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ES 2 appello 19/06/2019

$$W=\left\langle \underset{v_1}{\begin{pmatrix}1\\1\\1\end{pmatrix}} \underset{v_2}{\cancel{\begin{pmatrix}0\\1\\-2\end{pmatrix}}} \underset{v_3}{\begin{pmatrix}1\\3\\-3\end{pmatrix}} \right\rangle \qquad \underline{\underline{\varphi: W \longrightarrow \mathbb{R}^3}} \text{ t.c. } \varphi\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$$\varphi\begin{pmatrix}1\\3\\-3\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}$$

ⓐ $A_{B\mathcal{E}}(\varphi)=?$ $B, \mathcal{E}$ arbitrarie

ⓑ base e dim. di $\operatorname{Ker}(\varphi), \operatorname{Im}(\varphi)$?

ⓒ trovare $\phi: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ t.c. $\phi(w)=\varphi(w)\ \forall w \in W$

$\operatorname{Im}(\phi)=\operatorname{Im}(\varphi)$

ⓐ $W=\left\langle \begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\3\\-3\end{pmatrix} \right\rangle$ $\qquad \mathcal{E}$ scegliamo $\mathcal{E}=\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$

$\boxed{v_1 = v_3 - 2v_2}$ $\qquad \underline{\underline{B=\left\{\begin{pmatrix}1\\1\\1\end{pmatrix}\begin{pmatrix}1\\3\\-3\end{pmatrix}\right\}}}$

$$\underset{\underset{M_{3\times2}(\mathbb{R})}{\Uparrow}}{A_{B\mathcal{E}}(\varphi)} = \begin{pmatrix}2 & 1\\2 & 1\\0 & 0\end{pmatrix}$$

$$\boxed{u=\underset{\hookrightarrow \text{coo. in } B}{\begin{pmatrix}a\\b\end{pmatrix}} \in W \iff u = a\,v_1 + b\,v_3}$$

ⓑ $\operatorname{Im}(\varphi)=\left\langle \begin{pmatrix}2\\2\\0\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix} \right\rangle = \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rangle$ $\qquad \dim(\operatorname{Im}\varphi)=\operatorname{rk}\varphi=1$

$$\Downarrow$$

$$\dim(\operatorname{Ker}\varphi)=\dim W-\operatorname{rk}\varphi=2-1=1$$

$$\operatorname{Ker}\varphi=\left\langle \begin{pmatrix}1\\-2\end{pmatrix} \right\rangle_B = \langle 1v_1 - 2v_3 \rangle$$

$$= \left\langle \begin{pmatrix}-1\\-5\\7\end{pmatrix} \right\rangle_{\mathbb{R}^3}$$

$$\begin{pmatrix}2 & 1\\2 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}1\\-2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

ⓒ $\phi : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

$$\mathcal{B}' = \Big\{ \underbrace{\overset{v_1}{\begin{pmatrix}1\\1\\1\end{pmatrix}} \overset{v_3}{\begin{pmatrix}1\\3\\-3\end{pmatrix}}}_{\mathcal{B}} \overset{v_4}{\begin{pmatrix}0\\0\\1\end{pmatrix}} \Big\}$$

$$\underset{\substack{\cap\!\!\!\!| \\ M_{3\times3}(\mathbb{R})}}{A_{\mathcal{B}'\mathcal{E}}(\phi)} = \begin{pmatrix}2 & 1 & \\ 2 & 1 & v \\ 0 & 0 & \end{pmatrix} \quad \text{t.c.} \quad Im(\varphi) = Im(\phi)$$

$$Im(\varphi) = \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rangle = \left\langle \cancel{\begin{pmatrix}2\\2\\0\end{pmatrix}} \begin{pmatrix}1\\1\\0\end{pmatrix} v \right\rangle = Im(\phi)$$

$$\left\langle \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rangle = \left\{ a\begin{pmatrix}1\\1\\0\end{pmatrix} \Big| \, a \in \mathbb{R} \right\}$$

$$\left\langle \cancel{\begin{pmatrix}2\\2\\0\end{pmatrix}} \begin{pmatrix}1\\1\\0\end{pmatrix} v \right\rangle = \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix} v \right\rangle = \left\{ a\begin{pmatrix}1\\1\\0\end{pmatrix} + bv \, \Big| \, a,b \in \mathbb{R} \right\}$$

$\begin{pmatrix}1\\1\\0\end{pmatrix}$ e v sono lin. dip.

$$\Updownarrow$$

$$a\begin{pmatrix}1\\1\\0\end{pmatrix} + bv = 0 \Leftrightarrow a,b=0$$

$$\Updownarrow$$

$$v = -\frac{a}{b}\begin{pmatrix}1\\1\\0\end{pmatrix} = \alpha\begin{pmatrix}1\\1\\0\end{pmatrix}$$

con $\alpha = -\frac{a}{b}$

$\Rightarrow$ scegliamo $v = \begin{pmatrix}1\\1\\0\end{pmatrix}$

$\Rightarrow A_{\mathcal{B}'\mathcal{E}}(\phi) = \begin{pmatrix}2 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 0\end{pmatrix}$ ✓

$\phi(w) = \varphi(w) \quad \forall w \in W$

$Im\,\varphi = Im\,\phi$

$(0\ 0\ 0)$

$$\operatorname{Im}\varphi = \operatorname{Im}\phi$$

F10/E3 punto ⓔ

$$U = \left\langle \begin{pmatrix}2\\0\\1\\-1\end{pmatrix}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\begin{pmatrix}1\\-1\\-1\\0\end{pmatrix}\right\rangle \text{ in } \mathbb{R}^4$$

ⓔ trovare i vettori di $W = \left\langle \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right\rangle$ t.c. la proiez. $\perp$ su U sia in $V := \left\langle \begin{pmatrix}1\\0\\0\\1\end{pmatrix}\right\rangle$

modo 1

$$P_U : \mathbb{R}^4 \longrightarrow \mathbb{R}^4$$

$$\operatorname{Im}(P_U) = U$$

$v \in W$ e t.c. $\underbrace{P_U(v)}_{\in \operatorname{Im}(P_U)} \in V \Rightarrow P_U(v) \in V \cap \operatorname{Im}(P_U) = V \cap U$

$$\underbrace{\left\langle \overset{U}{\overset{v_1}{\begin{pmatrix}2\\0\\1\\-1\end{pmatrix}}\overset{v_2}{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}}\overset{v_3}{\begin{pmatrix}1\\-1\\-1\\0\end{pmatrix}}} \right\rangle} \cap \left\langle \overset{V}{\overset{v_4}{\begin{pmatrix}1\\0\\0\\1\end{pmatrix}}} \right\rangle = \langle 0 \rangle$$

$$\boxed{U = \left\langle \underset{v_1 - v_2}{\begin{pmatrix}1\\0\\0\\-1\end{pmatrix}}\underset{v_2}{\begin{pmatrix}1\\0\\1\\0\end{pmatrix}}\underset{v_3+v_2}{\begin{pmatrix}2\\-1\\0\\0\end{pmatrix}} \right\rangle}$$

$$\begin{cases} x = a + b + 2c \\ y = -c \\ z = b \\ t = -a \end{cases} \Rightarrow \overset{\text{eq. cart. di } U}{\left\{ x + 2y - z + t = 0 \right.}$$

v_4 non le risolve

$$1 + 2 \cdot 0 - 0 + 1 = 0$$
$$\Updownarrow$$
$$2 = 0 \quad ↯$$

$$\dim U = 3 \Rightarrow \dim U^\perp = 4 - 3 = 1$$

$v \in W$ t.c. $P_U(v) \in V$

$$\Updownarrow$$

$v \in W$ t.c. $P_U(v) = 0_{\mathbb{R}^4} \Leftrightarrow v \in \operatorname{Ker}(P_U) = U^\perp$

$\Rightarrow$ cerchiamo gli elementi di $W \cap U^\perp$

$\Rightarrow$ cerchiamo gli elementi di $W \cap U^\perp$

$$U^\perp : \begin{cases} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} = 0 \\ \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = 0 \\ \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} = 0 \end{cases} \Leftrightarrow \begin{cases} x - t = 0 \\ x + z = 0 \\ 2x - y = 0 \end{cases} \Rightarrow U^\perp = \left\langle \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

$$\Updownarrow$$

$$\begin{cases} t = x \\ z = -x \\ y = 2x \end{cases} \Leftrightarrow \left\{ \begin{pmatrix} x \\ 2x \\ -x \\ x \end{pmatrix} \middle| \; x \in \mathbb{R} \right\} = \left\langle \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

$$W \cap U^\perp = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle \cap \left\langle \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} \right\rangle = \langle 0 \rangle$$

$\Rightarrow$ l'unico $v \in W$ t.c. $P_U(v) \in V$

è $v = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

modo 2

$$v \in W \Leftrightarrow v = \begin{pmatrix} a \\ 0 \\ 0 \\ b \end{pmatrix} \qquad U^\perp = \left\langle \underset{u}{\begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}} \right\rangle$$

$$P_U(v) = v - \frac{v \cdot u}{\|u\|^2} u = v - \frac{a+b}{7} \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 6a - b \\ -2a - 2b \\ a + b \\ -a + 6b \end{pmatrix} \in V = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$$

$$\Downarrow$$

$$V : \begin{cases} x - t = 0 \\ y = 0 \\ z = 0 \end{cases}$$

$$\Rightarrow \begin{cases} (6a - b) - (-a + 6b) = 0 \\ -2a - 2b = 0 \\ a + b = 0 \end{cases}$$

} sono la stessa

$$\Rightarrow \begin{cases} \cancel{7}a - \cancel{7}b = 0 \\ a + b = 0 \end{cases} \Leftrightarrow \begin{cases} a = 0 \\ b = 0 \end{cases} \Rightarrow v = \begin{pmatrix} a \\ 0 \\ 0 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

F10/E4 Punto ⓑ

$\mathbb{R}^4$ trovare sistema lineare che abbia come soluzione V

con $V^{\perp} = \left\langle \begin{pmatrix}1\\1\\0\\1\end{pmatrix} \begin{pmatrix}1\\0\\1\\0\end{pmatrix} \right\rangle$

$$(V^{\perp})^{\perp} = V$$

$$V: \begin{cases} \begin{pmatrix}x\\y\\z\\t\end{pmatrix} \cdot \begin{pmatrix}1\\1\\0\\1\end{pmatrix} = 0 \\ \begin{pmatrix}x\\y\\z\\t\end{pmatrix} \cdot \begin{pmatrix}1\\0\\1\\0\end{pmatrix} = 0 \end{cases} \Rightarrow V: \begin{cases} x+y+t = 0 \\ x+z = 0 \end{cases} \Rightarrow V = \left\langle \begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \begin{pmatrix}1\\-1\\-1\\0\end{pmatrix} \right\rangle$$

FII / EI

$\mathbb{A}^3(\mathbb{R})$ $\quad r: \begin{cases} x-y+z=2 \\ 3x-y-z=2 \end{cases}$ $\quad S = \begin{pmatrix}1\\-1\\2\end{pmatrix} + \left\langle \overset{\checkmark}{\begin{pmatrix}1\\1\\0\end{pmatrix}} \right\rangle$

ⓐ r in parametriche ?

ⓑ eq. cart. di s ?

ⓒ posizione tra r e s ?

ⓓ piano π per $\begin{pmatrix}1\\1\\1\end{pmatrix}$ t.c. $\pi /\!/ r$, $\pi /\!/ S$

ⓐ $\mathrm{II} - 3\mathrm{I}$

$$\begin{cases} x-y+z=2 \\ 2y-4z=-4 \end{cases} \Rightarrow \begin{cases} x-y+z=2 \\ y=2z-2 \end{cases} \Rightarrow \begin{cases} x = 2z\cancel{-2}-z\cancel{+2} = z \\ y = 2z-2 \end{cases}$$

$$\Rightarrow \left\{ \begin{pmatrix}z\\2z-2\\z\end{pmatrix} \middle| z \in \mathbb{R} \right\} = \begin{pmatrix}0\\-2\\0\end{pmatrix} + \left\langle \overset{\checkmark}{\begin{pmatrix}1\\2\\1\end{pmatrix}} \right\rangle : r$$

ⓑ

$$\begin{cases} x = 1+a \\ y = -1+a \\ z = 2 \end{cases} \Rightarrow \begin{cases} a = x-1 \\ y = -1+x-1 \\ z = 2 \end{cases} \Rightarrow \begin{cases} z = 2 \\ x-y = 2 \end{cases} : S$$

ⓒ

$$r\cap s = \left\{\begin{array}{l} \left.\begin{array}{l} x-y+z=2\\ 3x-y-z=2\end{array}\right| \mapsto r \\ \left.\begin{array}{l} z=2\\ x-y=2\end{array}\right| \mapsto s\end{array}\right. \Rightarrow \left\{\begin{array}{l} x-y+\cancel{z}=\cancel{2} \\ \sim\sim\sim \\ \sim\sim\sim \\ x-y=2\end{array}\right. \quad ↯$$

$$\Rightarrow r\cap s=\emptyset$$

$$U\cap V = \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix}\right\rangle \cap \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}\right\rangle = \langle 0\rangle \Rightarrow$$ r e s sono sghembe

ⓓ $\pi: P + W$ t.c. $\dim W = 2$

- $\pi \parallel r \iff V \leq W$ $\qquad$ $V\cap U = \langle 0\rangle \Rightarrow \dim(V+U)=2$
- $\pi \parallel s \iff U \leq W$ $\qquad$ $\Rightarrow V+U \leq W$

$$\Downarrow$$

$$V+U = W$$

$$\Rightarrow \pi: P+(V+U) \Rightarrow P+\left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}\right\rangle$$

- $\begin{pmatrix}1\\1\\1\end{pmatrix} \in \pi \Rightarrow$ possiamo scegliere $P=\begin{pmatrix}1\\1\\1\end{pmatrix}$

$$\Rightarrow \pi: \begin{pmatrix}1\\1\\1\end{pmatrix} + \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}\right\rangle$$

FII/E2

$\mathbb{A}^3(\mathbb{R})$ $\qquad$ $\pi_1: 2x-y=2$ $\qquad$ $\pi_2: \overbrace{\begin{pmatrix}1\\-1\\1\end{pmatrix}}^{P} \overset{\in}{+} \overbrace{\left\langle \begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\right\rangle}^{V} = \begin{pmatrix}0\\0\\0\end{pmatrix} + V = V$

ⓐ posizione tra π_1 e π_2

ⓑ retta // a π_1 e π_2 complanare con $\begin{pmatrix}1\\-1\end{pmatrix} + \left\langle \begin{pmatrix}1\\1\end{pmatrix}\right\rangle$

ⓑ retta // a π_1 e π_2 complanare con $\begin{pmatrix}1\\-1\\2\end{pmatrix}+\left\langle\begin{pmatrix}1\\1\\0\end{pmatrix}\right\rangle$

ⓒ eq. cart. di σ piano t.c. $\sigma \supseteq \pi_1 \cap \pi_2$ e σ // a $\begin{pmatrix}1\\-1\\2\end{pmatrix}+\left\langle\begin{pmatrix}1\\1\\0\end{pmatrix}\right\rangle$

ⓐ $\pi_1 \cap \pi_2$

$$\pi_2 = \left\{ \begin{pmatrix}1+a\\-1+b\\1+a\end{pmatrix} \middle| \, a,b \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix}c\\d\\c\end{pmatrix} \middle| \, c,d \in \mathbb{R} \right\} = \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix} \begin{pmatrix}0\\1\\0\end{pmatrix} \right\rangle$$

$$: \begin{cases} x = c \\ y = d \\ z = c \end{cases} \Rightarrow \left\{ x - z = 0 \right.$$

$$\pi_1 \cap \pi_2 = \begin{cases} 2x - y = 2 \\ x - z = 0 \end{cases} \quad \begin{cases} y = 2x - 2 \\ z = x \end{cases} \Rightarrow \pi_1 \cap \pi_2 : \left\{ \begin{pmatrix}x\\2x-2\\x\end{pmatrix} \middle| \, x \in \mathbb{R} \right\}$$

$$= \begin{pmatrix}0\\-2\\0\end{pmatrix} + \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix} \right\rangle$$

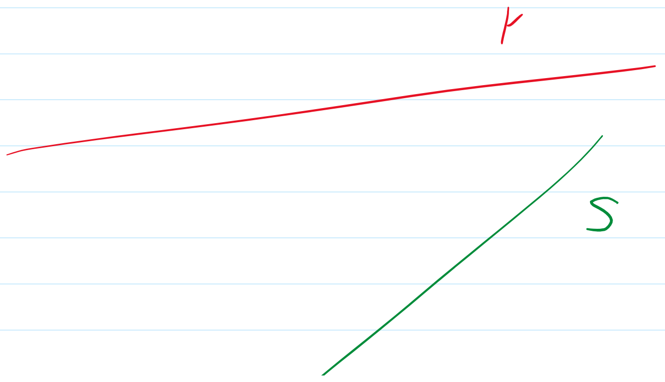
$\Rightarrow \pi_1$ e π_2 sono incidenti

ⓑ $r: P + W$ $\qquad W \leq \text{Giacitura}(\pi_1) \cap \text{Giacitura}(\pi_2)$

$r: P + \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix} \right\rangle$ $\qquad$ r sia complanare a $s: \begin{pmatrix}1\\-1\\2\end{pmatrix} + \left\langle \begin{pmatrix}1\\1\\0\end{pmatrix} \right\rangle$

prendiamo $P = \begin{pmatrix}1\\-1\\2\end{pmatrix}$

$r: \begin{pmatrix}1\\-1\\2\end{pmatrix} + \left\langle \begin{pmatrix}1\\2\\1\end{pmatrix} \right\rangle$



$r: \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\rangle$

ⓒ $\sigma \supseteq \pi_1 \cap \pi_2$

$\sigma: \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$

$$\begin{cases} x = a + b \\ y = -2 + 2a + b \\ z = a \end{cases}$$

$$\Rightarrow \begin{cases} x = z + y + 2 - 2z \\ b = y + 2 - 2z \\ a = z \end{cases} \rightarrow \sigma: \{\, x - y + z = 2$$

FII/E3
FII/E4