Superconductive Materials

Part 2

Fundamental Properties of Superconductivity

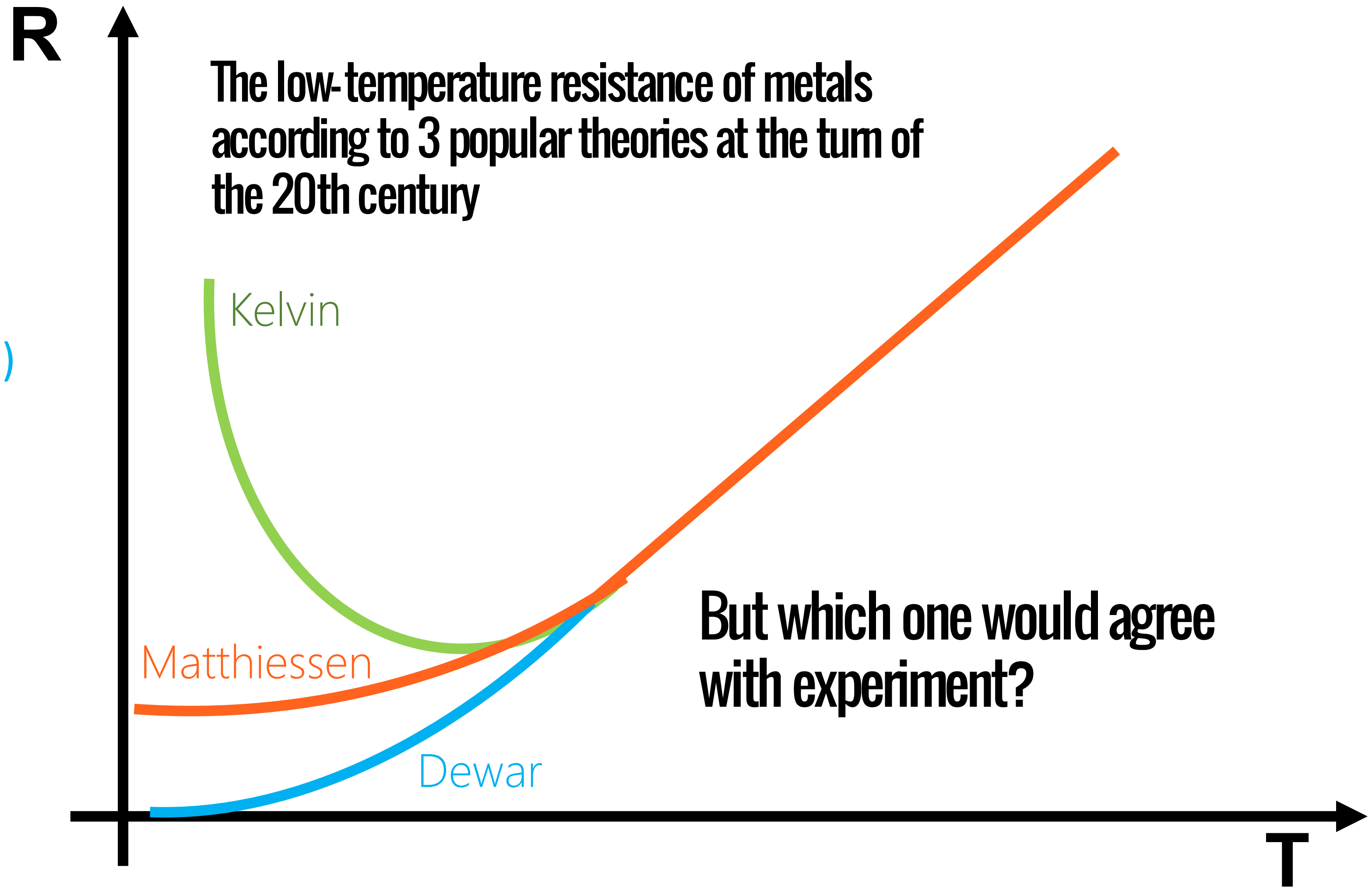
Leiden, the coldest place on earth (from 1908 to 1923)

Leiden, the coldest place on earth (from 1908 to 1923)

What would happen to the resistance of a metal as it was cooled to absolute zero?

THREE MAIN TEORIES IN 1908

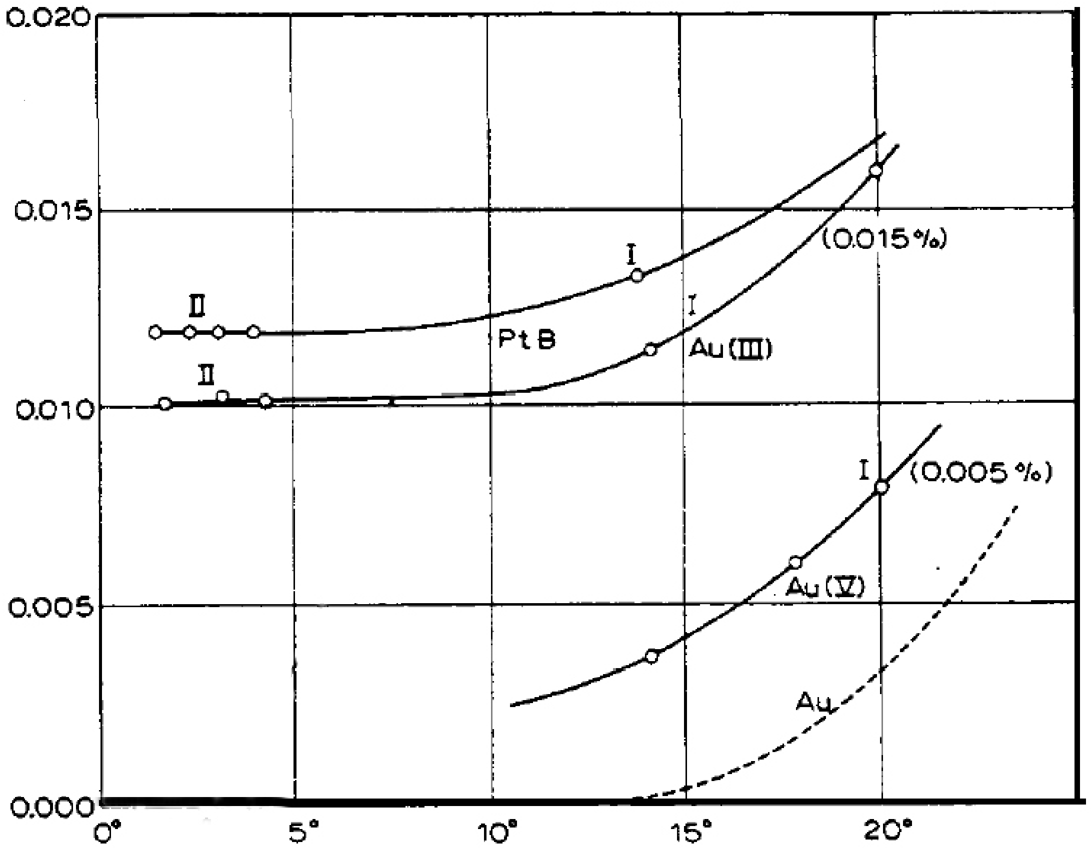
1. The resistance could approach zero value with decreasing temperature (James Dewar, 1904)
2. It could approach a finite limiting value (Heinrich Friedrich Ludwig Matthiesen, 1864)
3. It could pass through a minimum and approach infinity at very low temperatures (Lord Kelvin, 1902)



But which one would agree with experiment?

Au and Pt measurements

- Initially, Kamerlingh-Onnes studied Pt and Au samples (easy to purify)
- **Presence of Residual Resistance**
- **Residual Resistance related to sample purity (RRR concept)**
- At the Third International Congress of Refrigeration in Chicago in 1913, he reported on these experiments and arguments. There he said: *"Allowing a correction for the additive resistance I came to the conclusion that probably the resistance of absolutely pure platinum would have vanished at the boiling point of helium."*



The Kamerlingh Onnes plot from his Nobel lecture showing the low temperature measurements of the resistance in various metals (Au of different purity, Pt and Hg). The dashed curve shows the expectations for the pure gold. The original text describes the axis of the figure in the following way: "The resistance, in fractions of the resistance at zero Centigrade, is shown as the ordinate and the temperature as the abscissa."

https://arxiv.org/abs/1111.5318

Mercury

- Kamerlingh-Onnes decided to study **mercury**, the only metal at the time that he hoped could be extremely well **purified by means of multiple distillation**
- The initial experiments carried out by Kamerlingh-Onnes, together with his coworkers Gerrit Flim, **Gilles Holst**, and Gerrit Dorsman appeared to confirm the Onnes concepts (Dewar theory)

Mercury and discovery of Superconductivity (1911)

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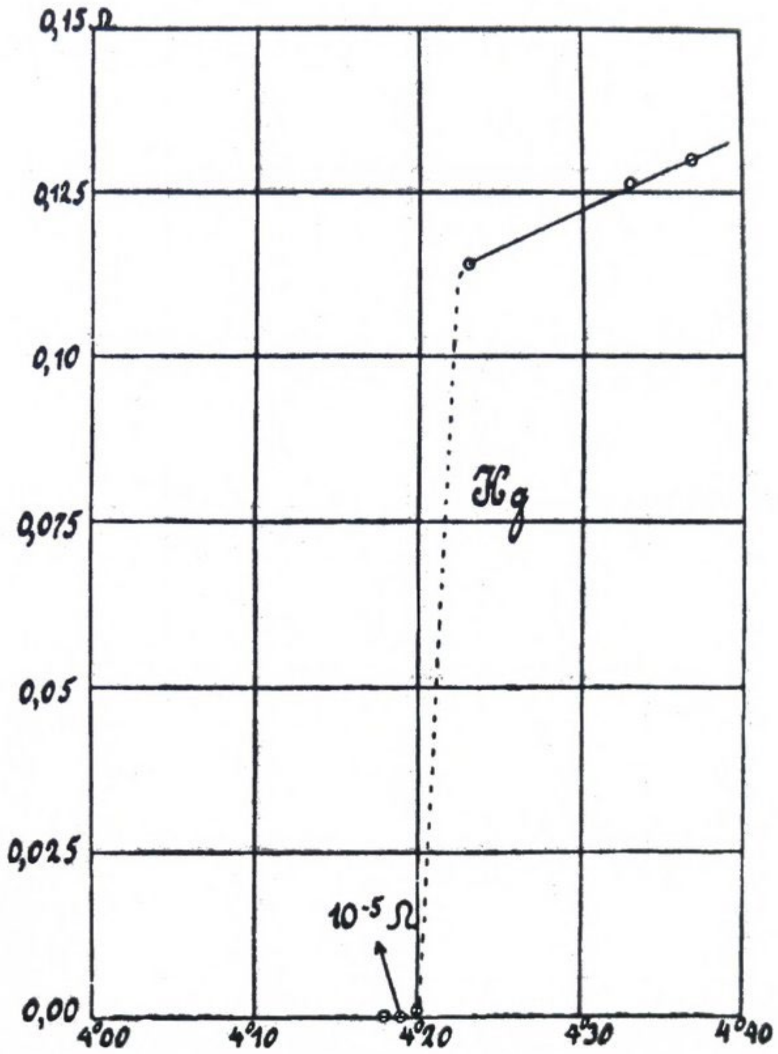
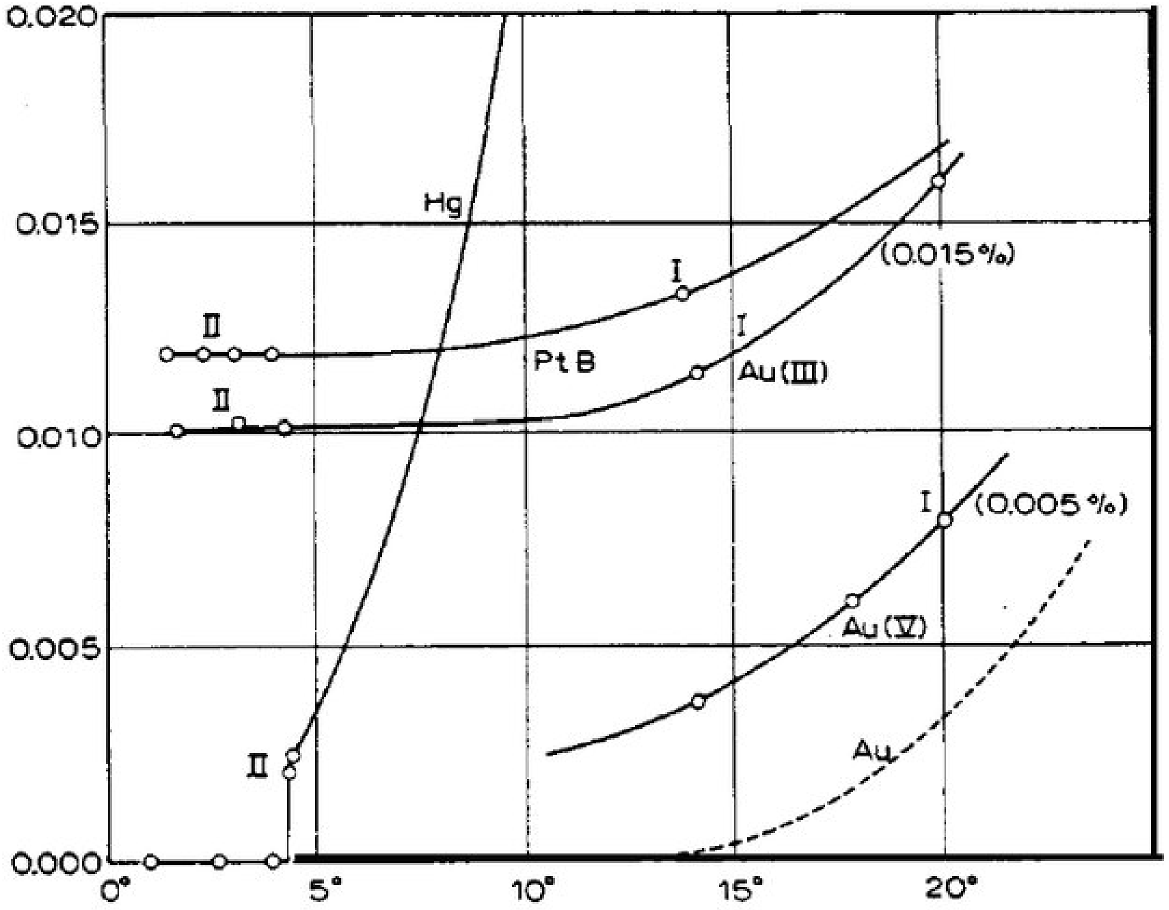
- The initial experiments carried out by Kamerlingh-Onnes, together with his coworkers Gerrit Flim, **Gilles Holst**, and Gerrit Dorsman appeared to confirm the Onnes concepts (Dewar theory)
- Further experiments using improved apparatus shows that the **behavior is completely different compared to Au and Pt**
- **A resistance jump to zero** appears slightly below 4,2 K



The discovery of Superconductivity (1911)

"At this point [slightly below 4.2 K] within some hundredths of a degree came a sudden fall not foreseen by the vibrator theory of resistance, that had framed, bringing the resistance at once less than a millionth of its original value at the melting point. . . . Mercury had passed into a new state, which on account of its extraordinary electrical properties may be called the superconductive state."

Kamerlingh-Onnes, 1911



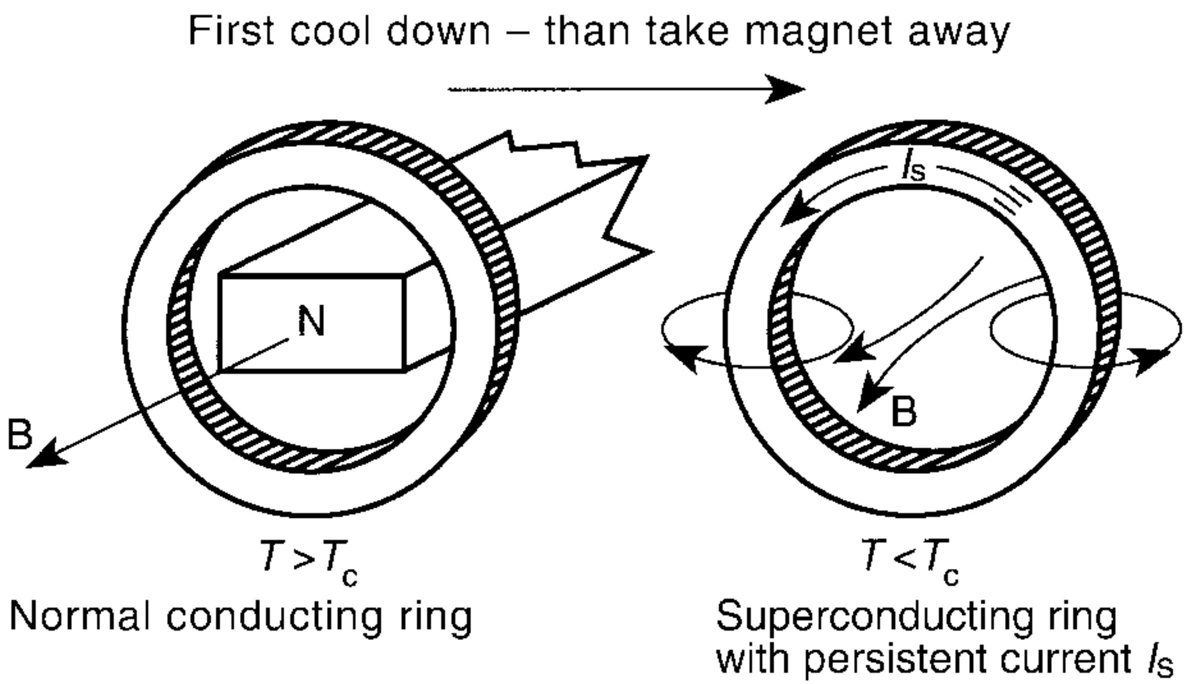
How to measure R=0?

If **R=0** ➡ **I flows permanently**

If not: $$I(t) = I_0 e^{-\frac{R}{L}t}$$

I(t) Current at time *t*
I(0) Current at time zero
L Self-induction coefficient
R Resistance of the ring

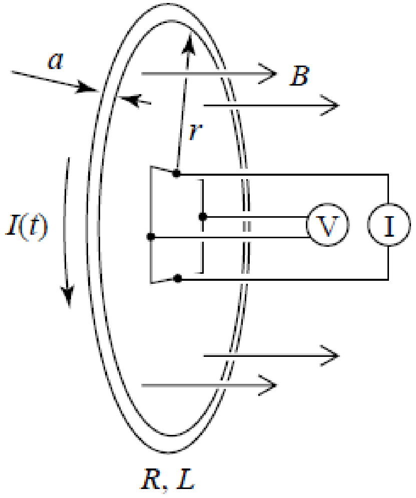
Induced current



How to measure R=0? Some numbers

$$I(t) = I_0 e^{-\frac{R}{L}t} \quad \Rightarrow \quad \boldsymbol{\tau = \frac{L}{R}}$$

I(t) Current at time *t*
I(0) Current at time zero
L Self-induction coefficient
R Resistance of the ring



Sketch of a simple setup to monitor the possible decay of current *I (t)* via its associated magnetic induction *B(t)* in a closed loop of inductance *L* and resistance *R*. The wire has a diameter *a* and a loop radius *r*. The field *B* is normal to the loop area. The magnetic sensor is a Hall probe.

Comes from the conservation of energy: $$\frac{d}{dt}\left(\frac{1}{2}LI^2\right) + RI^2 = 0$$

After t_1=1 year → I_1= I_0 Actually we can not say that

We must to take in account the **measure sensibility**

$$\delta I > I_0 - I_{t1} = I_0(1 - e^{-\frac{R}{L}t1})$$

$$\delta B > B_0 - B_{t1} = B_0(1 - e^{-\frac{R}{L}t1})$$

$$\boldsymbol{\frac{\delta I}{I_0} = \frac{\delta B}{B_0} > 1 - e^{-\frac{R}{L}t1}}$$

How to measure R=0? Some numbers

$$\frac{\delta I}{I_0} = \frac{\delta B}{B_0} > 1 - e^{-\frac{R}{L}t1}$$

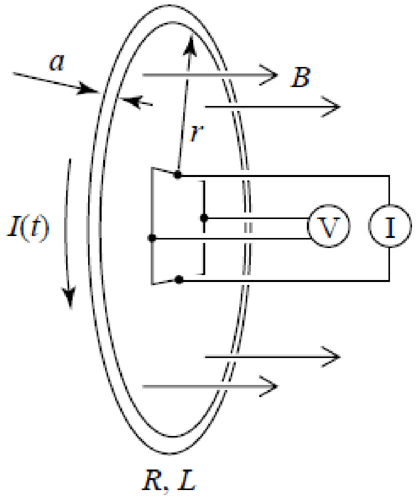
Solving for R

$$R < \frac{L}{t_1}\ln\left(1 - \frac{\delta B}{B_0}\right) = \frac{L}{t_1}\ln\left(1 - \frac{\delta I}{I_0}\right)$$

where all quantities on the right can be determined from the experiment

For ***r >> a*** ➡ $L \approx \mu_0 r \ln(^r/_a)$

$$R < \frac{\mu_0 r \ln(^r/_a)}{t_1}\ln\left(1 - \frac{\delta B}{B_0}\right)$$ ➡ $R < 10^{-19}\Omega$



r = 50 mm

a = 1 mm

t_1 = 1 year ~ $3x10^7$

L = $1,3x10^{-7}$ H

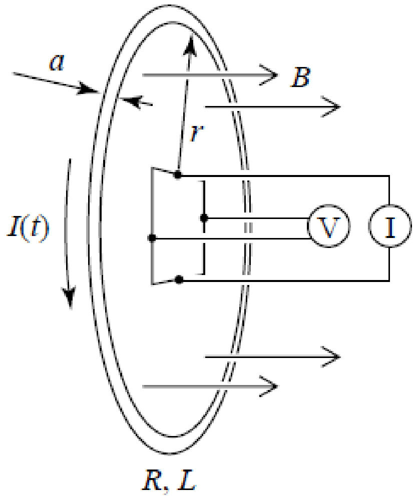
$\delta B/B_0 = 10^{-5}$

How to measure R=0? Some numbers

$$R < \frac{\mu_0 r \ln(r/a)}{t_1} \ln\left(1 - \frac{\delta B}{B_0}\right) \Rightarrow \mathbf{R < 10^{-19}\Omega}$$

$$R = \rho \frac{\ell}{A}$$

$$\rho < 10^{-19}\Omega \left(\frac{A}{\ell}\right) \approx \mathbf{2.5 \times 10^{-25}\Omega m}$$



$\rho_{Al}^{4,2K} = 10^{-12}\Omega m$ **13 order of magnitude less than Aluminum @4.2 K!**

$\rho_{Cu}^{273K} = 10^{-8}\Omega m$ **17 order of magnitude less than Copper @300 K!**

r = 50 mm

a = 1 mm

t_1 = 1 year ~ $3x10^7$

L = $1,3x10^{-7}$ H

$\delta B/B_0 = 10^{-5}$

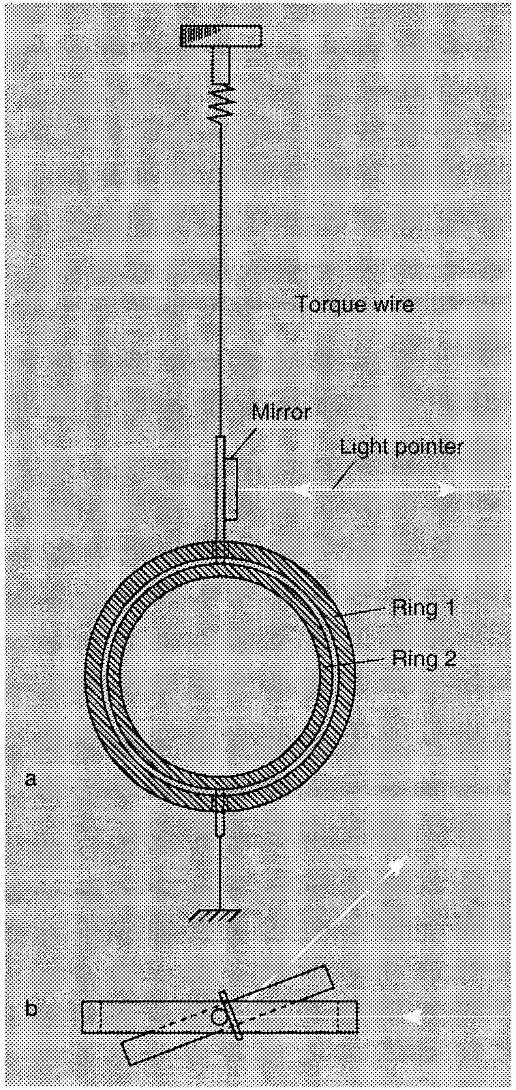
$$\tau = \frac{L}{R} > \mathbf{5.8 \times 10^{13} s} \Rightarrow$$ **More than 2 milion of years!**

But remember that the limit is the measure sensibility

With more sensible measure (SQUID), τ approach the lifetime of Universe

How to measure R=0? Further improvements

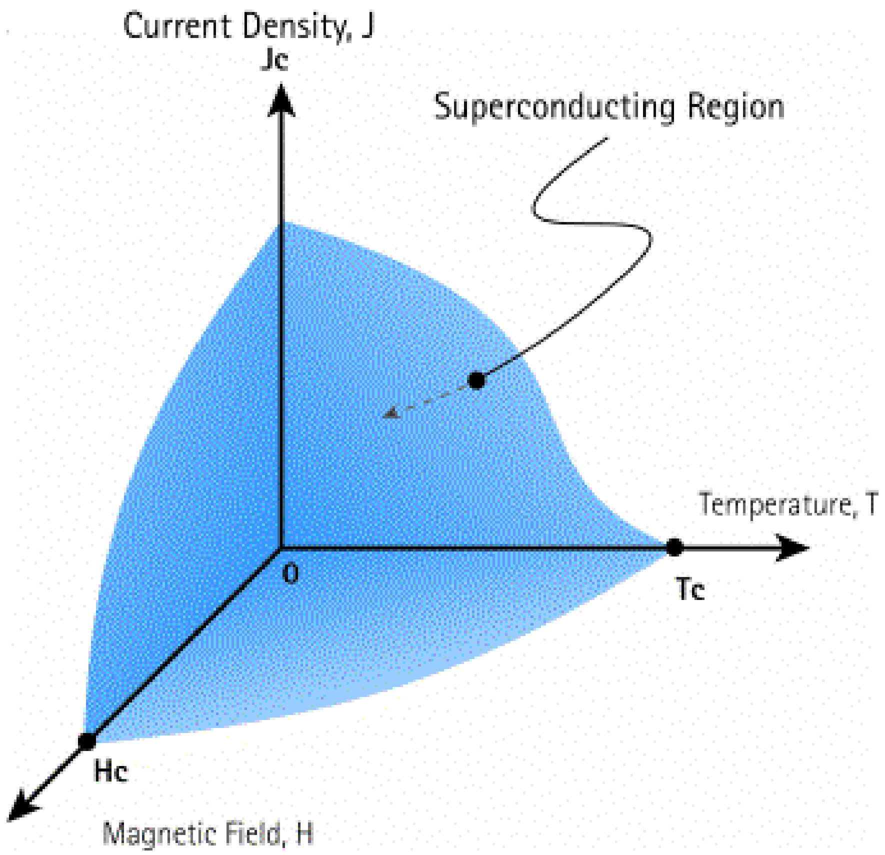
- Kamerlingh-Onnes enhanced the measure sensibility with a new set-up
- An equilibrium position is established in which the angular moments of the permanent current and of the torsion thread balance each other. This equilibrium position can be observed very sensitively using a light beam.
- During all such experiments, no change of the permanent current has ever been observed



- Today we know that a **SC** has a specific electric **resistance** about **17 orders of magnitude smaller** than the specific resistance of **Cu**!!!
- **The difference in resistance of a metal between the superconducting and normal states is at least as large as that between copper and a standard electrical insulator**

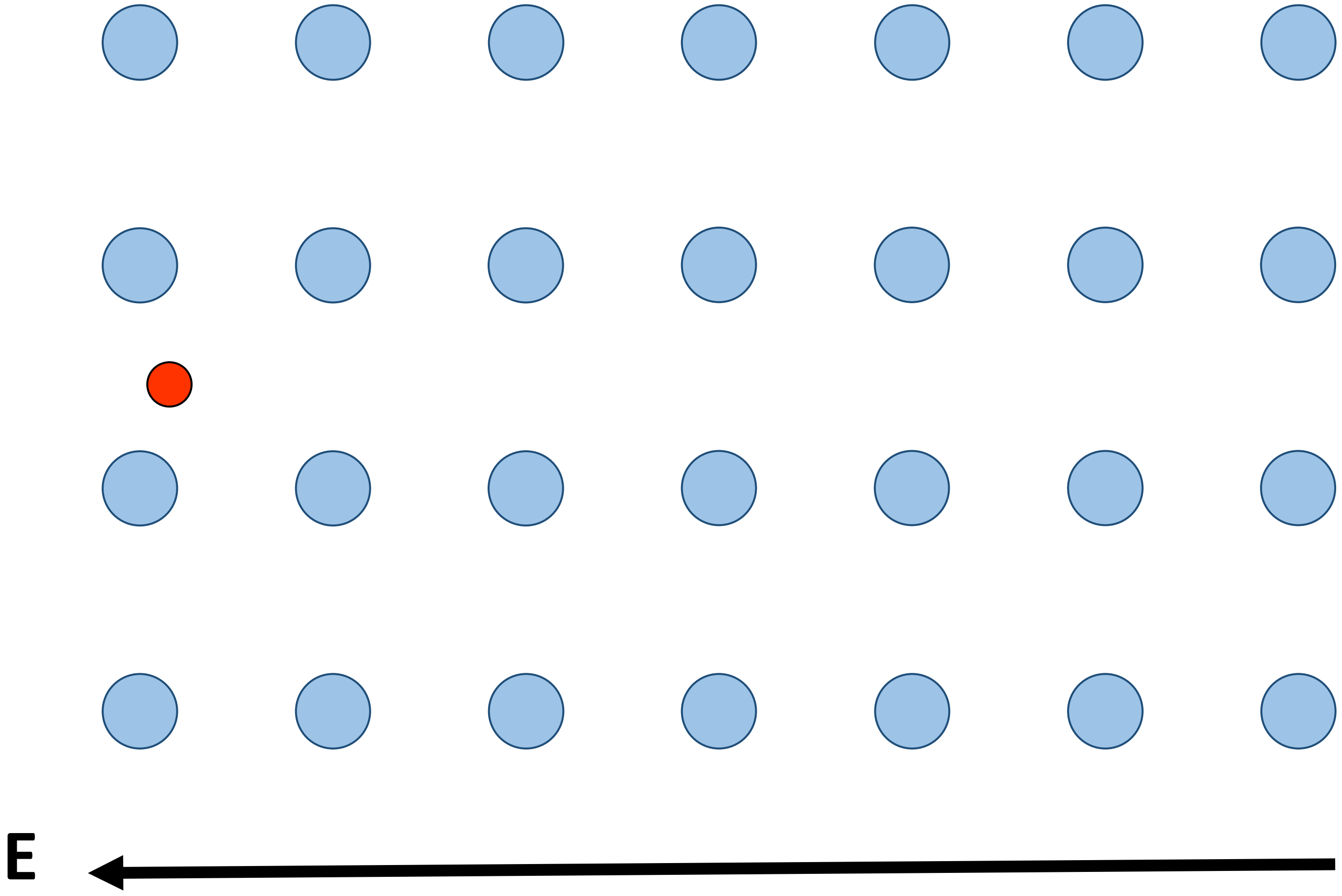
New superconductors and new critical parameters appears

- Onnes’ team discovered that **small impurities to mercury had no effect on Tc.** This implied that the effect was intrinsic to mercury.
- In 1912 Kamerlingh Onnes discover **superconductivity in Tin (3.7 K) and Lead (7.2 K)**
- Leiden group works to built high magnetic field superconducting coils (up to 10 T)
- However, experiments showed that **SC could be destroyed** if the material was subjected **to a sufficiently large magnetic field (Bc)**
- The **critical magnetic field** which would destroy superconductivity (in Sn and Pb) **was rather small**
- Francis Silsbee had showed in 1916 that **the critical current and critical magnetic field were two sides of the same coin**
- None of the discovered SC is a particularly good conductors of electricity **(Au, Ag, Cu are not SC)**. This in itself was an important piece of the puzzle, but no-one could understood its significance at the time
- Onnes died in 1926, but work on superconductivity continued at the Leiden laboratory
- In 1931, W. J. de Haas and W. H. Keesom, **discovered SC in an alloy**



And what about the Drude Model?

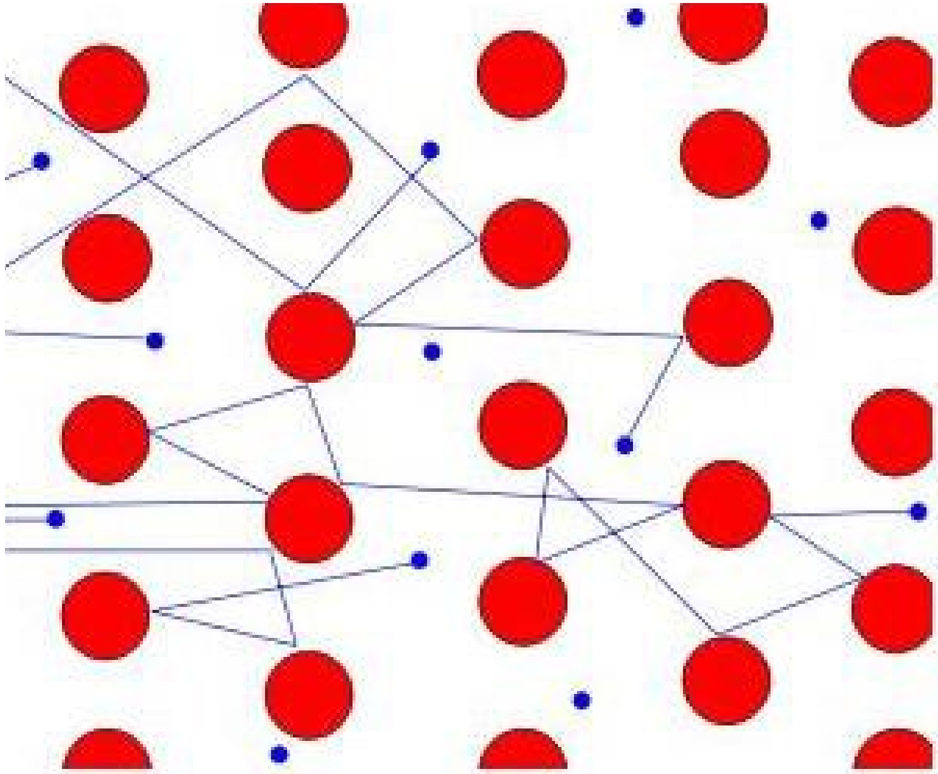
Electrons in Electrical Field



Ohm Law (for metal conductors)

- $V = RI$
- $R = \frac{\ell}{A}\rho$
- ρ?
- $\boldsymbol{p} = e\boldsymbol{E}\tau$
- $m\boldsymbol{v_d} = e\boldsymbol{E}\tau$
- $\boldsymbol{j} = -n_e e\boldsymbol{v_d} = \frac{ne^2\tau}{m}\boldsymbol{E}$

$\boldsymbol{\sigma = \frac{1}{\rho}}$



Drude Model

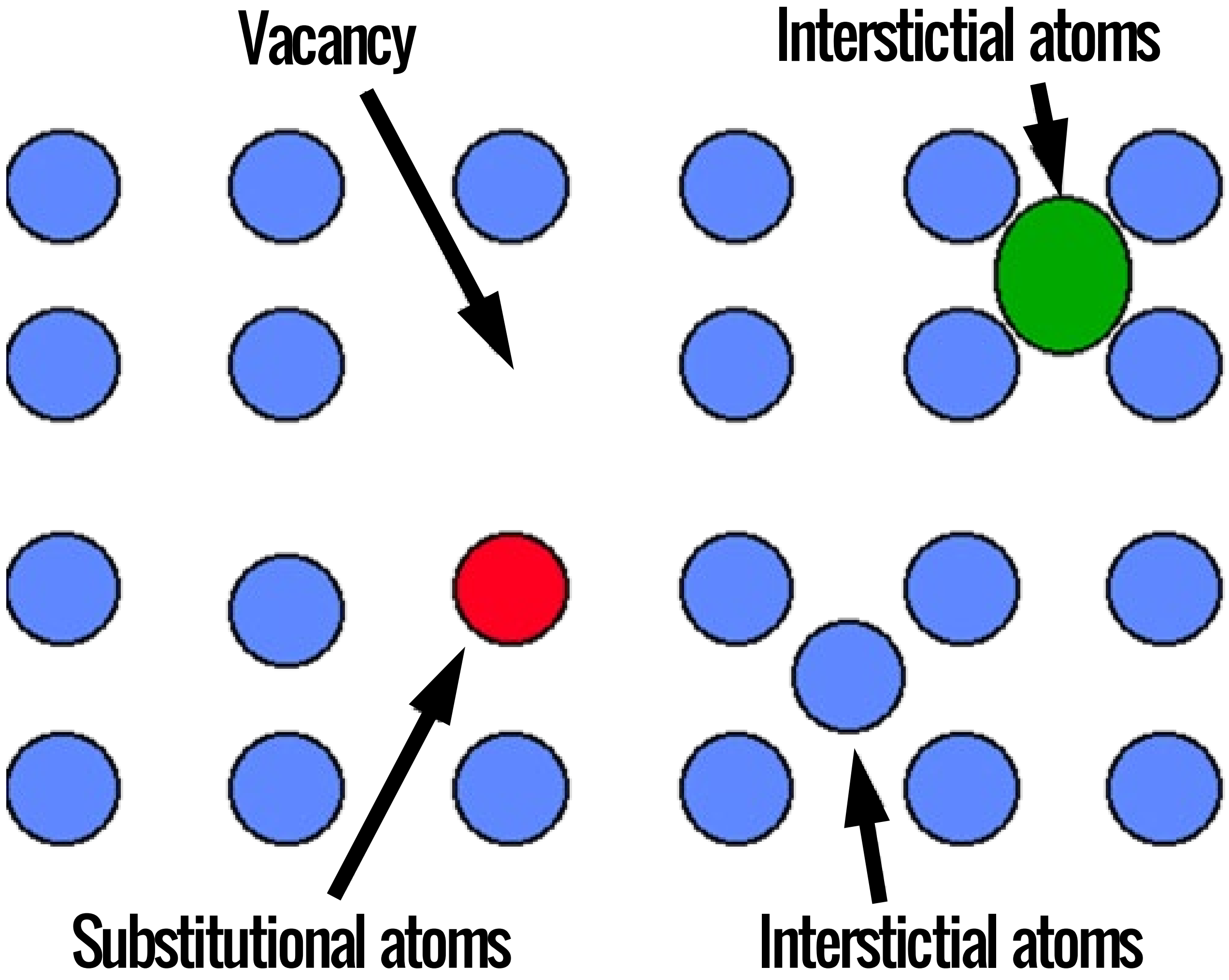
Electrical Resistance and Resistivity

Electrical Resitance is due to hit between electrons and lattice

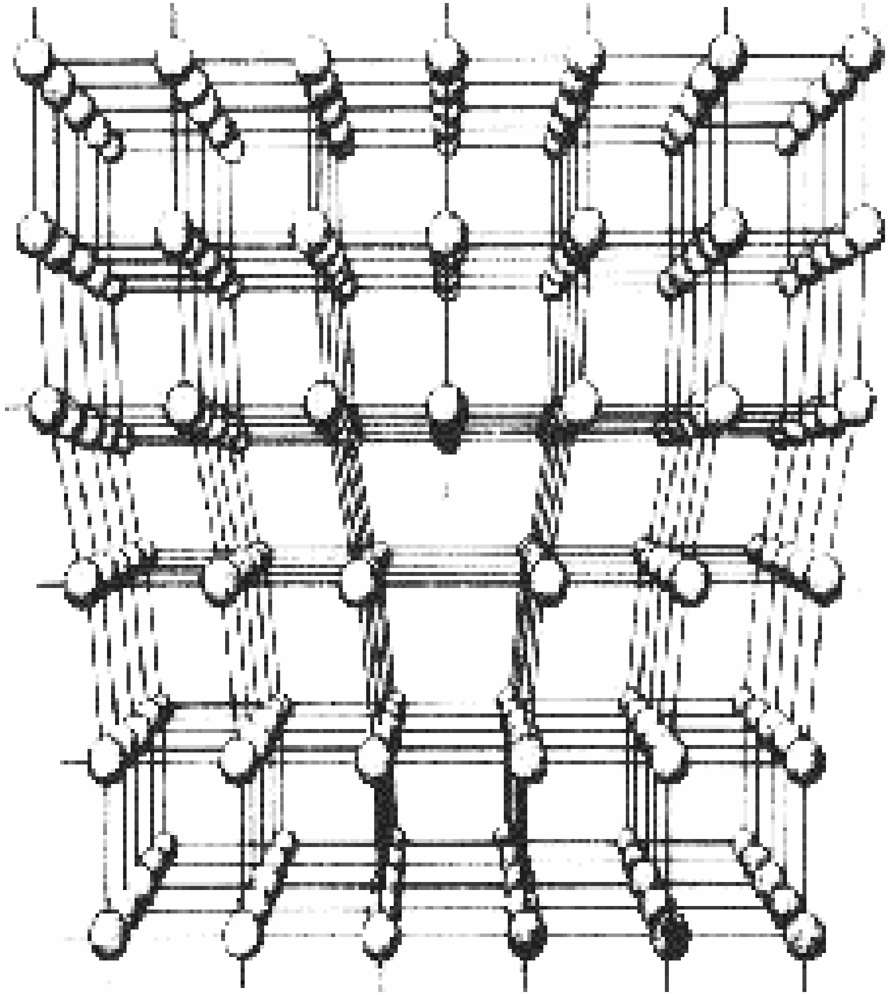
Sum of two contributions:

- **Lattice vibration (phonons)**
- **Defects and lattice distortions**

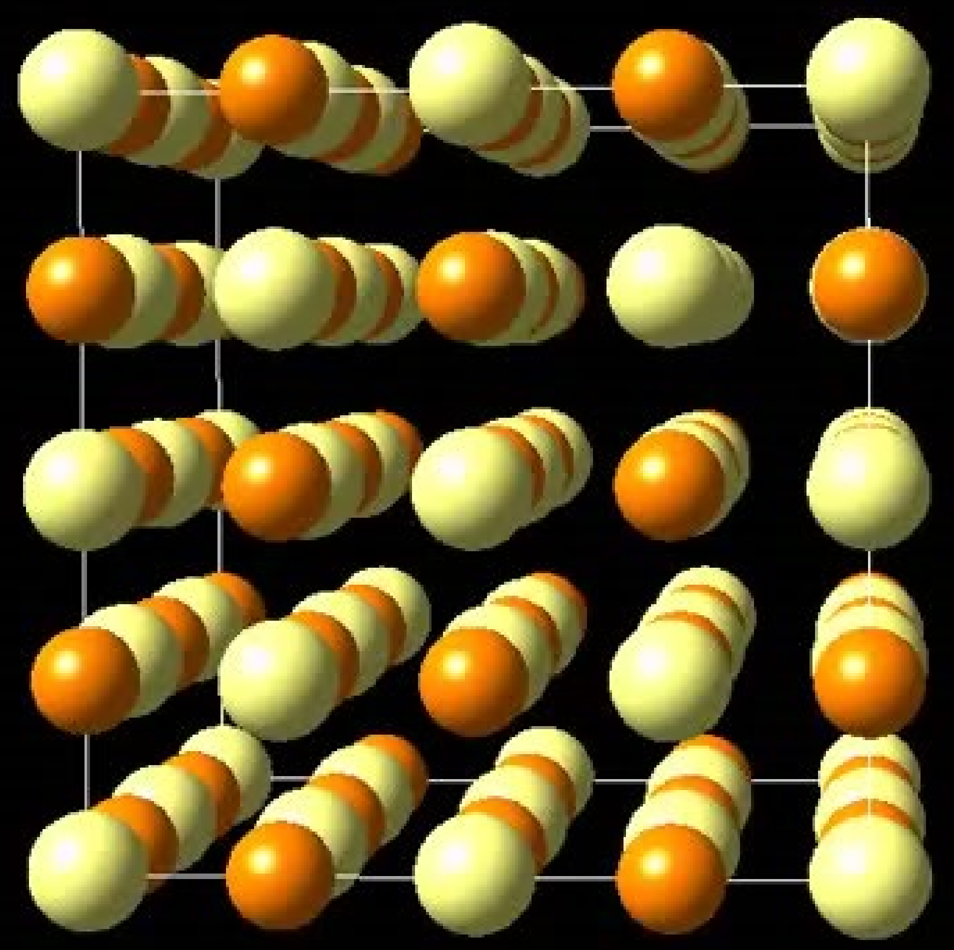
Puntual defects



Dislocations

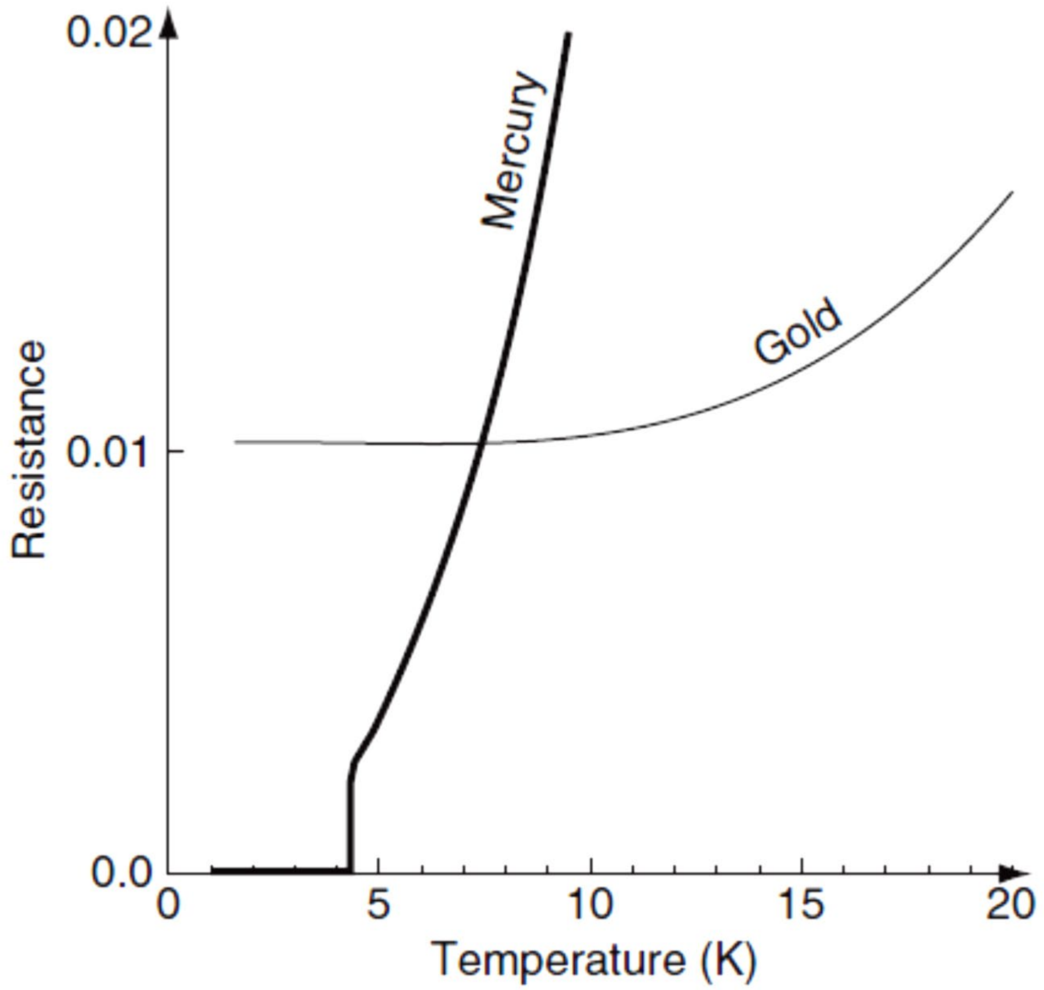


Lattice vibrations due to Phonons

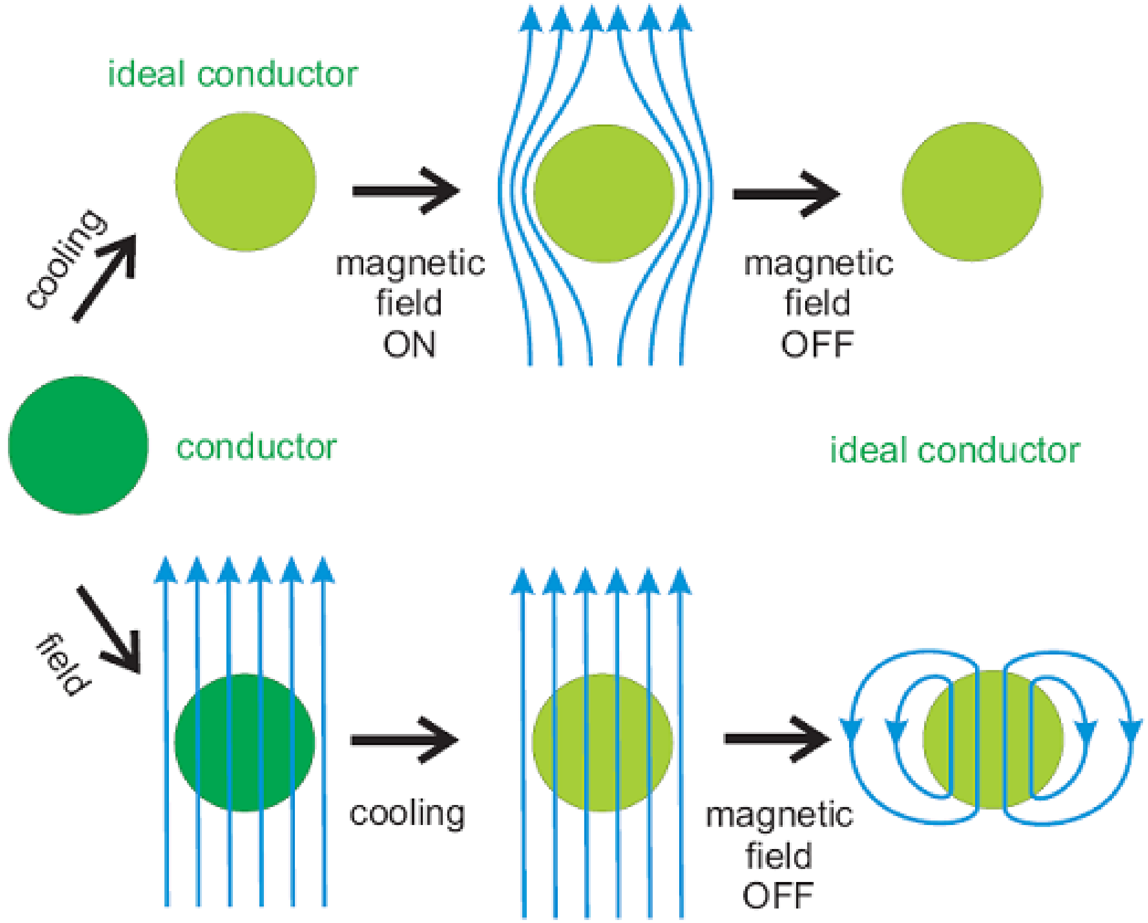


How to explain superconductivity?

- **Drude model can not explain superconductivity**
- Many scientist tried tried (without success) to develop a theory to explain superconductivity. Sir J. J. Thomson and Frederick Lindemann are two of those
- Bloch found his attempts to formulate a satisfactory theory of superconductivity were doomed to failure; Bloch concluded that *"the only theorem about superconductivity that can be proved is that any theory of superconductivity is refutable"*. His equally facetious second theorem was: *"Superconductivity is impossible"*
- Albert Einstein, reviewing the situation in 1922, concluded that *"with our wide-ranging ignorance of the quantum mechanics of composite systems, we are far from able to compose a theory out of these vague ideas. We can only rely on experiment."*
- **We must to wait other experiments and Quantum Mechanics (and a couple of weeks too…)**



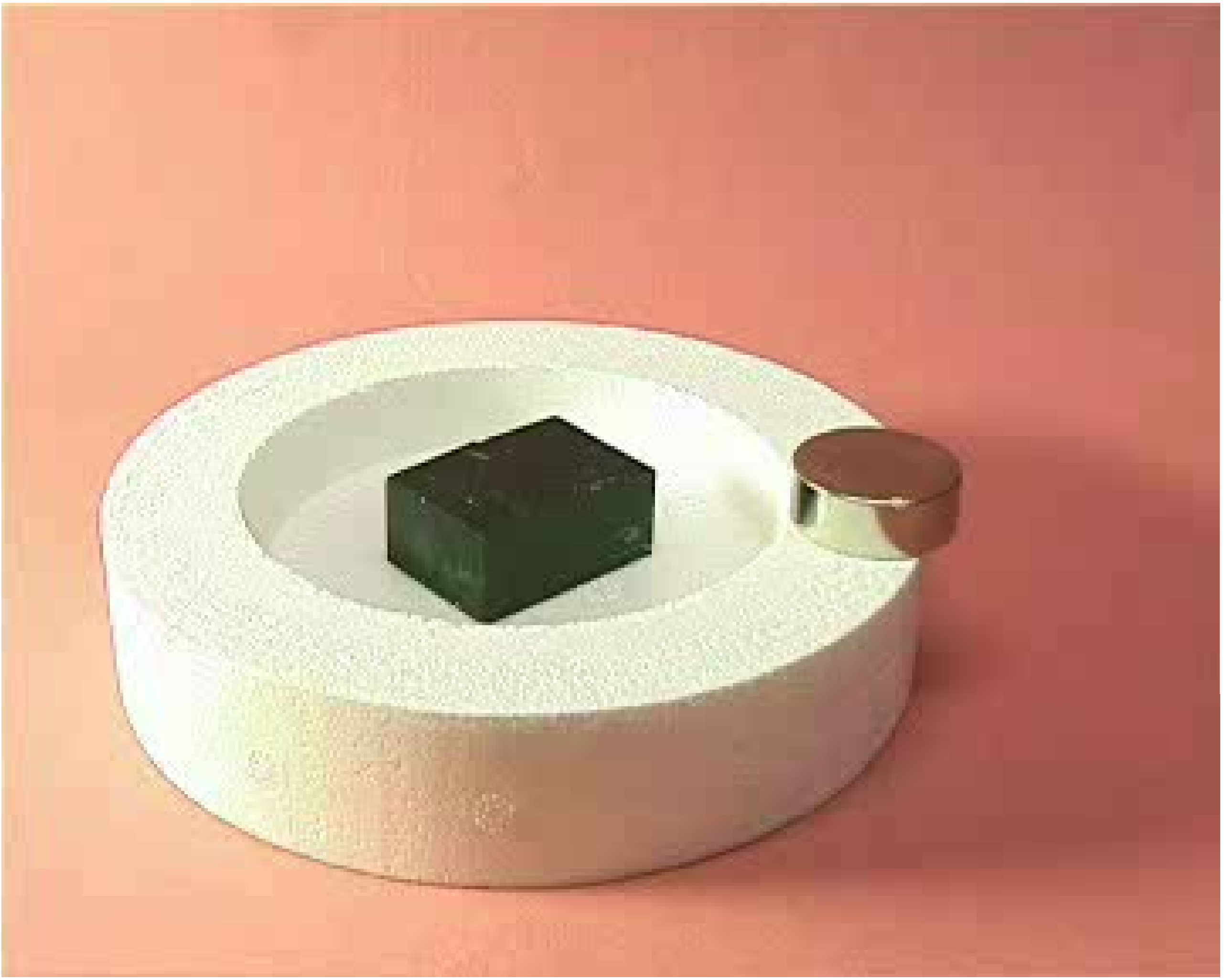
Superconductors are ideal conductors?

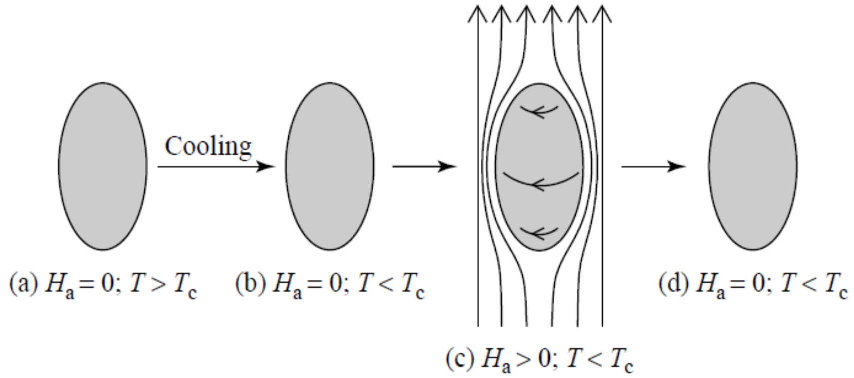


1

2

Superconductors are ideal conductors?





1

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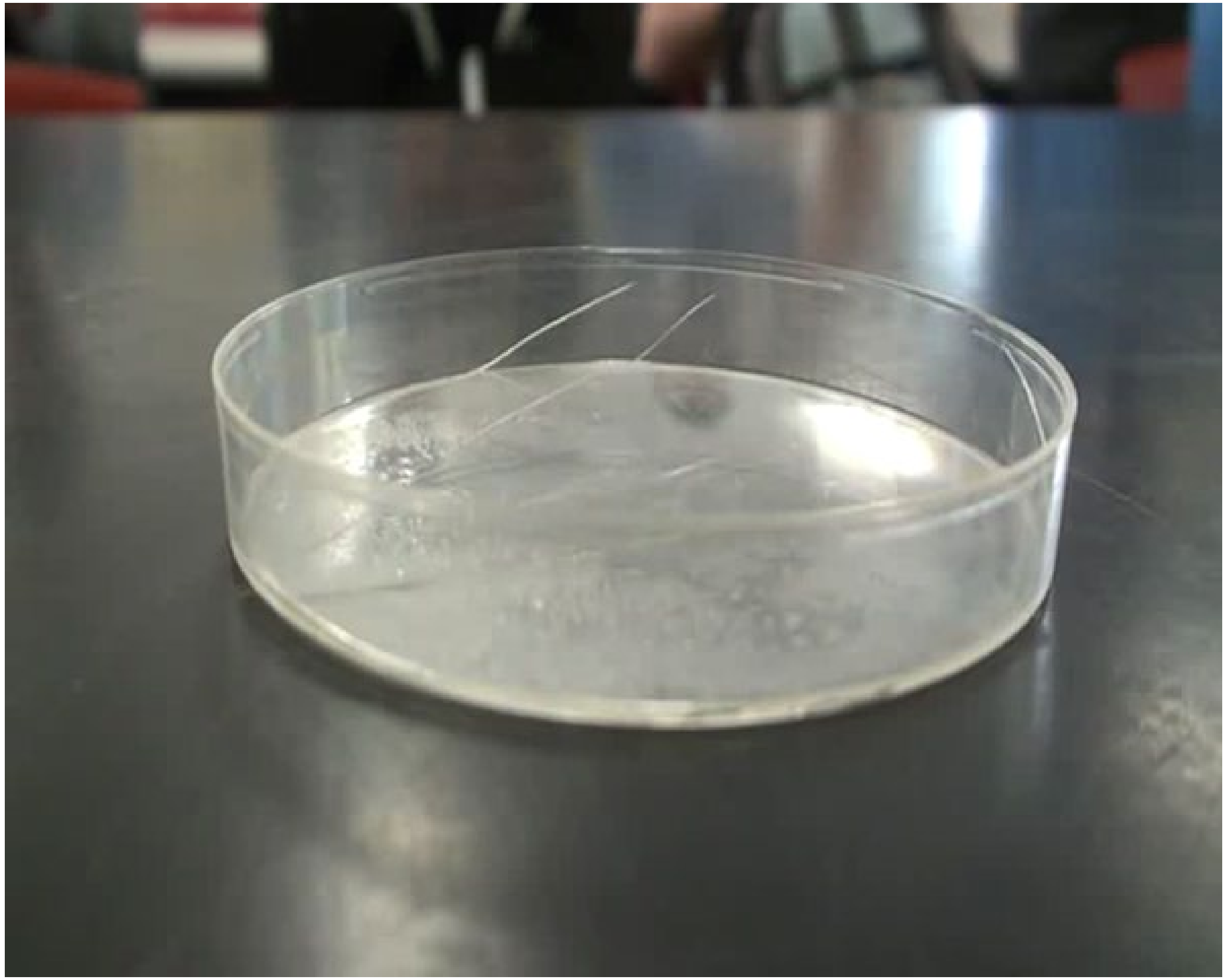

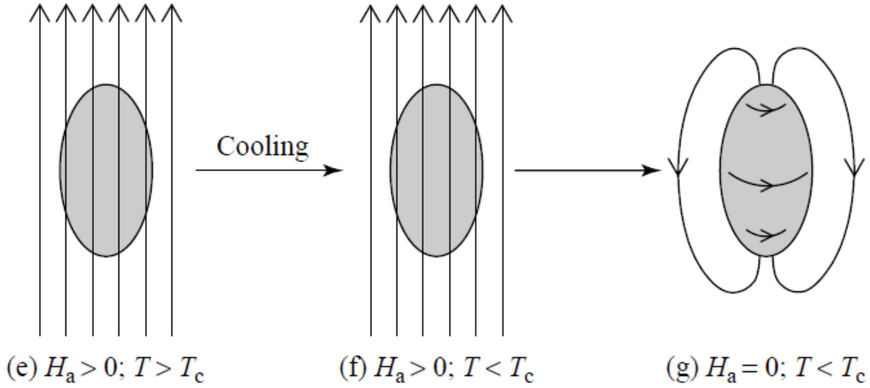

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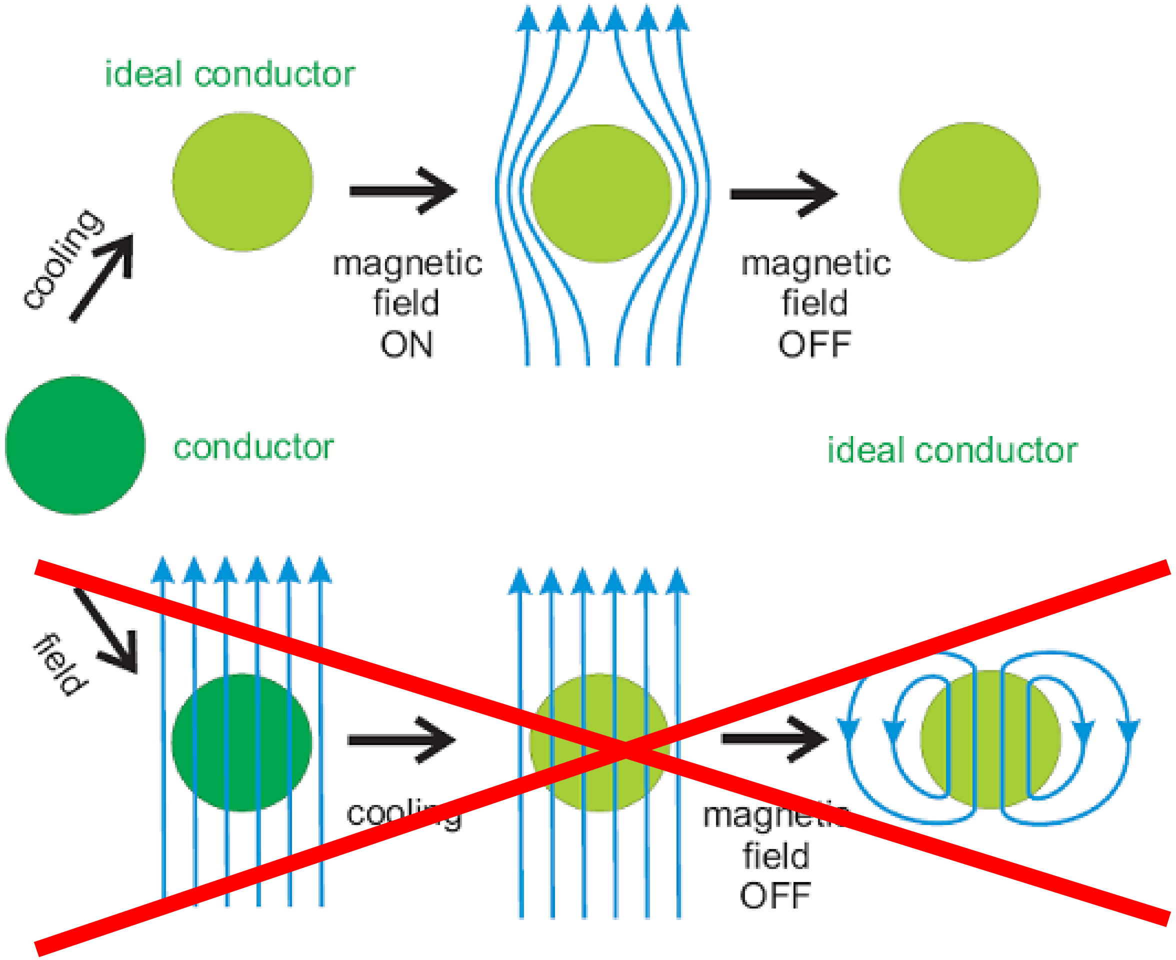
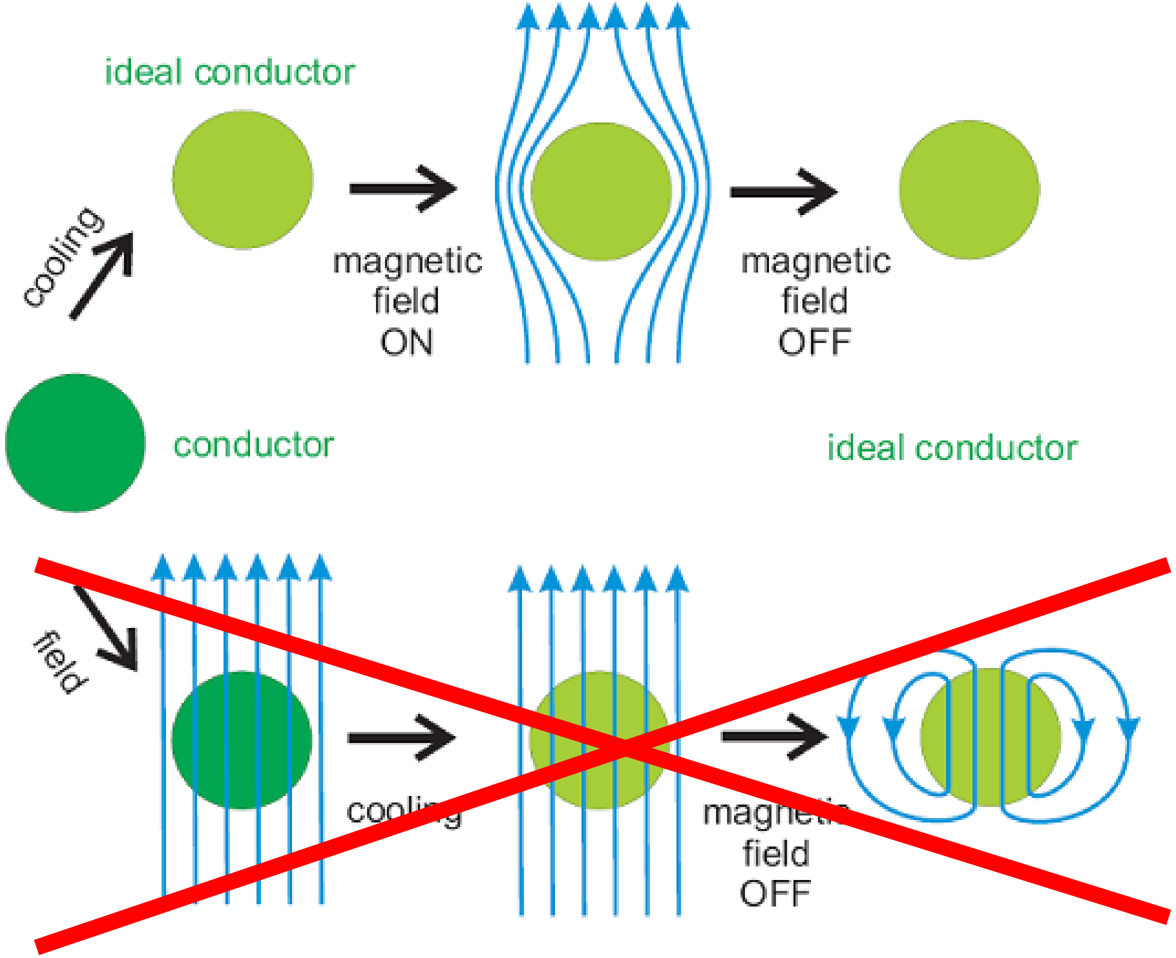
Superconductors are ideal conductors?

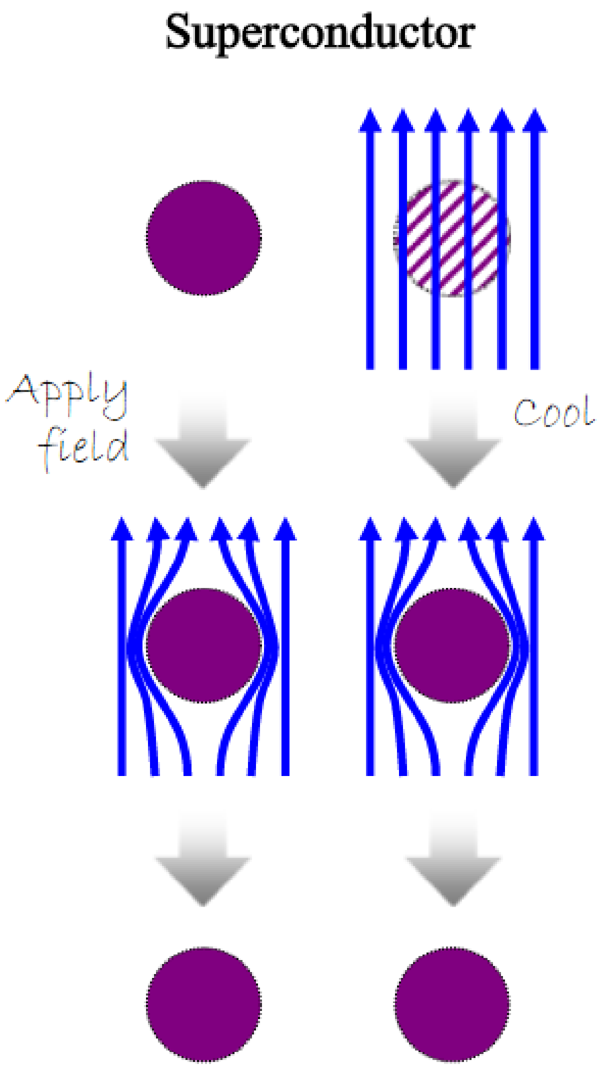


1

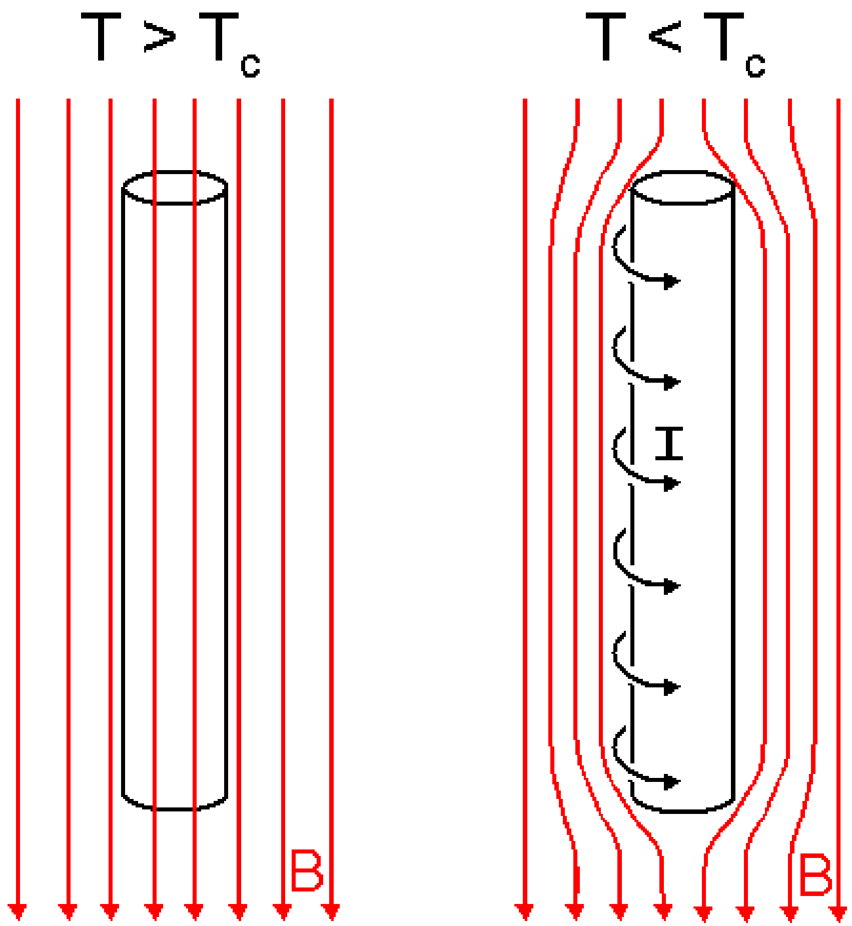
2

Perfect diamagnets

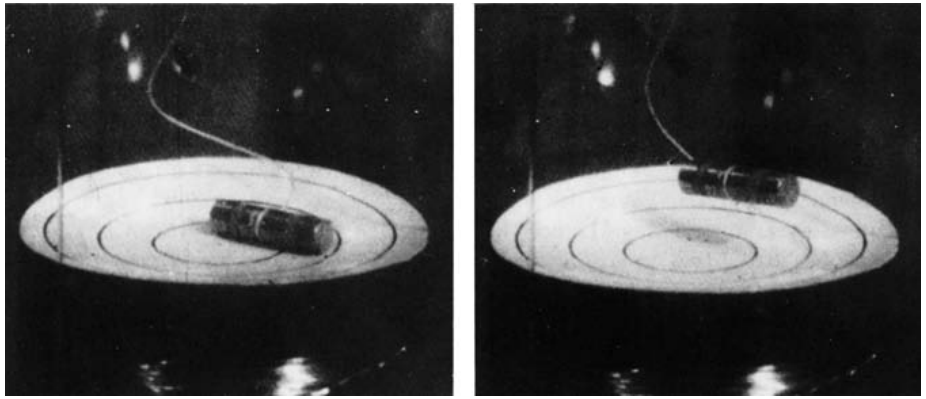




Meissner Effect



- **Ideal diamagnetic state**
- **Independent of the temporal sequence of cooling or magnetic field application**
- This ideal diamagnetism was discovered in 1933 by Meissner and Ochsenfeld for rods made of lead or tin
- Kamerlingh-Onnes in 1924 choosed the wrong shape to test it… a ring



“Levitated magnet” for demonstrating the Meissner-Ochsenfeld effect in the presence of an applied magnetic field.
Left: starting position at $T > Tc$. *Right*: equilibrium position at $T < Tc$

Perfect diagmanetism

Below Tc → **B=0** inside the superconductor

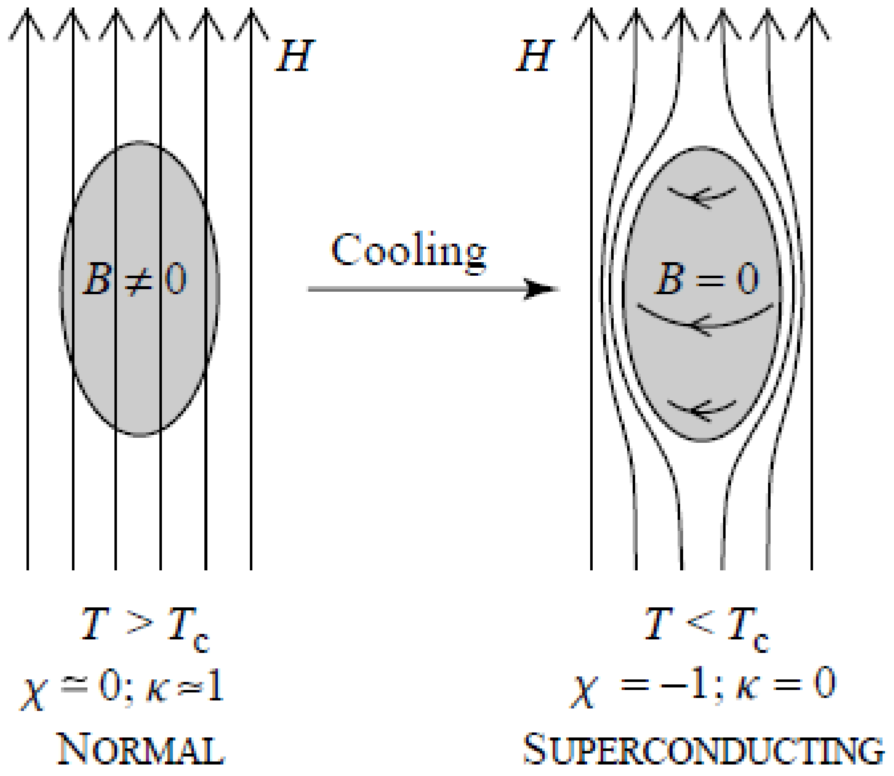
$$\boldsymbol{B} = \boldsymbol{\mu_0}(\boldsymbol{H} + \boldsymbol{M}) = \boldsymbol{0}$$

Which implies that in the superconductor

$$\boldsymbol{M} = -\boldsymbol{H}$$

And the susceptibily is

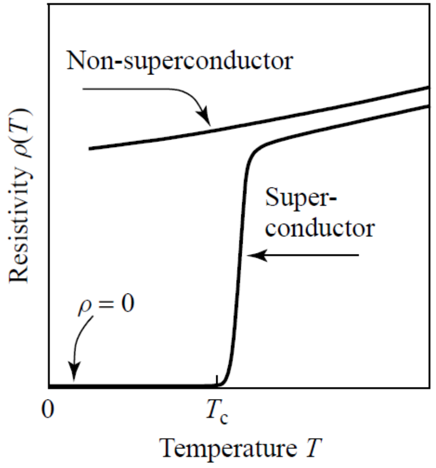
$$\boldsymbol{X} = \frac{\boldsymbol{dM}}{\boldsymbol{dH}} = -\boldsymbol{1}$$

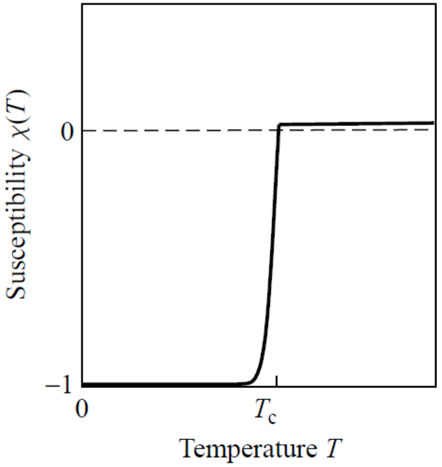


Consequences of Meissner effect

Superconductivity is
more than just ρ = 0

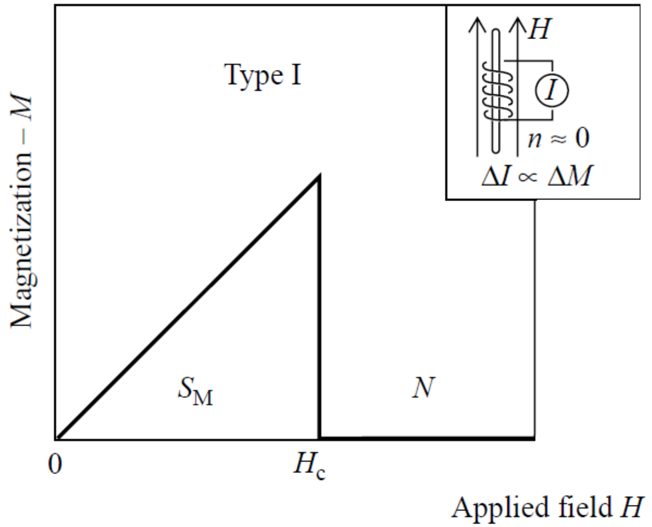
Superconductivity is a
thermodynamic state,
contrary to a state
characterized by just ρ = 0

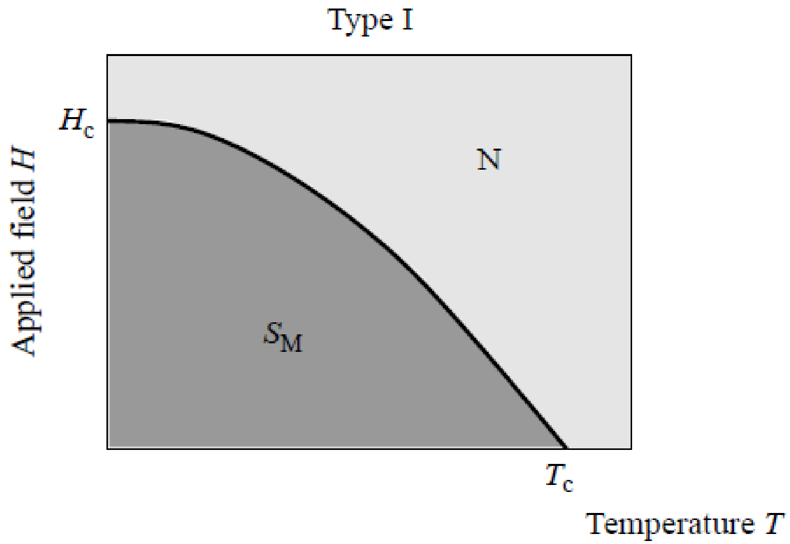




2 types of Superconductors

Type I The superconductor **switches abruptly** over from the Meissner state to one of ***full penetration of magnetic flux***, the normal state, at a well-defined critical field, H_c. Examples of such materials are Hg, Al, Sn, In





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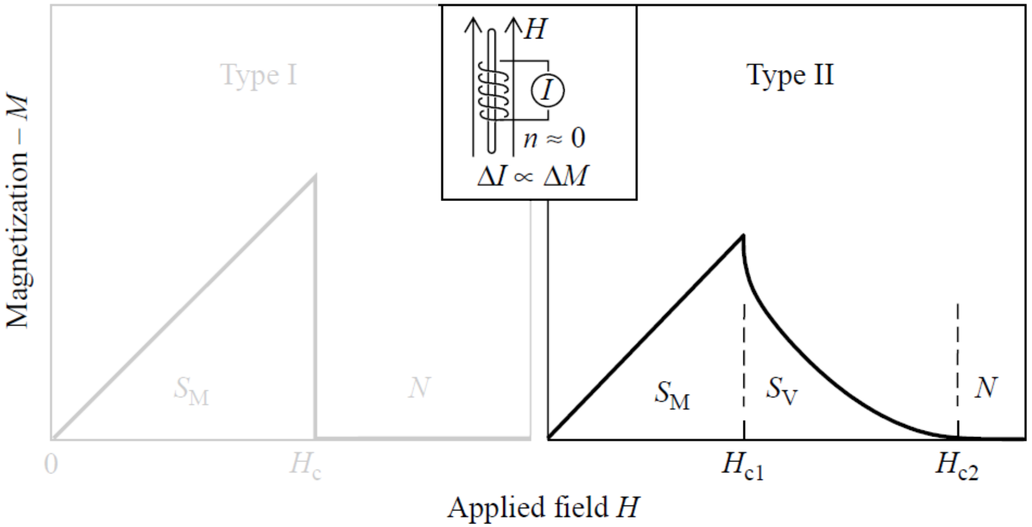
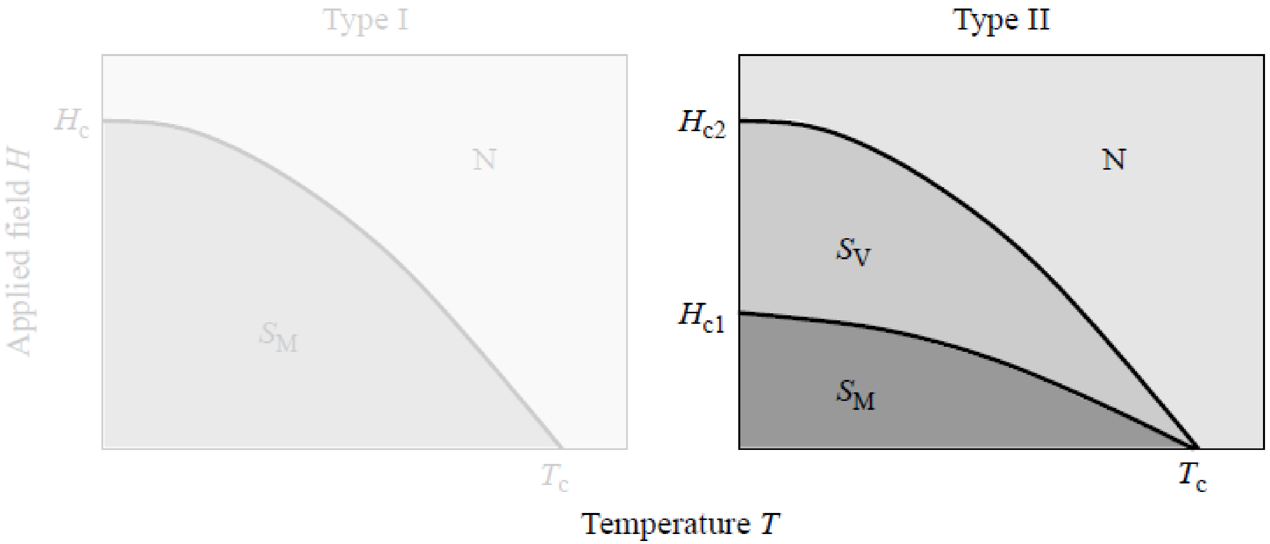
Type II The superconductor switches **from the Meissner state to a state of *partial penetration of magnetic flux*, the *mixed state* (**aka***Shubnikov phase*)**, at a critical field H_{c1}. Thereafter it ***crosses over continuously* to *full flux penetration***, the normal state, at an upper field H_{c2}. Examples: Nb, Nb_3Sn, NbTi, and all high-T_c cuprates





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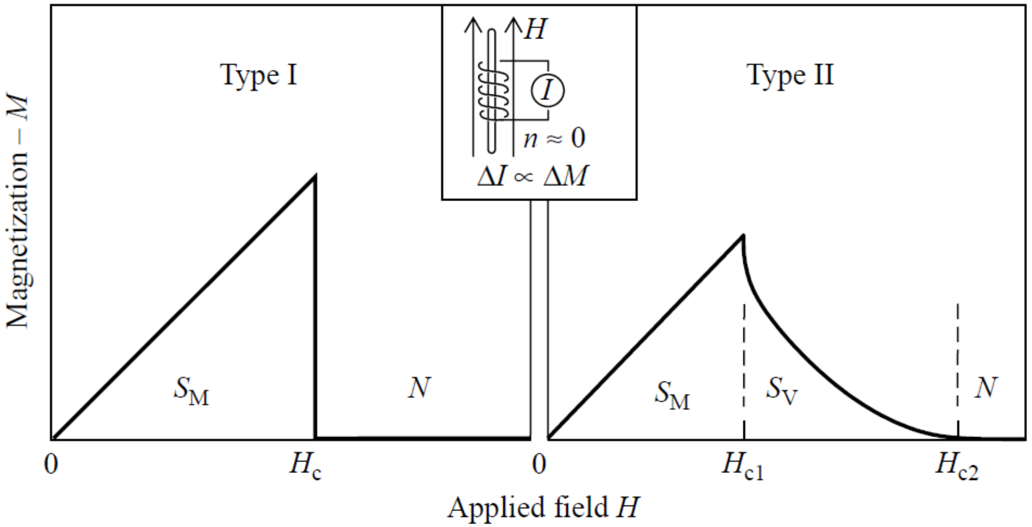
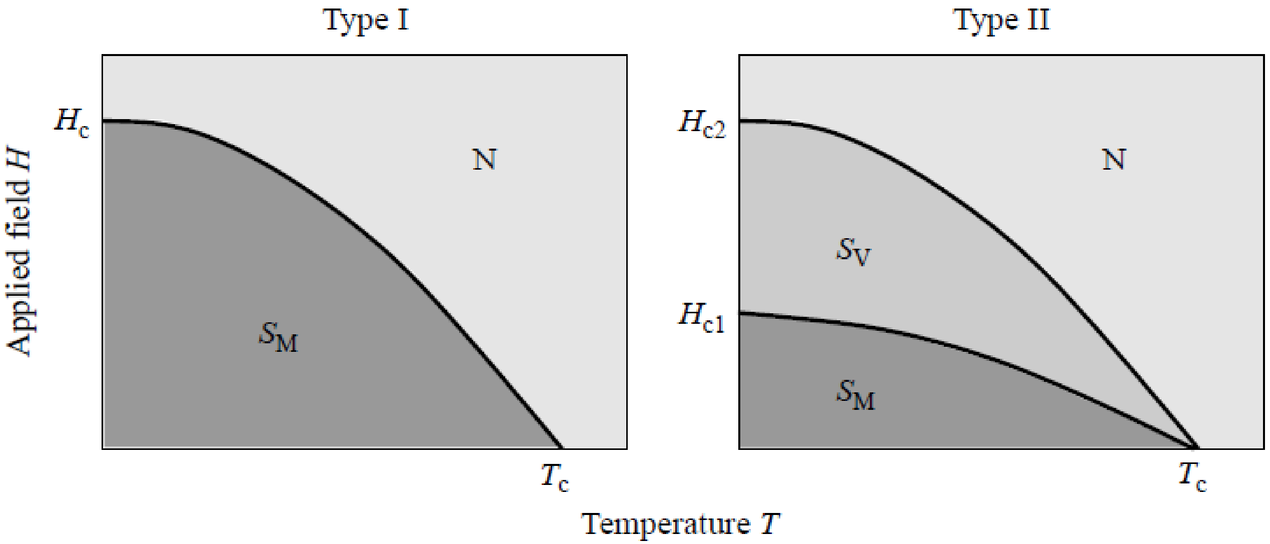




Vortex in Type II Superconductors

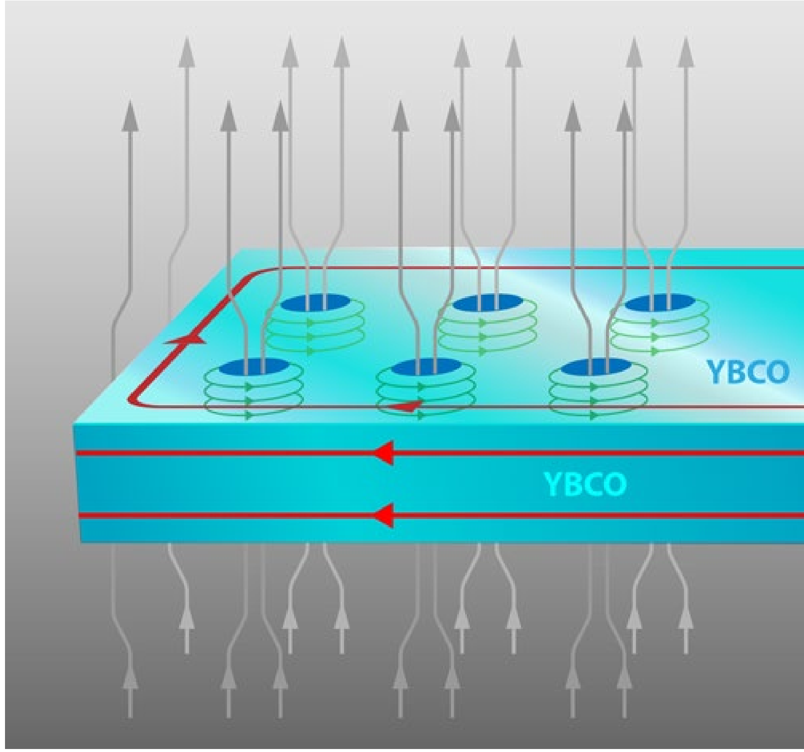
Abrikosov, Nobel Prize in 2003

Magnetic field only partly penetrates into the sample

Flux penetrates in tiny, precisely **quantized units of flux**

$$\Phi_0 = \frac{h}{2e}$$

h = *Planck's constant*
e = *magnitude of electronic charge*

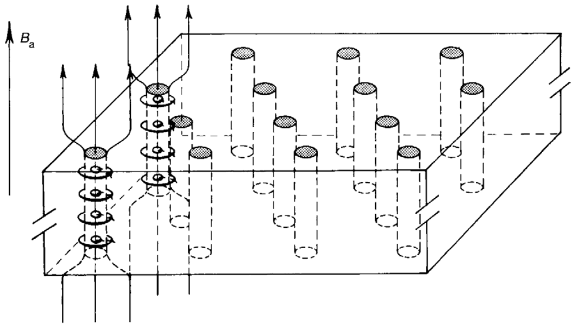


Shielding currents flow within the superconductor and concentrate the magnetic field lines

The **density of flux lines increases with increasing applied magnetic field**

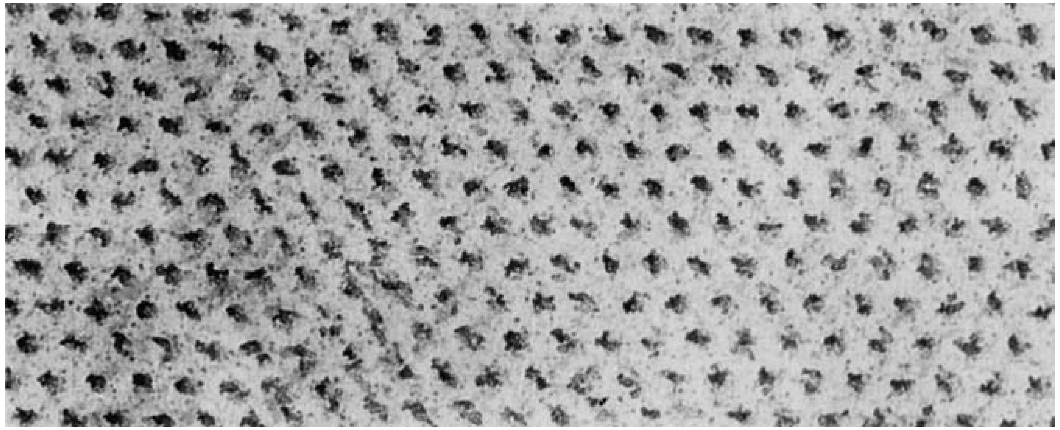
Vortex in Type II Superconductors (2)

- Abrikosov in 1953 predicted the existence of the so-called mixed phase
- Landau was not convinced (looks like pseudoscience for him)
- Richard Feynman 2 years later explained some properties of superfluid helium using vortices
- This work convinced Landau and Abrikosov's work was published in 1957
- Real images of the Shubnikov phase were generated by Essmann and Träuble using an ingenious decoration technique in 1967



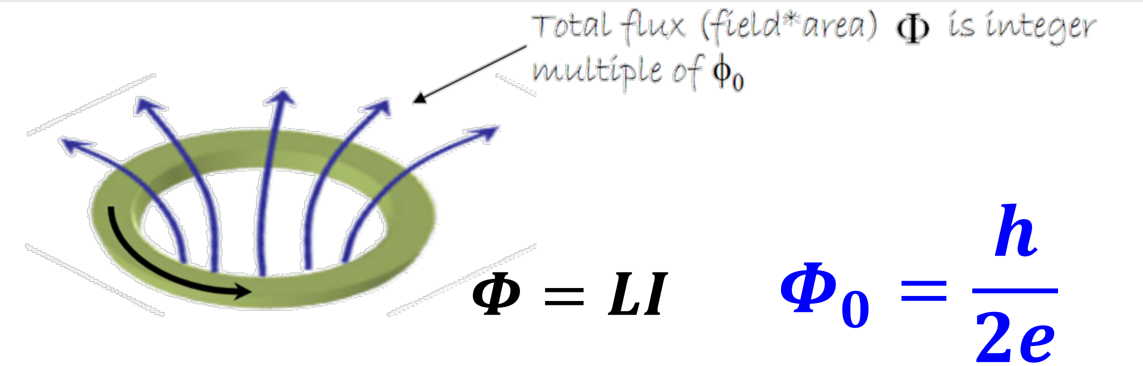
Magnetic flux structure images were obtained as follows:

- *Above the superconducting sample, iron atoms are evaporated from a hot wire*
- *During their diffusion through the helium gas in the cryostat, the iron atoms coagulate to form iron colloids*
- *These colloids have a diameter of less than 50 nm, and they slowly approach the surface of the superconductor*
- *At this surface the flux lines of the Shubnikov phase exit from the superconductor*
- *The ferromagnetic iron colloid is collected at the locations where the flux lines exit from the surface, since here they find the largest magnetic field gradients*
- *In this way the flux lines can be decorated. Subsequently, the structure can be observed in an electron microscope*



Quantization of the flux

Doll and Nabauer / Deaver and Fairbank, 1961



$$\Phi = LI \qquad \Phi_0 = \frac{h}{2e}$$

Already expected by Fritz London almost thirty years before

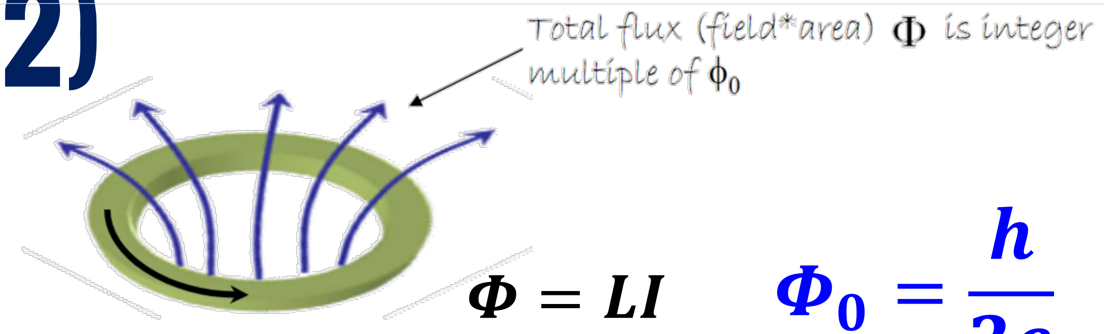
(We will met him soon)

In **macroscopic systems magnetic flux** through the ring **could take any arbitrary value**

In **type-II SC magnetic fields are concentrated in the form of flux lines**, each of which **carries a single flux quantum Φ_0**

Quantization of the flux (2)

Doll and Nabauer / Deaver and Fairbank, 1961



$$\boldsymbol{\Phi = LI} \qquad \boldsymbol{\Phi_0 = \frac{h}{2e}}$$

Extremelly difficult to prove due to small **B** necessary

$$\Phi_0 = \frac{h}{2e} = \frac{6.626 \cdot 10^{-34}\ J\ s\ [C\ V\ s]}{2 \cdot 1.602 \cdot 10^{-19}\ C} = 2.07 \cdot 10^{-15}\ V\ s = \mathbf{2.07 \cdot 10^{-15}\ T\ m^2}$$

$$\Phi_B = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{S}}\ (in\ the\ case\ of\ omogeneous\ B\ field)$$

Both groups used thin tubes with d ~ 10 μm

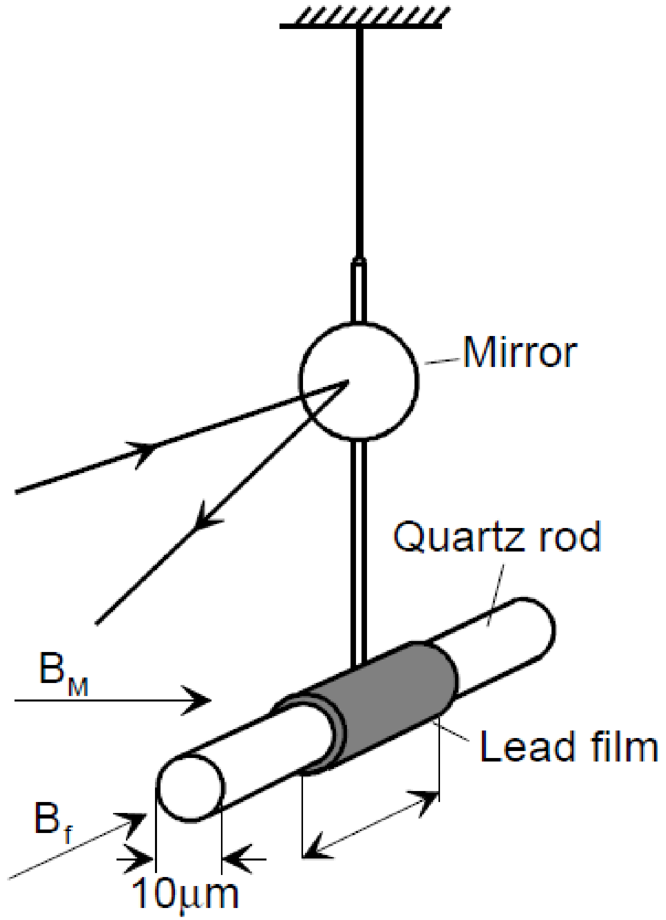
$$B = \frac{\Phi_0}{\pi r^2} = \frac{2.07 \cdot 10^{-15}\ T\ m^2}{7.85 \cdot 10^{-11}\ m} = \mathbf{2.6 \cdot 10^{-5}\ T}$$

Same order of magnitude of Earth's magnetic field

Quantization of the flux (3)

Doll and Nabauer experiment

- Permanent current is generated by cooling in a freezing field *Bf*
- Torque values were too small to be detected in a static experiment
- They measure the field to excite a torsional oscillation of the system



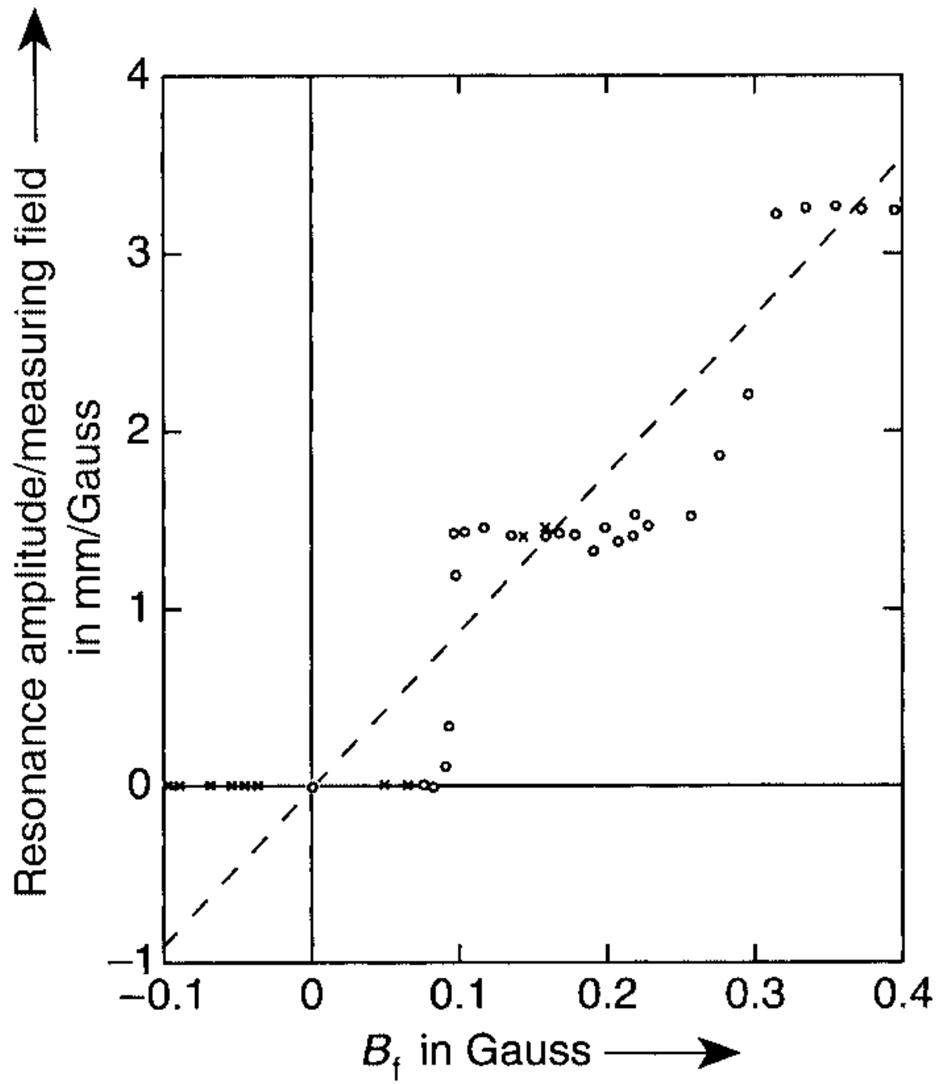
Schematics of the experimental setup of Doll and Näbauer

The quartz rod carries a small lead cylinder formed as a thin layer by evaporation. The rod vibrates in liquid helium

Quantization of the flux (4)

Doll and Nabauer experiment

- Permanent current is generated by cooling in a freezing field *Bf*
- Torque values were too small to be detected in a static experiment
- They measure the field to excite a torsional oscillation of the system
- The experiment clearly indicates a quantization of the flux in the SC cylinder



3 hallmarks of superconductor

1. **Zero resistance**

2. **Complete diagmanetism**

3. **Flux quantization**

But there is something more

No changes in crystal structure below Tc

The X-ray diffraction pattern does not change when crossing the transition temperature showing **no transition in the lattice structure**

No appreciable change in the reflectivity of the superconductor can be detected (Although the optical properties of normal metals are strictly connected with resistivity)

Photoelectric properties remain unchanged too

The elastic properties, the thermal expansion **does not change** with transition and no latent heat or volume change in absence of a magnetic field are observed.

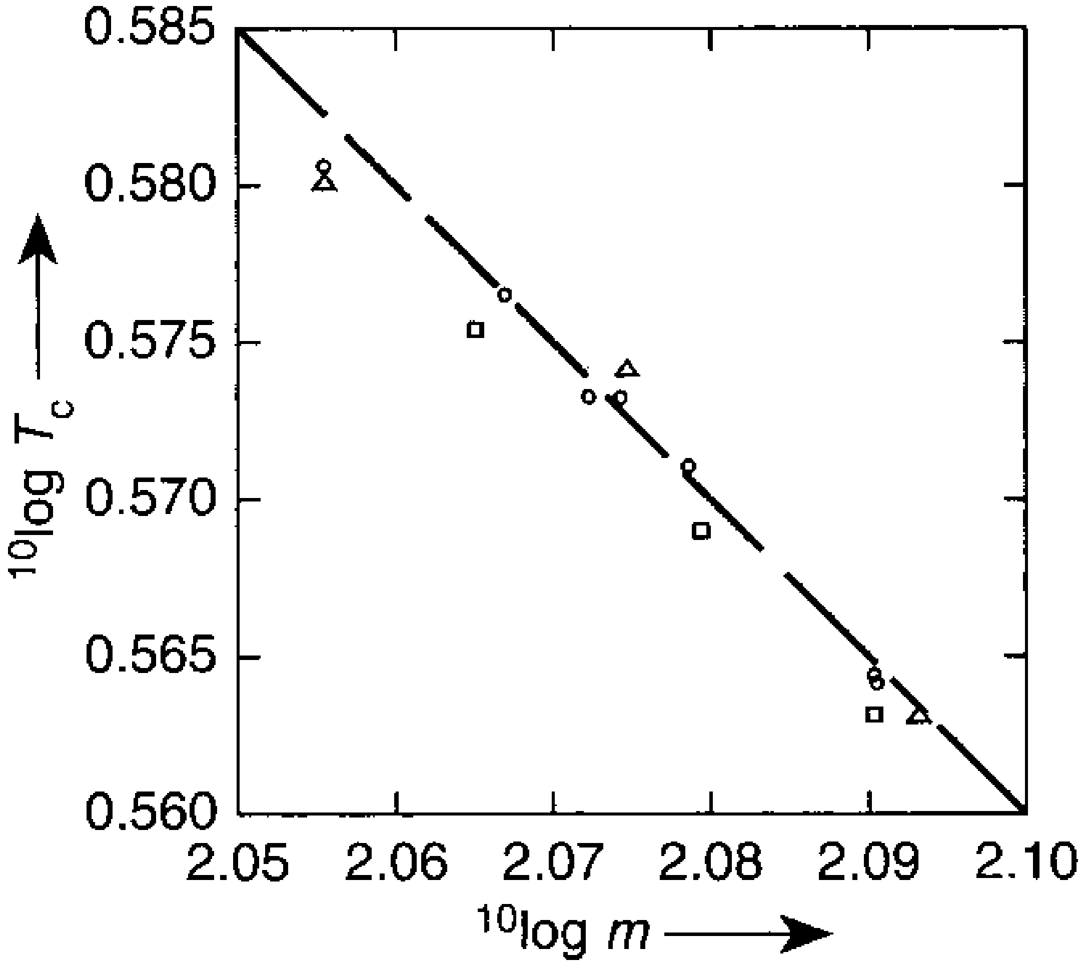
Isotopic effect, 1950

In 1950 Maxwell as well as Reynolds, Serin, Wright, and Nesbitt almost simultaneously observed in mercury a dependence of the Tc on nuclear mass

Average atomic mass	199.7	200.7	202.0	203.4
Transition temperature T_c in K	4.161	4.150	4.143	4.126

$$T_c \propto M^{-1/2}$$

As suggested by Frohlich and Bardeen



Isotope effect in tin: ○ Maxwell; □ T Lock, Pippard, and Shoenberg; D Serin, ∆ Reynolds, and Lohman

E. Maxwell: Phys. Rev. ***86****, 235 (1952); B. Serin, C. A. Reynolds, C. Lohman: Phys. Rev.* ***86****, 162 (1952); J. M. Lock, A. B. Pippard, D. Shoenberg: Proc. Cambridge Phil. Soc.* ***47****, 811 (1951).*

Isotopic effect (2)

$$T_c \propto M^{-1/2}$$

In a crystal lattice, the **frequency of vibration** is **inversely proportional** to the **square root of the mass** of the atoms (**Debye frequency**)

$$\omega_D = \pi \left(\frac{C}{M}\right)^{1/2}$$

The 'isotope effect' was a piece of evidence demonstrating that one should **not ignore the presence of the nuclei that make up most of the mass in atoms**

Whereas the non-transition metals display well the exponent **β = 1⁄2**, the transition metals show strong deviations from this value

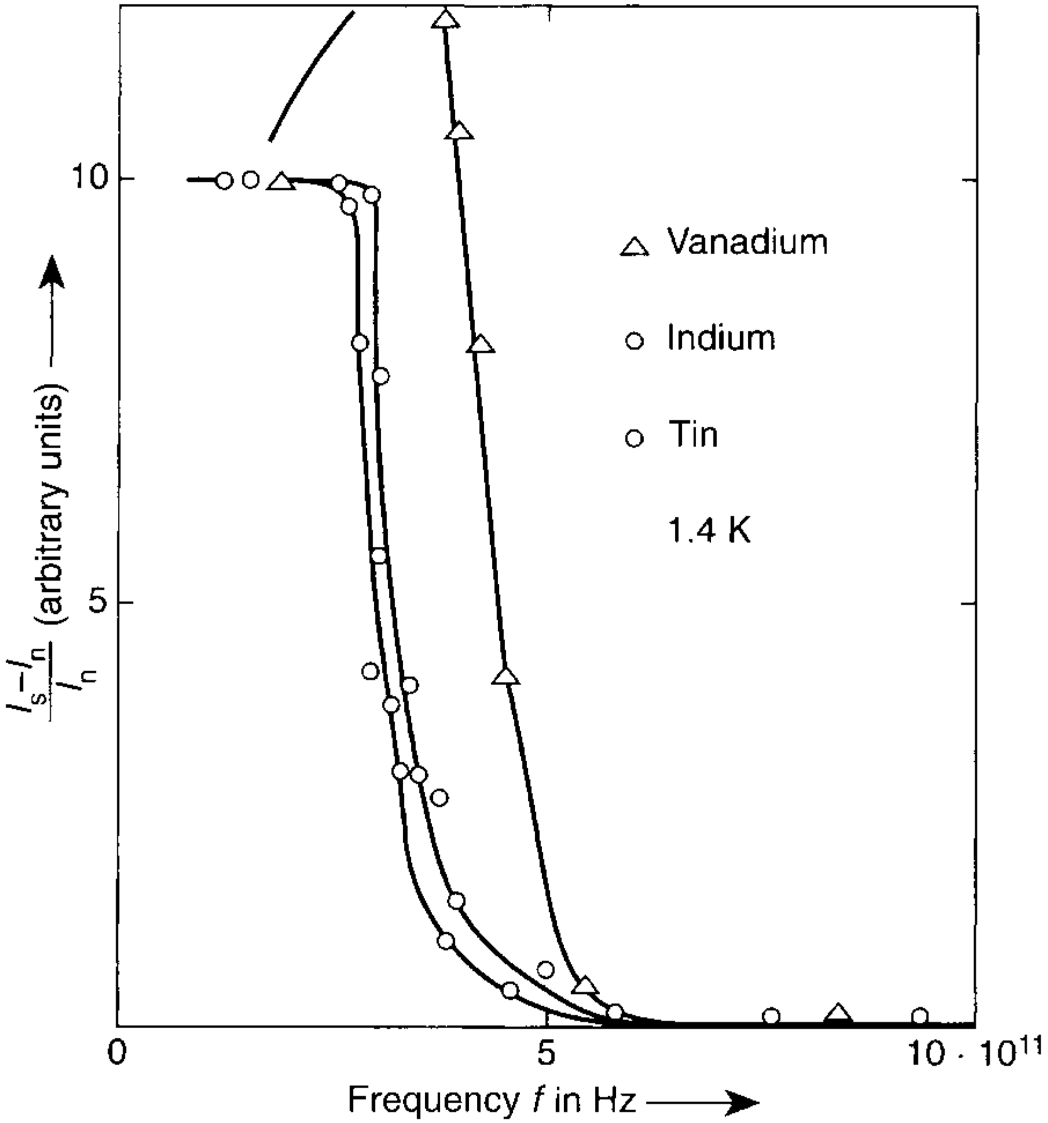
Element	Hg	Sn	Pb	Cd	Tl	Mo	Os	Ru
Isotope exponent β^*	0.50	0.47	0.48	0.5	0.5	0.33	0.2	0.0

*) The exponent β is obtained from experiment by fitting to the relation $T_c \propto M^{-\beta}$. The values shown are taken from R. D. Parks, "Superconductivity", Marcel Dekker, New York, 1969, p. 126.

Infrared absorbtion, 1956

In 1956, **Tinckham and Glover** demonstrated that **SC absorb IR radiation only above a certain threshold value**

The results of the experiment suggest the presence of an **Energy gap between the Normal and the Superconducting State**

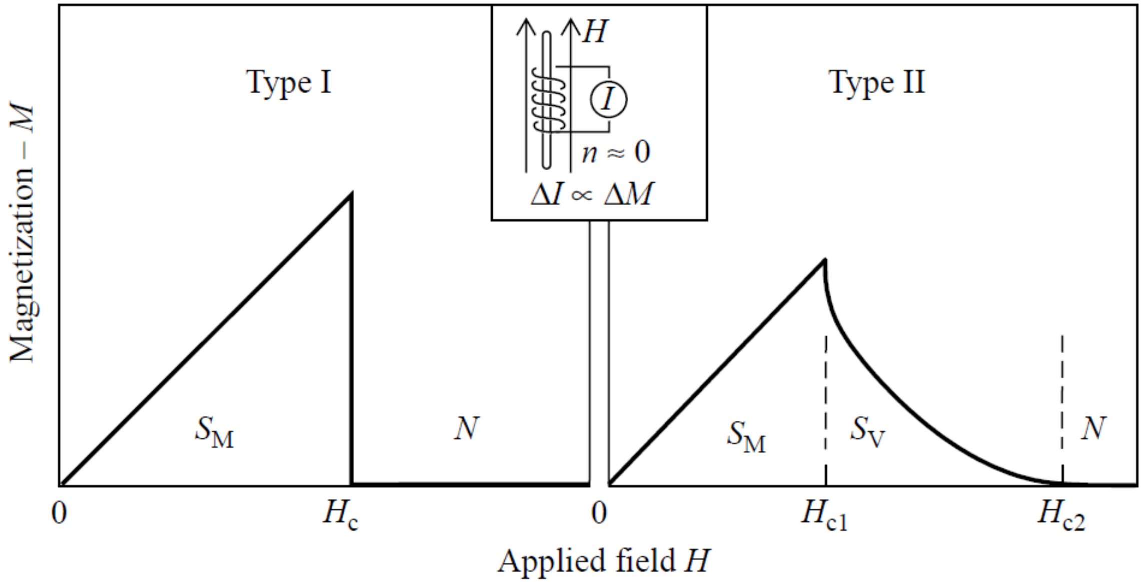


Infrared transmission of thin superconducting films

Thermodynamic of the SC state

SC state appears as a **thermodynamic state** with a **Energy gap** that **separates** it from the **NS**

Magnetic diagrams can be read as **phase diagrams**



Thermodynamic of the SC state (2)

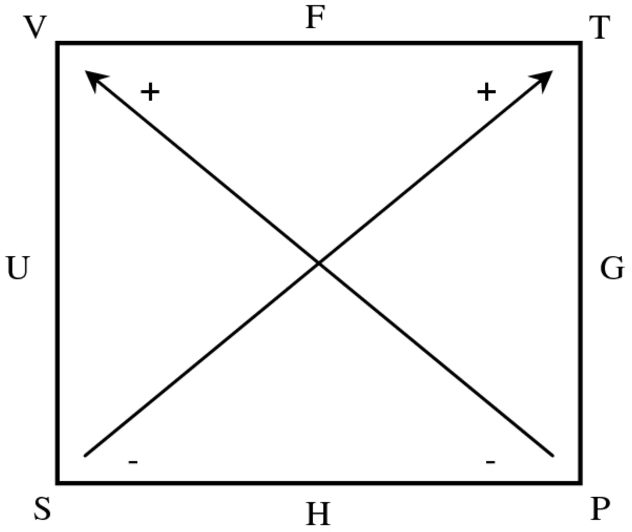
In thermodynamics you can derive
relationships between measurable quantities by analysis of free energy

$$\left(\frac{\partial G}{\partial P}\right)_T = \left(\frac{\partial H}{\partial P}\right)_S = V$$

$$\left(\frac{\partial G}{\partial T}\right)_P = \left(\frac{\partial F}{\partial T}\right)_V = -S$$

$$\left(\frac{\partial U}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_P = T$$

Thermodynamics square is a useful tool to remember the basic relationships



V Volume
F Helmotz free energy
T Absolute Temperature
G Gibbs free energy
P Pressure
H Enthalpy
S Entropy
U Internal Energy

Thermodynamic of the SC state (2)

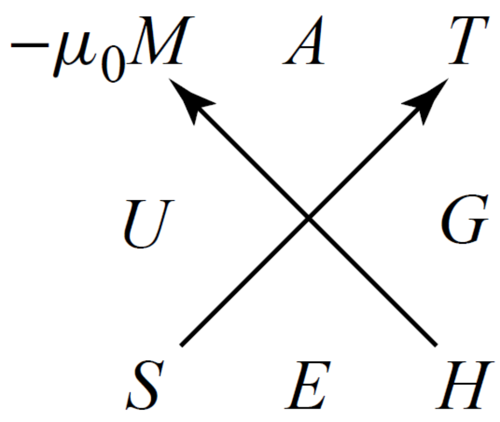
In thermodynamics you can derive
relationships between measurable quantities by analysis of free energy

$$\left(\frac{\partial G}{\partial H}\right)_T = -\mu_0 M$$

$$\left(\frac{\partial G}{\partial T}\right)_H = -S$$

$$\left(\frac{\partial U}{\partial S}\right)_M = T$$

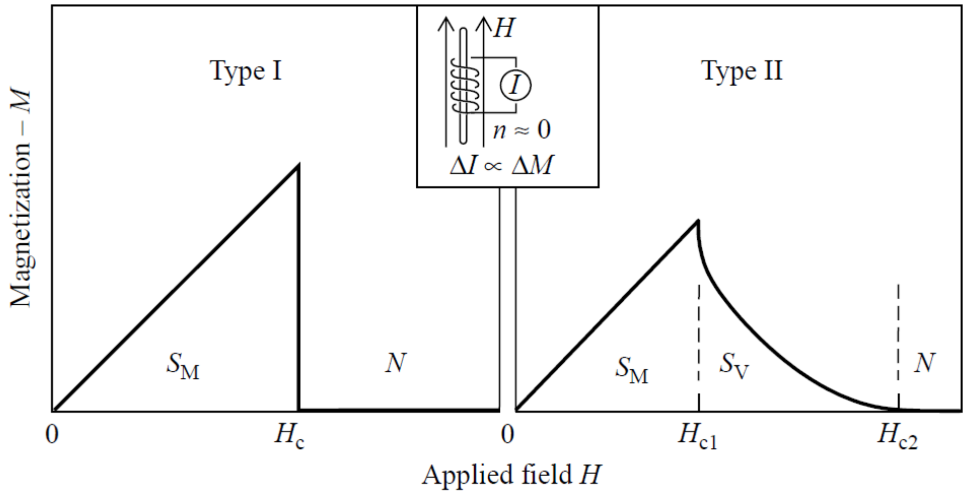
Thermodynamics square is a useful tool to remember the basic relationships



M Magnetization
A Helmotz free energy
T Absolute Temperature
G Gibbs free energy
H Applied magnetic field
E Enthalpy
S Entropy
U Internal Energy

Thermodynamic of the SC state (3)

We can obtain the Gibbs' energy *G* of the superconducting state starting from the magnetization, consider G dependent on *T* and *H*



$$\left(\frac{\partial G}{\partial H}\right)_T = -\mu_0 M \quad \Rightarrow \quad \mathrm{d}G = -\mu_0 M \mathrm{d}H$$

The relation is valid both in SC and NC. Integrate from 0 to H_c at constant T

$$\int_0^{H_c} dG_S = -\mu_0 \int_0^{H_c} M_S(T,H)dH$$

Thermodynamic of the SC state (4)

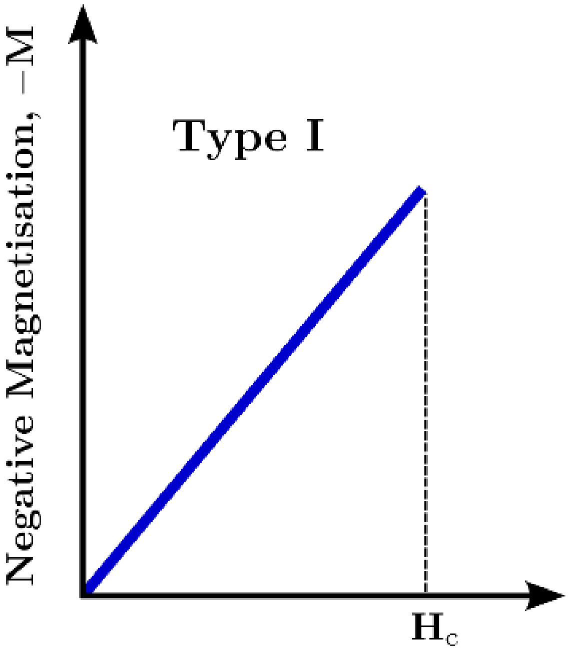
$$\int_0^{H_c} \mathrm{d}G_S = -\mu_0 \int_0^{H_c} M_S(T,H)\mathrm{d}H$$

In **type I superconductor**, the Meissner state is characterized by **M = –H**

$$\int_0^{H_c(T)} \mathrm{d}G_S = -\mu_0 \int_0^{H_c(T)} (-H)\mathrm{d}H$$

$$-\mu_0 \int_0^{H_c(T)} (-H)\mathrm{d}H = \frac{\mu_0}{2} H_c^2(T)$$

$$\int_0^{H_c(T)} \mathrm{d}G_S = G_S(T,H_C) - G_S(T,0)$$



Thermodynamic of the SC state (5)

$$G_S(T,H_C) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$$

If exist an Energy Gap, we can calculated it from the energy difference between the NS and the SC states

Remember that $dG_n = -\mu_0 M \mathrm{d} H$

For a normal metal **χ << 1** → $G_n(T,H_C) = G_n(T,0)$

$$\int_0^{H_c} dG_n = -\mu_0 \int_0^{H_c} M_n dH$$ → $\cong 0$

Thermodynamic of the SC state (5)

From the fact that the two phases coexist in the intermediate state of type I

(with a field Hc in the normal lamina and zero in the superconducting ones):

$$G_n(T,H_C) = G_S(T,H_C) \quad \text{and} \quad G_n(T,0) = G_S(T,H_C)$$

Substituting it in $G_S(T,H_C) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$

we obtain: $G_n(T,0) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$

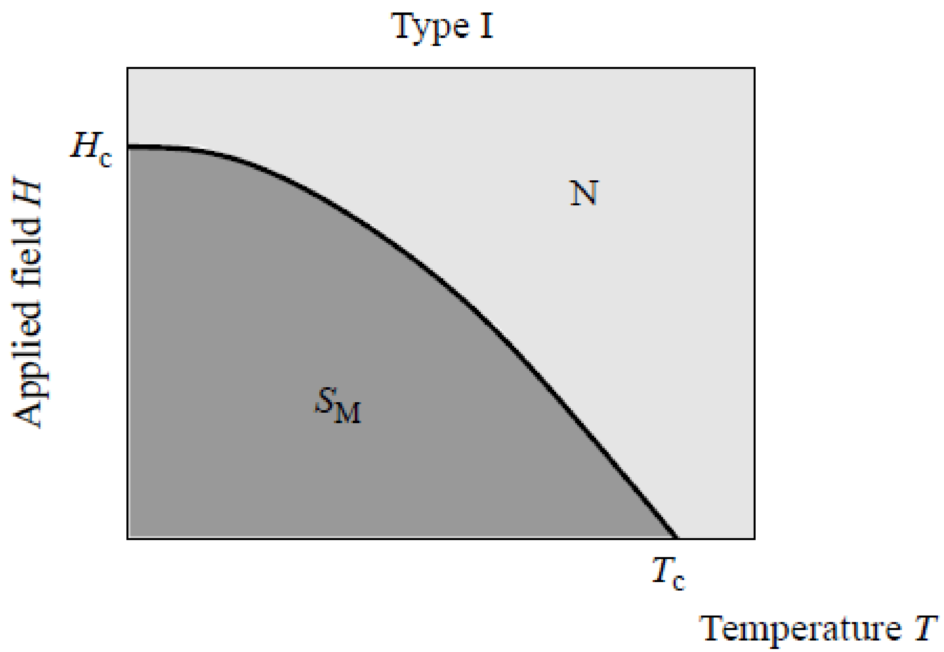
Thermodynamic of the SC state (6)

$$G_n(T,0) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$$

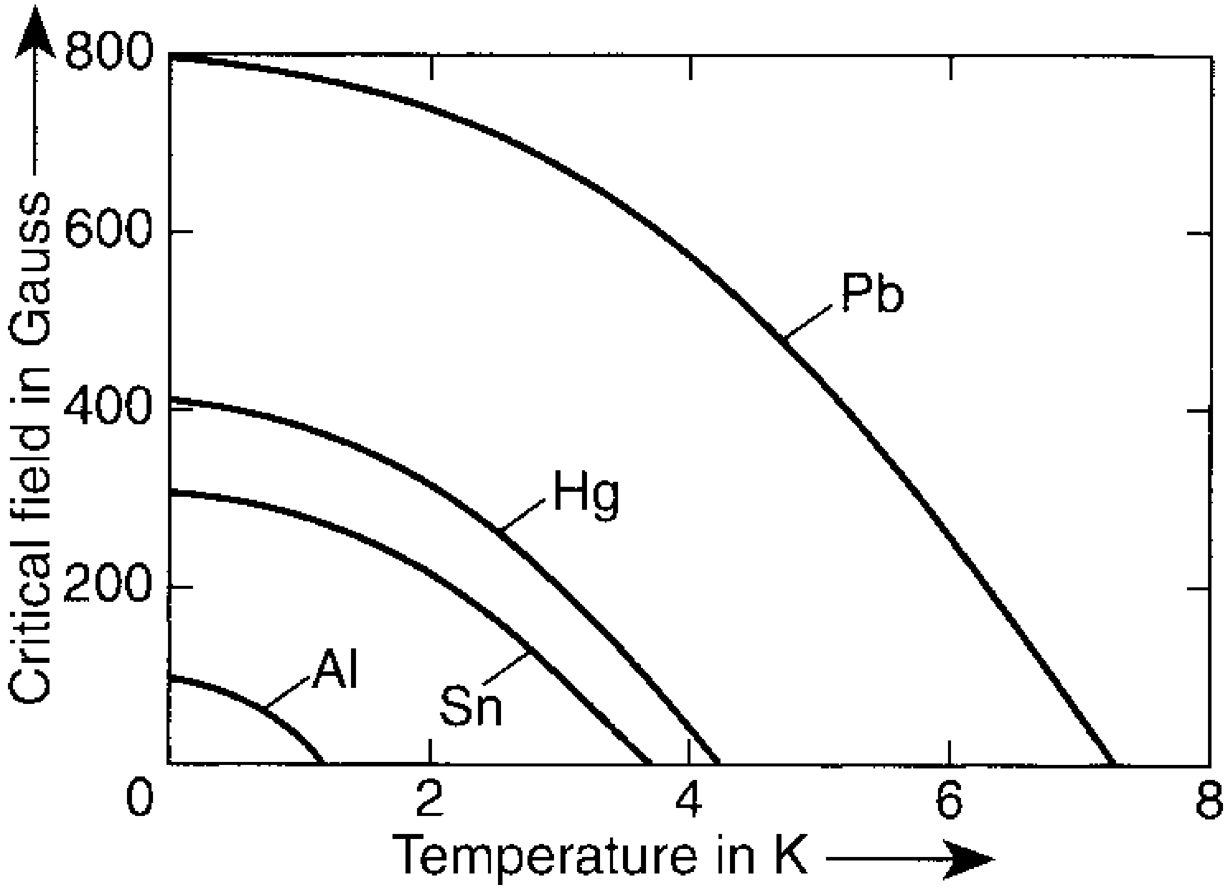
H_c^2 is in a deeper sense a measure of the "**condensation energy**" of the Meissner state

$H_c(T)$ from experimental data could be express as:

$$H_c(T) = H_C(0)\left(1 - \frac{T}{T_c}\right)^2$$



T dependence of Hc



$$H_c(T) = H_C(0)\left(1 - \frac{T}{T_c}\right)^2$$

Thermodynamic of the SC state - Entropy

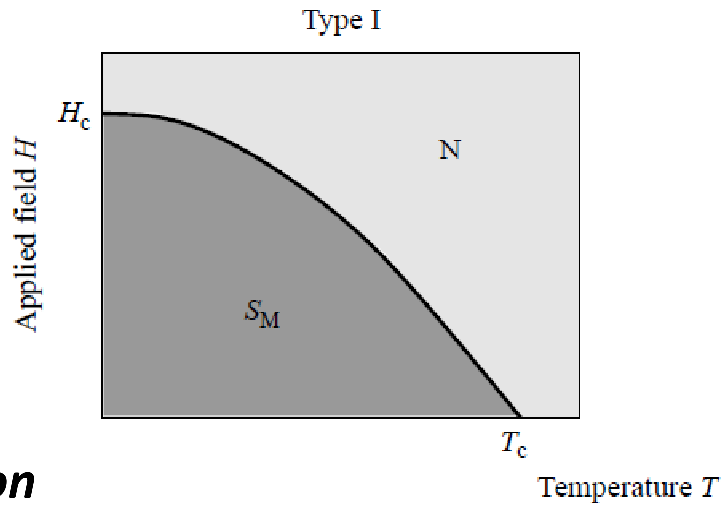
$$S_s = -\left(\frac{\partial G_s}{\partial T}\right)_H$$

We can now calculate entropy change

from the temperature derivate of: $G_n(T,0) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$

$$S_n(T,0) - S_S(T,0) = -\mu_0 H_C(T) \frac{\mathrm{d}H_c}{\mathrm{d}T}$$

$$-\mu_0 H_C(T) \frac{dH_c}{dT} \geq 0 \;\Rightarrow\; S_n(T,0) - S_S(T,0) \geq 0$$



At Tc, where Hc =0 → ΔS = 0 ⇒ ***Second order or continuous transition***

The superconducting state is characterized by greater order than the normal state

Thermodynamic of the SC state - Heat Capacity

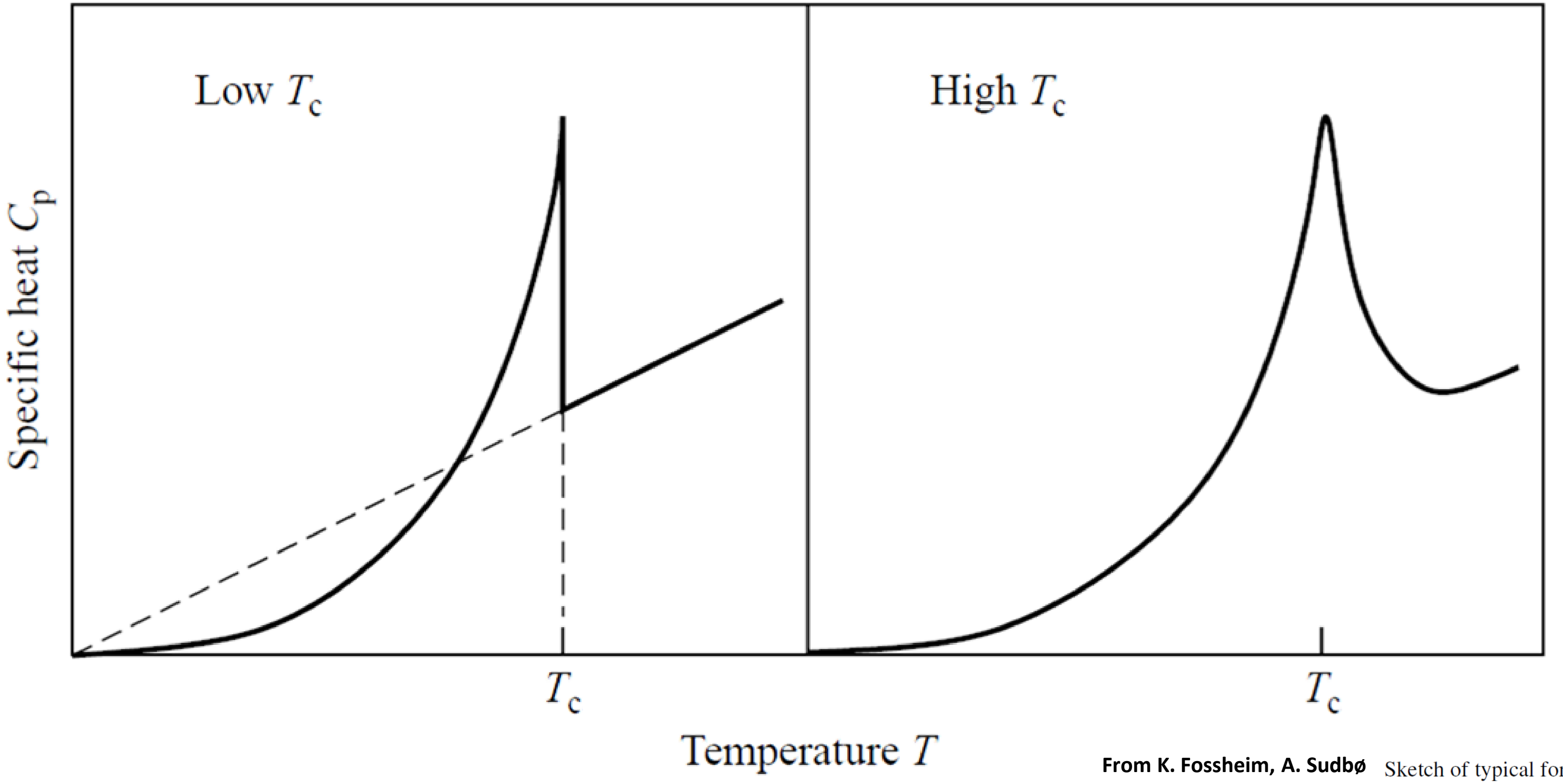
$C_H \equiv T\left(\frac{\partial S}{\partial T}\right)_H$ Remember that $S_s = -\left(\frac{\partial G_s}{\partial T}\right)_H$

We can calculate HEAT CAPACITY
from the 2nd temperature derivate of: $G_n(T,0) - G_S(T,0) = \frac{\mu_0}{2} H_c^2(T)$

$$C_S(T) - C_n(T) = \mu_0 T\left[\left(\frac{\mathrm{d}H_c}{\mathrm{d}T}\right)^2 + H_c \frac{d^2 H_c}{\mathrm{d}T^2}\right]$$

At $T_c \rightarrow H_c=0$ ➡ $C_S(T) - C_n(T) = \mu_0 T\left(\frac{dH_c}{dT}\right)^2$ **A discontinuity at Tc is expected**

Thermodynamic of the SC state - Heat Capacity



From K. Fossheim, A. Sudbø Sketch of typical forms of specific heat curves measured from above the superconducting transition in a mean-field low-T_c metallic superconductor like Al (left), and in a substance with strong superconductivity phase fluctuations like the high-T_c compound YBCO (right).

Discontinuity at Tc verified in the experiments

Heat capacity

Element	T_c in K	$(c_s - c_n)$ thermal data in 10^{-3} W · s/(mol · K)	$(c_s - c_n)$ magnetic data in 10^{-3} W · s/(mol · K)
Sn[a)]	3.72	10.6	10.6
In[a)]	3.40	9.75	9.62
Tl[b)]	2.39	6.2	6.15
Ta[a)]	4.39	41.5	41.6
Pb[b)]	7.2	52.6	41.8

a) Mapother, D. E.: IBM Journal **6**, 77 (1962).
b) Shoenberg, D.: »Superconductivity«. Cambridge University Press 1952.

From Buckel-Kleiner

Bibliography of this part

- W. Buckel, R. Kleiner, "*Superconductivity - Fundamentals and Applications*", Wiley

  Introduction

  Chapter 1 - Fundamental Properties of Superconductors

  Paragraph 4.6.1 Critical Field and Magnetization of Rod-Shaped Samples

- K. Fossheim, A. Sudbø, "*Superconductivity - Physics and applications*", Wiley

  I – Basic topics

  Chapter 1 - What is superconductivity? A brief overview