lez. 02 : riassunto

rappresentazione di sistemi dinamici NON LINEARI

$\longrightarrow$ rappresentazione interna

linearizzazione $\sim$ approssimazione di un sistema non lineare con un sistema lineare intorno a una <u>certa condizione operativa</u>

$\llcorner$ punti di equilibrio

$\bar{x} \in \mathbb{R}^n \qquad x(t) = \bar{x} \qquad \forall\, t \geq 0$

- semplicemente stabili
- asintoticamente stabili

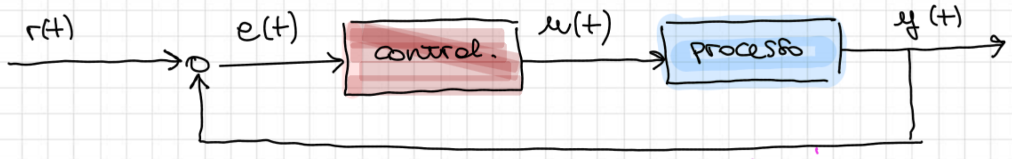
$\dot{x}(t) = f(x, u) \qquad \begin{matrix} x \in \mathbb{R}^n \\ u \in \mathbb{R}^m \end{matrix}$

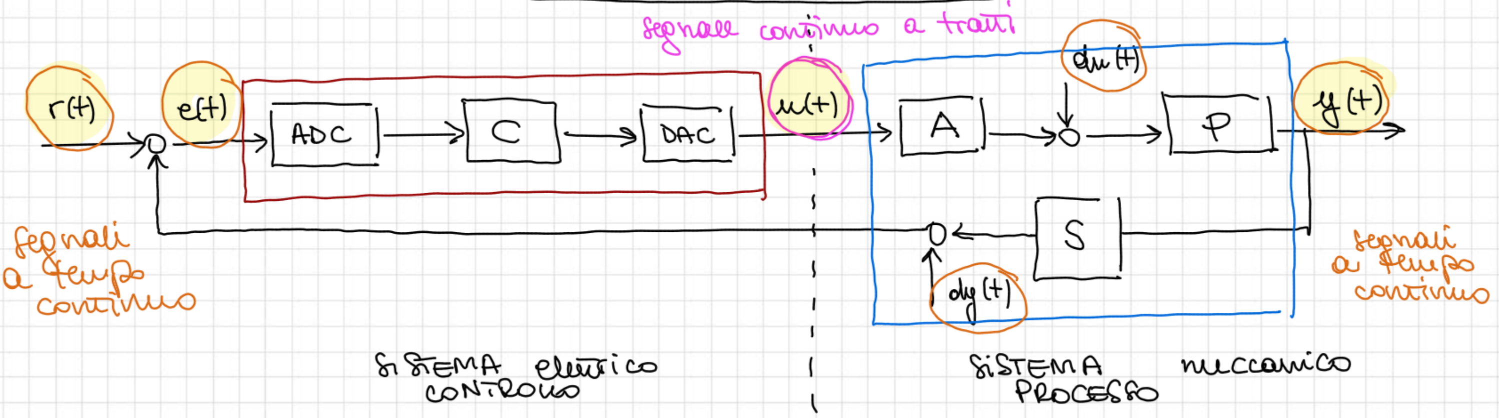
$\hookrightarrow (\bar{x}, \bar{u}) \quad : \quad \dot{\delta}_x = F_\delta\, \delta_x + G_\delta\, \delta_u \qquad \begin{matrix} \delta_x = x - \bar{x} \\ \delta_u = u - \bar{u} \end{matrix}$

$F_\delta = J_f^{(x)}(\bar{x}, \bar{u}) \in \mathbb{R}^{n \times n}$

$G_\delta = J_f^{(u)}(\bar{x}, \bar{u}) \in \mathbb{R}^{n \times m}$

MODELLIZZAZIONE di un SISTEMA DINAMICO REALE (controllato)



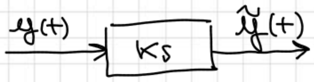


SISTEMA PROCESSO

P : processo vero e proprio – sistema dinamico descritto da equazioni integro/differenziali

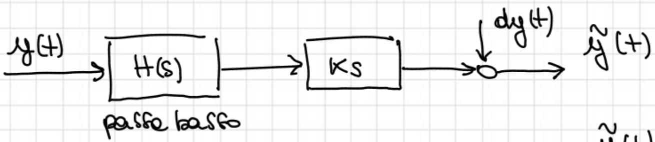
S : sensore – convertitore di una grandezza fisica in uscita dal processo in un segnale elettrico compatibile con il sistema di controllo

- modello ideale



$$\tilde{y}(t) = K_S \, y(t)$$

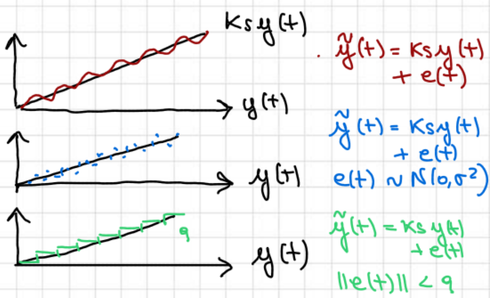
- modello reale



$$\tilde{y}(t) = H(S)\, K_S \, y(t) + d_y(t)$$

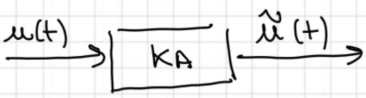
$$H(S) = \frac{1}{1+ST_b} \qquad T_b \propto \text{banda passante}$$

$d_y(t)$: combinazione di errori dovuti
- accuratezza
- precisione
- quantizzazione



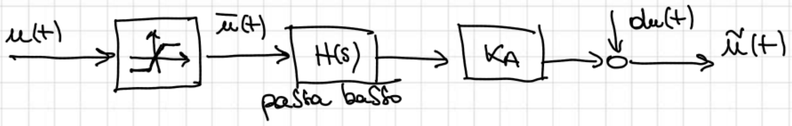
A : attuatore – convertitore di un segnale elettrico in uscita dal controllore in una grandezza fisica compatibile con il processo

- modello ideale



$$\tilde{u}(t) = K_A \, u(t)$$

- ~~mo~~ modello reale



$$\tilde{u}(t) = H(S)\, K_A \, \bar{u}(t) + d_u(t)$$

$$H(S) = \frac{1}{1+ST_b} \qquad T_b \propto \text{banda passante}$$

$d_u(t)$: combinazione di errori

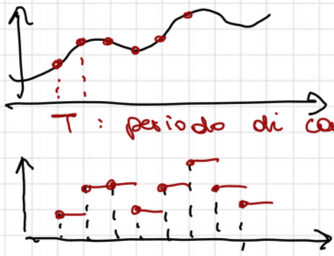
$$\bar{u}(t) = \begin{cases} u_{max} & u(t) \geq u_{max} \\ u(t) & u_{min} < u(t) < u_{max} \\ u_{min} & u(t) \leq u_{min} \end{cases}$$

SISTEMA CONTROLLO

C : controllore digitale

ADC : convertitore analogico - digitale

DAC : convertitore digitale - analogico



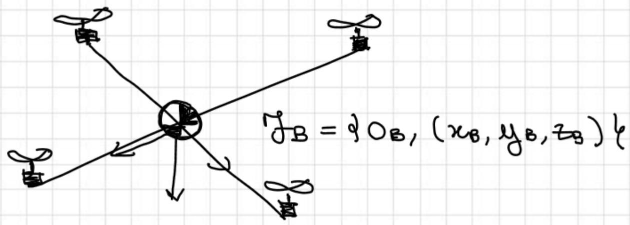
		$x(t)$: $\mathbb{R}$	$x(t)$: $\mathbb{Z}$
t	$\mathbb{R}^+$	segnale analogico	segnale a valori finiti
		ADC →	
t	$\mathbb{Z}^+$	segnale continuo a tratti	segnale digitale
		← DAC	

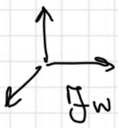
MODELLO del QUADROTOR

UAV - Unmanned Aerial Vehicle

4 attuatori = 4 propellers
= motore + disco

→ corpo rigido nello spazio 3D
- m : massa
- $J = \text{diag}(J_x, J_y, J_z)$





$\mathcal{F}_W = \{O_W, (x_W, y_W, z_W)\}$

NED : North East Down

$p \in \mathbb{R}^3$: posizione O_B in $\mathcal{F}_W$

$R \in SO(3)$: rotazione tra $\mathcal{F}_B$ e $\mathcal{F}_W$

$\delta \in \mathbb{R}^3$

per convenzione

$\mathcal{F}_W = \{O_W, (x_W, y_W, z_W)\}$ con $O_W = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$

$(x_W, y_W, z_W) = (e_1, e_2, e_3)$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\phi & -s\phi \\ 0 & s\phi & c\phi \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix}$$

$$R_z(\psi) = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = {}^{W}R_B = R_z(\psi) R_y(\theta) R_x(\phi)$$

$$\stackrel{!}{=} R(\psi, \theta, \phi) = R(\delta) \qquad \delta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \begin{matrix} \text{roll} \\ \text{pitch} \\ \text{yaw} \end{matrix}$$

- modello cinematico

$p \in \mathbb{R}^3$: $v \in \mathbb{R}^3$: velocità lineare in $\mathcal{F}_W$

$\eta \in \mathbb{R}^3$: $\omega \in \mathbb{R}^3$: velocità angolare in $\mathcal{F}_B$

$$\boxed{\begin{aligned} \dot{p} &= v \\ \dot{\eta} &= T\omega \end{aligned} \qquad T = \begin{bmatrix} 1 & s\phi\, t\theta & c\phi\, t\theta \\ 0 & c\phi & -s\phi \\ 0 & \frac{s\phi}{c\theta} & \frac{c\phi}{c\theta} \end{bmatrix} \in \mathbb{R}^{3\times 3}}$$

$$> \dot{R}(\eta) = \frac{\partial R}{\partial \eta} \cdot \dot{\eta} = \frac{\partial R}{\partial \phi}\dot{\phi} + \frac{\partial R}{\partial \theta} \cdot \dot{\theta} + \frac{\partial R}{\partial \psi} \cdot \dot{\psi} = \begin{bmatrix} \frac{\partial R}{\partial \phi} & \frac{\partial R}{\partial \theta} & \frac{\partial R}{\partial \psi} \end{bmatrix} \underbrace{\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}}_{\dot{\eta}}$$

$$> \dot{R} = R[\omega]_\times \quad \text{con} \quad [\omega]_\times = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

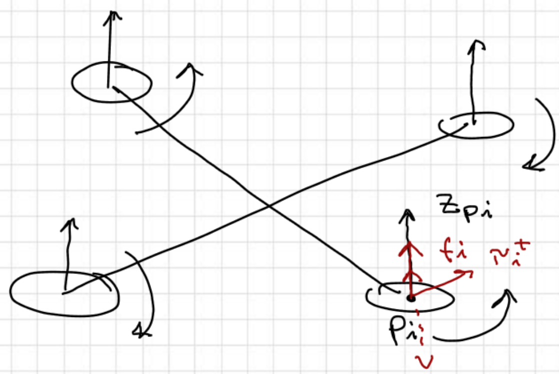
- modello dinamico

$z_{pi} \in \mathbb{R}^3$: asse di spinning

$z_{pi} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \forall i = 1 \dots 4$

$p_i \in \mathbb{R}^3$: com propeller in $\mathcal{F}_B$

$\omega_i \in \mathbb{R}$: velocità di spinning



ogni i-esimo attuatore genera

- forze di thrust : $f_i = c_f \cdot \omega_i^2 \cdot z_{pi} = c_f \cdot \omega_i^2 \cdot e_3 = \begin{bmatrix} 0 \\ 0 \\ c_f \cdot \omega_i^2 \end{bmatrix}$
- coppie di thrust : $\tau_i^t = c_f \cdot \omega_i^2 (p_i \times z_{pi}) = \begin{bmatrix} c_f \cdot \omega_i^2 p_{i2} \\ -c_f \omega_i^2 p_{i1} \\ 0 \end{bmatrix}$
- coppie di drag : $\tau_i^d = \pm c_\tau \omega_i^2 z_{pi} = \begin{bmatrix} 0 \\ 0 \\ \pm c_\tau \omega_i^2 \end{bmatrix}$

nel com del quadrotor agisce

- forze di controllo $f_c \in \mathbb{R}^3$: $f_c = \sum_{i=1}^4 f_i = \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^4 c_f \cdot \omega_i^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ F \end{bmatrix}$
  $\mathcal{F}_B$
- coppie di controllo $\tau_c \in \mathbb{R}^3$: $\tau_c = \sum_{i=1}^4 (\tau_i^d + \tau_i^t) = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$
  $\mathcal{F}_B$

$$\boxed{\begin{aligned} m\dot{v} &= -mge_3 + R(\eta) f_c \\ J\dot{\omega} &= -\omega \times J\omega + \tau_c \end{aligned}}$$

LINEARIZZAZIONE

hovering statico: $p = \bar{p}$, $\delta = \bar{\delta}$, $v = 0$, $\omega = 0$
$(\bar{\delta} = 0)$

$$x = \begin{bmatrix} p \\ \delta \\ v \\ \omega \end{bmatrix} \in \mathbb{R}^{12} \qquad u = \begin{bmatrix} F \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \|f_c\| \\ \tau_c \end{bmatrix} \in \mathbb{R}^4$$

modello non lineare cinematico + dinamico

$$\dot{x} = \begin{bmatrix} \dot{p} \\ \dot{\delta} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \underbrace{\begin{bmatrix} v \\ T\omega \\ -ge_3 + m^{-1}R(\delta)f_c \\ J^{-1}(-\omega \times J\omega + \tau_c) \end{bmatrix}}_{f(x,u)} = \underbrace{\begin{bmatrix} v \\ T\omega \\ -ge_3 \\ J^{-1}(-\omega \times J\omega) \end{bmatrix}}_{f_1(x)} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ m^{-1}R(\delta)f_c \\ J^{-1}\tau_c \end{bmatrix}}_{f_2(x,u)}$$

$f_1(x)$: dipendenza NON LINEARE dallo stato

$f_2(x,u)$: dipendenza NON LINEARE dallo stato ma LINEARE dall'ingresso

$$f_2(x,u) = \begin{bmatrix} \quad \\ \quad \\ \quad \end{bmatrix} \begin{bmatrix} F \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = F_2(x)\,u$$

$$\bar{x} = \begin{bmatrix} \bar{p} & 0 & 0 & 0 \end{bmatrix}$$
$$\bar{u} = \begin{bmatrix} mg & 0 & 0 & 0 \end{bmatrix}$$

$$F = J_f^{(x)}(\bar{x},\bar{u}) = \left.\frac{\partial f_1(x)}{\partial x}\right|_{\substack{x=\bar{x}\\u=\bar{u}}} + \left.\frac{\partial f_2(x,u)}{\partial x}\right|_{\substack{x=\bar{x}\\u=\bar{u}}}$$

$$= \begin{bmatrix} 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \\ 0 & F_{32} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad F_{32} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial \dot{p}}{\partial p} & \frac{\partial \dot{p}}{\partial \delta} & \frac{\partial \dot{p}}{\partial v} & \frac{\partial \dot{p}}{\partial \omega} \\ \frac{\partial \dot{\delta}}{\partial p} & \frac{\partial \dot{\delta}}{\partial \delta} & \frac{\partial \dot{\delta}}{\partial v} & \frac{\partial \dot{\delta}}{\partial \omega} \\ \frac{\partial \dot{v}}{\partial p} & \frac{\partial \dot{v}}{\partial \delta} & \frac{\partial \dot{v}}{\partial v} & \frac{\partial \dot{v}}{\partial \omega} \\ \frac{\partial \dot{\omega}}{\partial p} & \frac{\partial \dot{\omega}}{\partial \delta} & \frac{\partial \dot{\omega}}{\partial v} & \frac{\partial \dot{\omega}}{\partial \omega} \end{bmatrix} \quad — \quad T(\delta) = T(0) = I$$

$$f_2(x,u) = F_2(x)\,u$$

$$G = J_f^{(u)}(\bar{x},\bar{u}) = \left.\frac{\partial f_1(x)}{\partial u}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}} + \left.\frac{\partial f_2(x,u)}{\partial u}\right|_{\substack{x=\bar{x}\\ u=\bar{u}}}$$

$$= 0 + F_2(x)\Big|_{\substack{x=\bar{x}\\ u=\bar{u}}}$$

$$= \begin{bmatrix} 0 \\ 0 \\ G_3 \\ G_4 \end{bmatrix}$$

$$12 \times 4$$

$$G_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/m & 0 & 0 & 0 \end{bmatrix}$$

$$G_4 = \begin{bmatrix} 0 & J_x^{-1} & 0 & 0 \\ 0 & 0 & J_y^{-1} & 0 \\ 0 & 0 & 0 & J_z^{-1} \end{bmatrix}$$

$$\boxed{\dot{x} = Fx + Gu}$$