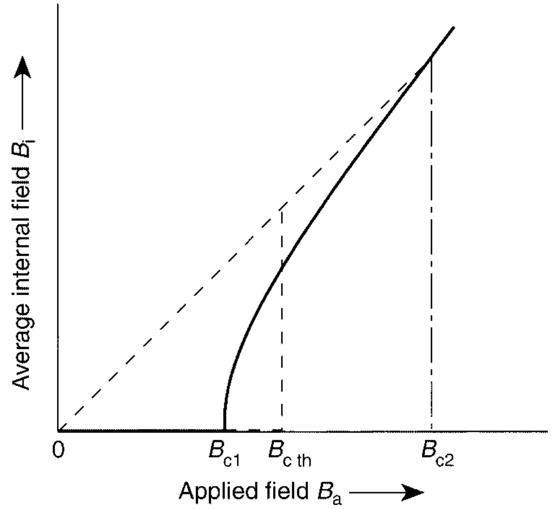
The Wall Energy in TYPE II Superconductor

In a **type II** superconductor **SC state remains at B > B_{cth}**

The **areas** under both magnetization curves must be **equal**

The “corresponding” **type-I SC has the same difference of the free enthalpies G_n–G_s as the type-II SC**

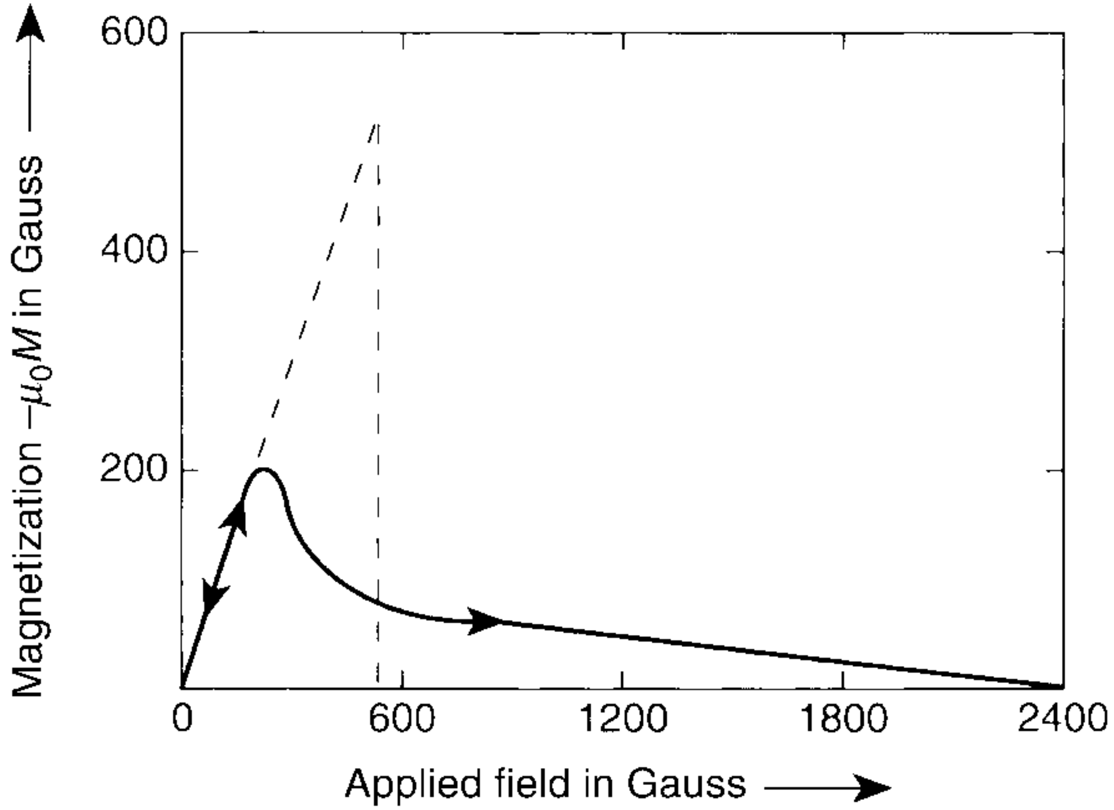




Turning TYPE I into TYPE II SC

From GL Theory we have that a **Type-I SC can turn into a type-II SC** if the **electron mean free path is reduced sufficiently**

Confirmed Experimentally



Magnetization curve of lead with 13.9 at.% of indium (solid line). Rod shaped sample with a small demagnetization coefficient. The dashed line shows the ideal curve of pure lead (1 G = 10–4 T.)

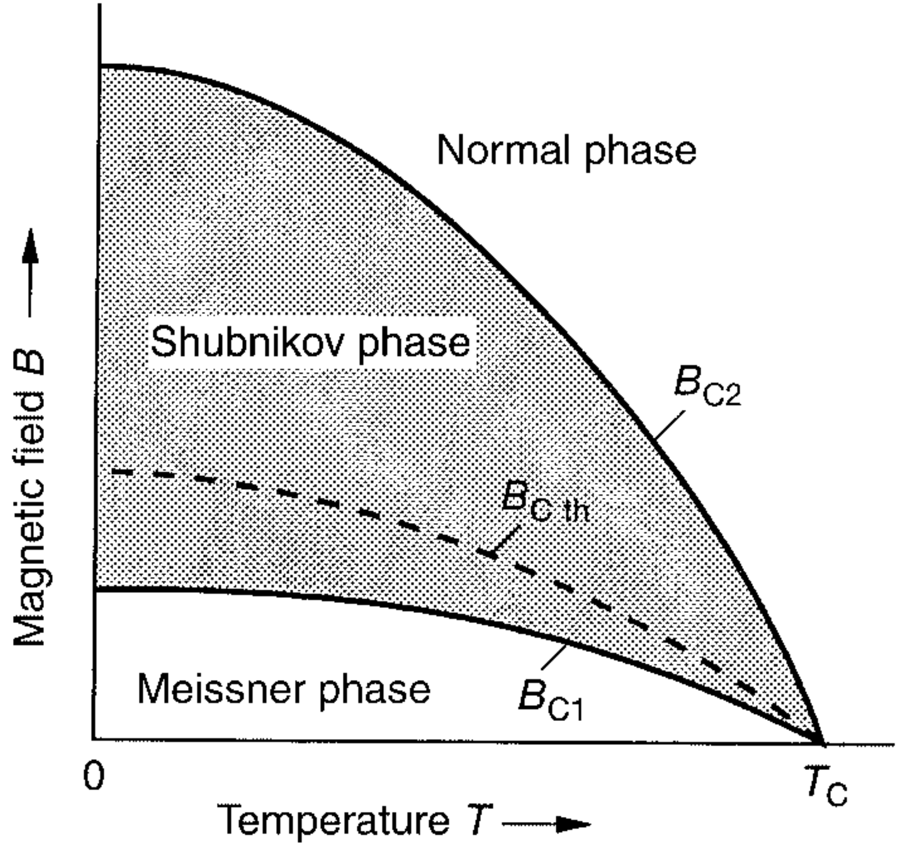
B_{c1} and B_{c2} expression in GL Theory

$$\kappa = \frac{\lambda_L}{\xi_{GL}}$$ **Ginzburg Landau Parameter**

$$B_{c2} = \sqrt{2}\kappa B_{cth}$$

$$B_{c1} = \frac{1}{2\kappa}(\ln\kappa + 0.08)B_{cth}$$

κ ↑ B_{c1} ↓ B_{c2} ↑



B_{c1} and B_{c2} are **temperature dependent** since are **related to B_{cth}**

B_{c1} and B_{c2} expression in GL Theory

$$\kappa = \frac{\lambda_L}{\xi_{GL}}$$ **Ginzburg Landau Parameter**

$$B_{c2} = \sqrt{2}\kappa B_{cth}$$

$$B_{c2} = \frac{\Phi_0}{2\pi\xi_{GL}^2}$$

$$B_{c1} = \frac{1}{2\kappa}(\ln \kappa + 0.08)B_{cth}$$

$$B_{c1} = \frac{\Phi_0}{4\pi\lambda_L^2}(\ln \kappa + 0.08)$$

Type I and Type II SC in GL Theory

In alloys the **mean free path ℓ decreases monotonically with increasing concentration of the impurities**

The "critical" concentration at which type-I turns into a type-II SC is defined by:

$$\boldsymbol{B_{c2} \geq B_{cth}}$$

$$B_{c2} = \sqrt{2}\kappa B_{cth}$$

$$\boldsymbol{\kappa \geq \frac{1}{\sqrt{2}}}$$

TYPE-I Superconductors $\kappa < \frac{1}{\sqrt{2}}$

TYPE-II Superconductors $\kappa \geq \frac{1}{\sqrt{2}}$

Ginzburg Landau Parameter

$$\kappa = \frac{\lambda_L}{\xi_{GL}}$$

Table 4.3 The κ_0 values* of superconducting elements.

Element	Al	In	Pb	Sn	Ta	Tl	Nb	V
κ_0 at T_c	0.03	0.06	0.4	0.1	0.35	0.3	0.8	0.85

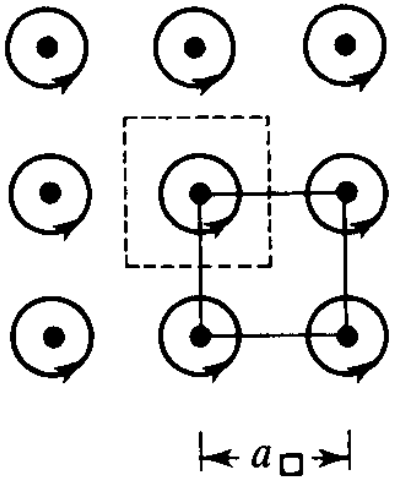
* See also "Superconductivity Data", H. Behrens, G. Ebel, eds., Fachinformationszentrum Energie, Physik, Mathematik Karlsruhe (1982).

Ordinate vortex lattice

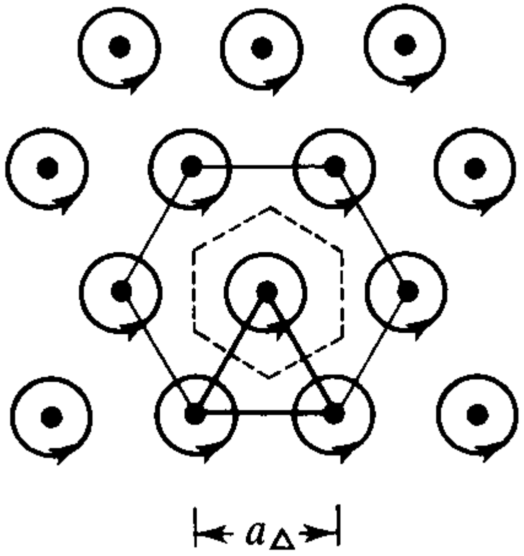
With complex calculation Abrikosov demonstrate that the **vortex arrange in ordinate pattern arrays**

$$a_{FL} = a_0 \sqrt{\frac{\Phi_0}{B}}$$

intervortex distance



$a_0 = 1$



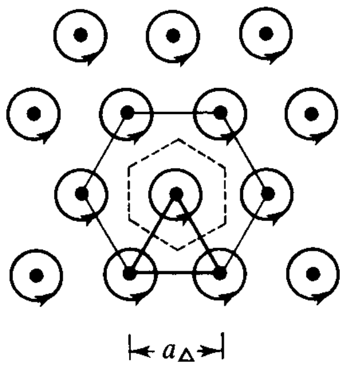
$a_0 = \sqrt[4]{4/3}$

(Thinkam, Introduction to Superconductivity, 2nd edition)

Triangular vortex lattice

The **hexagonal** (triangular) **lattice** has the **lowest energy** and gives a stable configuration

(as confirmed by experiments)



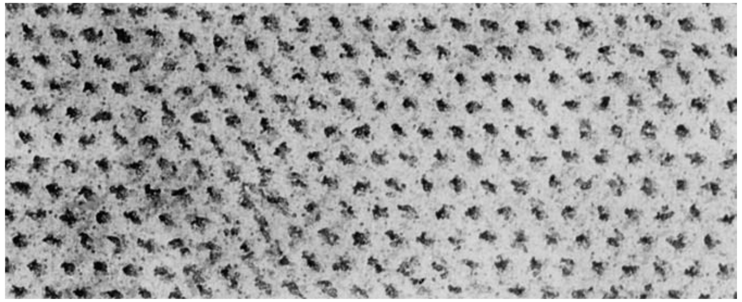
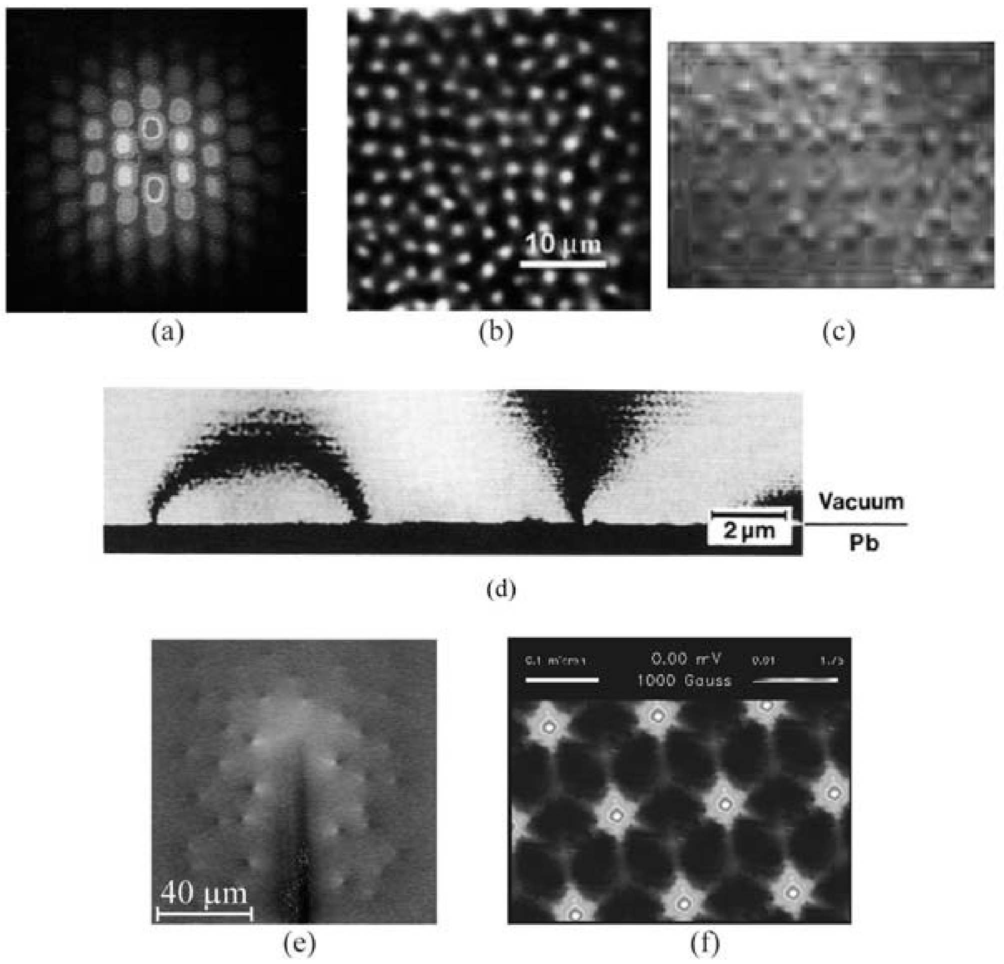


Image of the vortex lattice obtained with an electron microscope following the decoration with iron colloid. Frozen-in flux after the magnetic field has been reduced to zero. Material: Pb + 6.3 at.% In; temperature: 1.2 K;
sample shape: cylinder, 60 mm long, 4 mm diameter; magnetic field B*a* parallel to the axis. Magnification: 8300V. (Reproduced by courtesy of Dr. Essmann).



Methods for the imaging of flux lines. (a) Neutron diffraction pattern of the vortex lattice in niobium. (b) Magneto-optical image of vortices in $NbSe_2$. (c) Lorentz microscopy of niobium. (d) Electron holography of Pb. (e) Low-temperature scanning electron microscopy of $YBa_2Cu_3O_7$ (f) Scanning tunneling microscopy of $NbSe_2$.

(from Buckel-Kleiner book)

Relations between λ, ξ and H_{cth}

By combining the equations containing $\mathbf{H_{cth}}$, $\boldsymbol{\xi_{GL}}$ and $\boldsymbol{\lambda_L}$ we obtain the interesting result that:

$$H_{cth}\lambda_L\xi_{GL} = \frac{\Phi_0}{2\pi\sqrt{2\mu_0}}$$

Since the right-hand side is a constant, we see that on varying the temperature, the three quantities $\mathbf{H_{cth}}$, $\boldsymbol{\xi_{GL}}$ and $\boldsymbol{\lambda_L}$ are predicted to vary in such a way that their product remains fixed

Final remarks on GL Theory (1)

The GL theory pictures a SC as a flexible physical system responding to the applied currents and magnetic fields by adjusting its spatial distribution of order

The equilibrium configuration of order, current and field is the one that minimizes the total energy of the system

The theory provides two coupled equations with boundary condition giving the spatial distribution of the order parameter $\boldsymbol{\psi}$ and the vector potential **A** in terms of the GL parameter $\boldsymbol{\kappa}_{\mathrm{GL}}$ defined as the ratio between $\boldsymbol{\lambda_L}$ and $\boldsymbol{\xi_{GL}}$

For different values of $\boldsymbol{\kappa}_{\mathrm{GL}}$ there exist different thermodynamical states

Final remarks on GL Theory (2)

The importance of Ginzburg-Landau theory is incommensurable since it permits a deep understanding of the Type II superconductors and, by means of this, the development of all the technology using this kind of superconducting materials.

The great limit of the GL theory is that it is a local theory. That determines its failure at high frequencies and at temperatures far from the critical temperature.

Indeed, for T ~ Tc (that is the range of validity of GL theory) the penetration depth λ is larger than the coherence length ξ and the non-local aspect of the electronic interaction becomes of secondary importance. That is why GL theory describes Type II superconductors better than Type I.

Bibliography of this part

- W. Buckel, R. Kleiner, "*Superconductivity - Fundamentals and Applications*", Wiley

  4.4 Ginzburg-Landau Theory

  4.5 Characteristic Lengths of the Ginzburg-Landau Theory

  4.6 Type-I Superconductors in a Magnetic Field

  4.7-4.7.1 Type-II Superconductors in a Magnetic Field

- K. Fossheim, A. Sudbø, "*Superconductivity - Physics and applications*", Wiley

  4.7 The Ginzburg–Landau theory

  7.5 The length ξ and the upper critical field Bc2