Ex1

$k \in \mathbb{R}$

$$\begin{cases} x + z = 2 \\ 2x - y + kz = 1 \\ (1+k)x - ky + z = 2 \end{cases}$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 1 \\ 2 & -1 & k \\ 1+k & -k & 1 \end{pmatrix}}_{A_k} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

$$\det A_k = \begin{vmatrix} -1 & k \\ -k & 1 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ 1+k & -k \end{vmatrix} = k(k-1)$$

$$\det A_k \neq 0 \iff \boxed{k \neq 0, 1}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = A_k^{-1} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{k+1}{k} & \frac{-1}{k-1} & \frac{1}{k(k-1)} \\ \frac{k+2}{k} & -\frac{1}{k-1} & \frac{-k+2}{k(k-1)} \\ -\frac{1}{k} & \frac{1}{k-1} & -\frac{1}{k(k-1)} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2k+2}{k} - \frac{1}{k-1} + \frac{2}{k(k-1)} \\ \frac{2k+4}{k} - \frac{1}{k-1} + \frac{-2k+4}{k(k-1)} \\ -\frac{2}{k} + \frac{1}{k-1} - \frac{2}{k(k-1)} \end{pmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

$$1 = \frac{2\overbrace{(k+1)(k-1)}^{k^2-1} - k + 2}{k(k-1)}$$

$$= \frac{2k^2 - k}{k(k-1)} = \frac{\cancel{k}(2k-1)}{\cancel{k}(k-1)} = \frac{2k-1}{k-1}$$

$$2 = \frac{2(k+2)(k-1) - k - 2k + 4}{k(k-1)} =$$

$$= \frac{2(k^2+k-2)-3k+4}{k(k-1)} = \frac{2k^2-k}{k(k-1)} =$$

$$= \frac{\cancel{k}(2k-1)}{\cancel{k}(k-1)} = \frac{2k-1}{k-1}$$

(3) $$= \frac{-2(k-1)+k-2}{k(k-1)} = -\frac{1}{k-1}$$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{2k-1}{k-1} \\ \frac{2k-1}{k-1} \\ -\frac{1}{k-1} \end{pmatrix} \quad k \neq 0, 1$$

Ex2

$k \in \mathbb{R}$

$$\begin{cases} -x+ky+2z=1 \\ -kx+y+(1+k)z=2 \\ y+z=2 \end{cases}$$

$$\underbrace{\begin{pmatrix} -1 & k & 2 \\ -k & 1 & 1+k \\ 0 & 1 & 1 \end{pmatrix}}_{A_k} \begin{pmatrix} x \\ y \\ t \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}}_{b}$$

$\rightarrow \det A_k = 0$

$\rightarrow rk(A_k) < rk(A_k|b)$

$$\det A_k = -\begin{vmatrix} -1 & 2 \\ -k & 1+k \end{vmatrix} + \begin{vmatrix} -1 & k \\ -k & 1 \end{vmatrix} =$$

$$= -(-1-k+2k) + (-1+k^2) =$$

$$= k^2 - k = k(k-1)$$

$$\det A_k = 0 \Leftrightarrow k=0 \lor k=1 .$$

• $\boxed{k=0}$.

$$A_0 = \begin{pmatrix} -1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$III = -2I + II$

$$Im(A_0) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \overset{?}{\in} Im(A_0)$$

$$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \overset{?}{=} a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} a = 1 \\ b = 2 \\ b = 2 \end{cases} \Longrightarrow \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \in Im(A_0)$$

$\Longrightarrow$ INF. SOL.

$\cdot \boxed{K = 1}$

$$A_1 = \begin{pmatrix} -1 & 1 & 2 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\mathrm{Im}(A_1) = \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \overset{?}{\in} \mathrm{Im}(A_1)$$

$$\begin{cases} a + b = 1 \\ a + b = 2 \\ b = 2 \end{cases} \qquad \begin{matrix} a = -1 \\ -1 + 2 \neq 2 \\ b = 2 \end{matrix}$$

$$\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \notin \mathrm{Im}(A_1) \Longrightarrow \text{NON HA SOL.}$$

$\boxed{K = 1}$.

Tutorato 8°

Def. Sia L un endomorfismo di V_K.

Se $\bar{v} \in V$ soddisfa

$$L(\bar{v}) = \lambda \bar{v} \qquad \bar{v} \neq \bar{0} \qquad \lambda \in K$$

$\bar{v}$ si dice AUTOVETTORE di L e λ si dice AUTOVALORE di L associato a $\bar{v}$

$\boxed{Es\ 3}$ $f: \mathbb{R}^3 \to \mathbb{R}^3 : (x, y, z) \to (x+y-z,\ 2y,\ -x+y+z)$

$$\mathcal{B} = \left\{ \underset{\bar{v}_1}{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}} \underset{\bar{v}_2}{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}} \underset{\bar{v}_3}{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}} \right\}$$

a) $\mathcal{N} = e_1, e_2, e_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$f(e_1) = (1, 0, -1) \neq \lambda e_1 \quad \forall \lambda \in \mathbb{R} \Rightarrow$ NO AUTOVETTORE

$f(e_2) = (1, 2, 1) \neq \lambda e_2$

$f(e_3) = (-1, 0, 1) \neq \lambda e_3$

b) $f(\bar{v}_1) = (0, 2, 2) = \lambda \bar{v}_1 \qquad \lambda = 2$

$f(\bar{v}_2) = (0, 0, 0) = \lambda \bar{v}_2 \qquad \lambda = 0$

$f(\bar{v}_3) = (2, 2, 0) = \lambda \bar{v}_3 \qquad \lambda = 2$

c) $$A_f{}^{\mathcal{N}}_{\mathcal{N}} = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

per trovare $A_f{}^{\mathcal{B}}_{\mathcal{B}}$ vale

$f(\bar{v}_1) = a_{11} \bar{v}_1 + a_{21} \bar{v}_2 + a_{31} \bar{v}_3 = 2\bar{v}_1 + 0\bar{v}_2 + 0\bar{v}_3$

$f(\bar{v}_2) = a_{12} \bar{v}_1 \quad a_{22} \bar{v}_2 \quad a_{32} \bar{v}_3 = 0\bar{v}_1 + 0\bar{v}_2 + 0\bar{v}_3$

$f(\bar{v}_3) = a_{13} \bar{v}_1 \quad a_{23} \bar{v}_2 \quad a_{33} \bar{v}_3 = 0\bar{v}_1 + 0\bar{v}_2 + 2\bar{v}_2$

$$A_f{}^{B}_{B} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(c) $A_f{}^{B}_{B}$ e' diagonale $\Rightarrow$ diagonalizzabile

$\boxed{Ex\ 4}$

$A \in M_2(\mathbb{R})$

$\rightarrow$ Autovettori

$$w_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

$$\lambda_1 = 0 \qquad \lambda_2 = 2$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

- $A w_1 = \lambda_1 w_1 \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
- $A w_2 = \lambda_2 w_2 \longrightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ -3 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$$\begin{cases} 3a + b = 0 \longrightarrow b = -3a \\ 3c + d = 0 \longrightarrow d = -3c \\ a - 3b = 2 \longrightarrow a + 9a = 2 \\ c - 3d = -6 \longrightarrow c + 9c = -6 \end{cases}$$

$$\begin{cases} a = 1/5 \\ c = -3/5 \\ b = -3/5 \\ d = 9/5 \end{cases} \Rightarrow A = \begin{pmatrix} 1/5 & -3/5 \\ -3/5 & 9/5 \end{pmatrix}$$