5° TUTORATO

1)

$$A_k = \begin{pmatrix} 1 & 2-k & 1 \\ -1 & -k & -3 \\ k & 1 & k+1 \end{pmatrix}$$

a) Determinare k t.c. A_k abbia Rango 3

k | i vettori sono linearmente indipendenti

$$\begin{cases} a + (2-k)b + c = 0 \quad \rightarrow \quad c = -a + (k-2)b \\ -a - kb - 3c = 0 \quad \rightarrow \quad -a - kb + 3a - 3kb + 6b = 0 \\ ka + b + (k+1)c = 0 \end{cases}$$

$$\cancel{ka} + b + (k+1)(k-2)b - (\cancel{k}+1)a = 0$$

$$2a = 4kb - 6b$$

$$a = 2kb - 3b = (2k-3)b$$

$$c = (-2k + 3 + k - 2)b = (1-k)b$$

$$b = b$$

$$b + (k+1)(k-2)b - (2k-3)b = 0$$

$$(k^2 - k - 2 - 2k + 3 + 1)b = 0$$

$$(k^2 - 3k + 2)b = 0$$

$$\neq 0$$

$$\Leftrightarrow \quad (k-1)(k-2) \neq 0 \quad \Leftrightarrow \quad k \neq 1 \vee k \neq 2$$

$\mathrm{rk}\, A_k = 3$ se $k \neq 1, 2$

(b) $rk\, A_k \neq 3$ $k = 1, 2$ determinare $\ker L$ $\mathrm{Im}\, L$

$\boxed{k=1}$

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -3 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\ker A_1 \begin{cases} x + y + z = 0 \\ -x - y - 3z = 0 \\ x + y + 2z = 0 \end{cases} \longrightarrow \begin{array}{l} y = -x \\ \cancel{+y} + 2z \cancel{-y} - 3z = 0 \quad \boxed{-z = 0} \\ x = -y - 2z \end{array}$$

$$\begin{cases} x = x \text{ qualsiasi} \\ y = -x \\ z = 0 \end{cases} \Rightarrow \ker A_1 = \left\langle \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$

$$\mathrm{Im}\, A_1 = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \right\rangle$$

$k = 2$

$$A_2 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & -2 & -3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$\begin{array}{c} I \\ I+II \\ I+III \end{array} \begin{pmatrix} 1 & 0 & 1 \\ 0 & -2 & -2 \\ 0 & 1 & 1 \end{pmatrix} \underset{-2III-II}{\leadsto} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x + z = 0 \\ y + z = 0 \end{cases} \quad \begin{cases} x = -z \\ y = -z \end{cases}$$

$$\ker A_2 = \langle (-1, -1, 1) \rangle$$

$$\operatorname{Im} A_2 = \left\langle \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \right\rangle$$

(c) per $k=0$ determinare

$$A_0^{-1}(\{(1,0,0)\})$$

$$A_0 = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & -3 \\ 0 & 1 & 1 \end{pmatrix} \qquad v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$A_0 \cdot v = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} x + 2y + z = 1 \\ -x - 3z = 0 \\ y + z = 0 \end{cases} \longrightarrow \begin{array}{l} x = 3/4 \\ y = 1/4 \\ z = -1/4 \end{array}$$

$$\Rightarrow \begin{pmatrix} 3/4 \\ 1/4 \\ -1/4 \end{pmatrix}$$

(d) Per i valori di k per i quali A_k non ha $\operatorname{rk} A_k = 3$

determinare $A^{-1}(\{(2,-4,5)\})$

$k=1$

$$A_1 = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -3 \\ 1 & 1 & 2 \end{pmatrix}$$

$$\operatorname{Im} A_1 = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \right\rangle$$

$$\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} = a \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + b \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$$

$$-2+b-3b=-4$$
$$-2b=-2$$
$$\boxed{b=1}$$

$$a+b=2 \longrightarrow a=2-b=1$$
$$-a-3b=-4 \longrightarrow -(2-b)-3b=-4$$
$$a+2b=5 \longrightarrow (1)+2(1)\neq 5$$

$$A_1^{-1}(\{(2,-4,5)\}) = \emptyset$$

$\boxed{e=2}$

$$\operatorname{Im} A_2 = \left\langle \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \right\rangle$$

$$\boxed{b=1}$$

$$\begin{cases} a=2 \\ -a-2b=-4 \longrightarrow -2-2b=-4 \\ 2a+b=5 \longrightarrow 2(2)+1=5 \ \checkmark \end{cases}$$

$$A_2 = \begin{pmatrix} 1 & 0 & 1 \\ -1 & -2 & -3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$A_2 \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ -1 & -2 & -3 & -4 \\ 2 & 1 & 3 & 5 \end{array}\right)$$

$$\Longrightarrow \quad \begin{matrix} I + II \\ III - 2II \end{matrix} \left(\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 2 \\ 0 & \boxed{1} & 1 & 1 \\ \cancel{0} & \cancel{1} & \cancel{1} & \cancel{1} \end{array}\right)$$

$$\begin{cases} x + z = 2 \\ y + z = 1 \end{cases} \qquad \begin{aligned} x &= 2 - z \\ y &= 1 - z \\ z &= z \end{aligned}$$

$$A_2^{-1}(\{(2,-4,5)\}) = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

$$A_k = \begin{pmatrix} \boxed{1} & 0 & 1 \\ 2 & -1 & k \\ 1+k & -k & +1 \end{pmatrix}$$

$k \in \mathbb{R}$ A_k è invertibile e per tali valori trovare A_k^{-1}

$$A_k^{-1} = \frac{1}{\det(A_k)} \cdot S^T$$

$$= \left(\begin{matrix} 1 & 0 & 1 \\ 2 & -1 & k \\ 1+k & -k & 1 \end{matrix}\right) \begin{matrix} 1 & 0 \\ 2 & -1 \\ 1+k & -k \end{matrix}$$

$$(-1) + 0 - 2k - [-(1+k) - k^2 + 0]$$

$$= \cancel{-1} - 2k + \cancel{1} + k + k^2 + 0 = \boxed{k^2 - k}$$

$$\text{DET } A_k = k^2 - k \neq 0$$

$$k(k-1) \neq 0$$

$$\boxed{\begin{matrix} k \neq 0 \\ k \neq 1 \end{matrix}}$$

$$\text{Cof}(a_{ij}) = (-1)^{i+j} \cdot C_{ij}$$

minimo complementare

$$\begin{pmatrix} a_{ij} & a_{ij} \cdots & \cdots \\ & & \\ & & \end{pmatrix}$$

$$S = \begin{pmatrix} k^2-1 & k^2+k-2 & 1-k \\ -k & -k & k \\ 1 & -k+2 & -1 \end{pmatrix}$$

$$S_{11} = (-1)^{1+1} \cdot \begin{vmatrix} -1 & k \\ -k & 1 \end{vmatrix} = 1 \cdot (-1 + k^2) = k^2 - 1$$

$$S_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & k \\ 1+k & 1 \end{vmatrix} = (-1)(2 - k(1+k)) = -2 + k + k^2$$

$$S_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & -1 \\ 1+k & -k \end{vmatrix} = -2k + 1 + k = -k + 1$$

$$S_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 1 \\ -k & 1 \end{vmatrix} = 0(-1)(k) = -k$$

$$S_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 1 \\ 1+k & 1 \end{vmatrix} = 1 - (1+k) = \boxed{-k}$$

$$S = \begin{pmatrix} k^2-1 & k^2+k-2 & 1-k \\ -k & -k & k \\ 1 & -k+2 & -1 \end{pmatrix}$$

$$S^T = \begin{pmatrix} k^2-1 & -k & 1 \\ k^2+k-2 & -k & -k+2 \\ 1-k & k & -1 \end{pmatrix}$$

Invertire righe con colonne!

$$A_k^{-1} = \frac{1}{k(k-1)} \begin{pmatrix} k^2-1 & -k & 1 \\ k^2+k-2 & -k & -k+2 \\ 1-k & k & -1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{k+1}{k} & \frac{-1}{k-1} & \frac{1}{k(k-1)} \\ \frac{k+2}{k} & -\frac{1}{k-1} & \frac{-k+2}{k(k-1)} \\ \frac{-k+1}{k(k-1)} & \frac{1}{k-1} & -\frac{1}{k(k-1)} \end{pmatrix}$$

$$\boxed{k \neq 0, 1}$$