LCD (05/03/2024)

* Calculus of communicating systems

toy example

$$\begin{aligned} CS &= \overline{pub}.CS1 \\ CS1 &= \overline{coin}.CS2 \\ CS2 &= coffee.CS \end{aligned} \qquad \begin{aligned} CM &= coin.CM1 \\ CM1 &= \overline{coffee}.CM \end{aligned}$$

$$\begin{tikzcd} Off = (CS \mid CM) \setminus L \arrow[d, "\overline{pub}"] \\ (CS1 \mid CM) \setminus L \arrow[d, "\tau"] \\ (CS2 \mid CM1) \arrow[uu, bend left=60, "\tau"] \end{tikzcd} \qquad L = \{coin, coffee\}$$

$$Spec = \overline{pub}.Spec$$

$$Off \approx Spec$$

- → Syntax
- → Operational behaviour
- → Program equivalence

Syntax

set of channel / ports (infinite denumerable)

$\mathcal{A}$ $a, b, c, \ldots$

$\overline{\mathcal{A}} = \{ \bar{a} \mid a \in \mathcal{A} \}$

$\mathcal{L} = \mathcal{A} \cup \overline{\mathcal{A}}$ (interactions input /output)

$Act = \mathcal{L} \cup \{\tau\}$ $\tau \notin \mathcal{L}$

process constants

K $\ni K$

CCS processes

$$P, Q ::= K \quad \mid \quad \alpha . P \quad \mid \quad \sum_{i \in I} P_i \quad \mid \quad P \mid Q$$

$K \stackrel{def}{=} P$

$(K \in \mathcal{K})$

$\alpha \in Act$

(it can be τ)

I possibly infinite

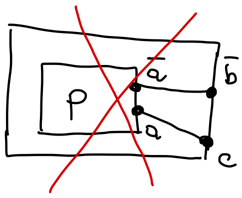
$$\mid \quad P[f] \qquad \qquad \mid \quad P \setminus L$$

$f : Act \to Act$

$f(\tau) = \tau$

$f(a) = b \implies f(\bar{a}) = \bar{b}$

$L \subseteq \mathcal{A}$



Remarks

→ inaction missing ?

$$0 \rightsquigarrow \sum_{i \in \emptyset} P_i$$

→ non-deterministic choice

$$P_1 + P_2 \longrightarrow \sum_{i \in \{1,2\}} P_i$$

$$\cong$$

$$P_2 + P_1$$

→ relabelling

$$\begin{array}{ccc} a_1 & \dots & a_n \\ \downarrow & & \downarrow \\ b_1 & \dots & b_n \end{array}$$

$$P[b_1/a_1, \dots, b_n/a_n] \rightsquigarrow P[f]$$

$f(\tau) = \tau$

$f(a_i) = b_i \qquad f(\bar{a}_i) = \bar{b}_i$

$f(x) = x \qquad \forall x \in \mathcal{L} \setminus \{a_i, \bar{a}_i \mid i = 1, \dots, n\}$

- restriction

$P \setminus \{a, b\}$ $\qquad$ $P \setminus a$

- priority

highest $\quad \_ \setminus \_ \qquad \_[f] \qquad \alpha.\_ \qquad \_ \mid \_ \qquad \_ + \_ \quad$ lowest

$$a.0 + (b.0 \mid c.0)$$

* Operational Behaviour

$$P \xrightarrow{\alpha} P'$$

syntax driven rules (structural operational semantics , Plotkin '81)

$$\frac{\dfrac{}{P_1 \xrightarrow{\alpha_1} P_1'} \qquad \dfrac{}{P_2 \xrightarrow{\alpha_2} P_2'}}{P_1 \text{ op } P_2 \xrightarrow{\alpha} P'}$$

depending on α_1, α_2

depends on P_1', P_2'

CCS Rules

* ACT

$$\frac{}{\alpha.P \xrightarrow{\alpha} P}$$

example

$$\frac{}{\overline{pub}.CS1 \xrightarrow{\overline{pub}} CS1}$$

* <u>SUM</u>

$$\frac{P_J \xrightarrow{\alpha} P_J'}{\sum_{i\in I} P_i \xrightarrow{\alpha} P_J'} \quad J \in I$$

$$\left(\frac{P_1 \xrightarrow{\alpha} P_1'}{P_1 + P_2 \longrightarrow P_1'} \quad \frac{P_2 \xrightarrow{\alpha} P_2'}{P_2 + P_2 \longrightarrow P_2'} \right)$$

$$\cancel{\frac{}{\sum_{i\in I} P_i \longrightarrow P_J}}$$

<u>Example</u>

$$\text{SUM}\ \dfrac{\text{ACT}\ \dfrac{}{\overline{coffee}.CTM \xrightarrow{\overline{coffee}} CTM}}{\overline{coffee}.CTM + \overline{tea}.CTM \xrightarrow{\overline{coffee}} CTM}$$

$$\text{SUM}\ \dfrac{\text{ACT}\ \dfrac{}{\overline{tea}.CTM \xrightarrow{\overline{tea}} CTM}}{\overline{coffee}.CTM + \overline{tea}.CTM \xrightarrow{\overline{tea}} CTM}$$

$$BC = \overline{tick}.(BC + 0)$$

$$(BC + 0 \overset{?}{\approx} BC)$$

$$\frac{0 \not\longrightarrow}{BC + 0 \longrightarrow}$$

$$BC = \overline{tick}.BC + \overline{tick}.0$$

$$\dfrac{\text{ACT}\ \dfrac{}{\overline{tick}.0 \xrightarrow{\overline{tick}} 0}}{\overline{tick}.BC + \overline{tick}.0 \xrightarrow{\overline{tick}} 0}$$

* <u>Parallel composition</u>

$$\frac{P \xrightarrow{\alpha} P'}{P \mid Q \xrightarrow{\alpha} P' \mid Q} \qquad \frac{Q \xrightarrow{\alpha} Q'}{P \mid Q \xrightarrow{\alpha} P \mid Q'} \qquad \frac{P \xrightarrow{a} P' \quad Q \xrightarrow{\overline{a}} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'}$$

notation

$$\alpha = \overline{a} \qquad \overline{\alpha} = \overline{\overline{a}} = a$$

$$\frac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM} \qquad \dfrac{}{coffee.CS \xrightarrow{coffee} CS}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\tau} CM \mid CS}$$

$$\frac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\overline{coffee}} CM \mid coffee.CS}$$

* <u>RES</u>

$$\frac{P \xrightarrow{\alpha} P'}{P \setminus L \xrightarrow{\alpha} P' \setminus L} \qquad \alpha, \overline{\alpha} \notin L$$

$L = \{ coin, coffee \}$

$$\frac{\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM} \qquad \dfrac{}{coffee.CS \xrightarrow{coffee} CS}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\tau} CM \mid CS}}{(\overline{coffee}.CM \mid coffee.CS) \setminus L \xrightarrow{\tau} (CM \mid CS) \setminus L}$$

$$\frac{\dfrac{\dfrac{}{\overline{coffee}.CM \xrightarrow{\overline{coffee}} CM}}{\overline{coffee}.CM \mid coffee.CS \xrightarrow{\overline{coffee}} CM \mid coffee.CS}}{(\overline{coffee}.CM \mid coffee.CS) \setminus L \;\cancel{\xrightarrow{\overline{coffee}}}}$$

$\cancel{coffee \notin L}$?

<u>Example</u>

$(\overline{coffee}.CTM + \overline{tea}.CTM \mid coffee.CS) \setminus \{coffee, tea\} \xrightarrow{\tau} (CTM \mid CS)$

external choice

$(\tau.\overline{coffee}.CTM + \tau.\overline{tea}.CTM \mid coffee.CS) \setminus \{coffee, tea\} \xrightarrow{\tau} (\overline{tea}.CTM \mid coffee.CS) \setminus \{coffee, tea\}$

a = random()
if a > 3

internal choice