2020/05/05

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F7/E1

svolgere $\begin{pmatrix}2&1\\-1&0\end{pmatrix}\begin{pmatrix}1&4&2&0\\0&-1&2&1\end{pmatrix}\begin{pmatrix}3&-2&1\\0&0&2\\1&-1&4\\5&1&-3\end{pmatrix}$

$$\begin{pmatrix}\bar{u}_1 & \bar{u}_2 \cdots & \bar{u}_n\end{pmatrix}\begin{pmatrix}x_1\\ \vdots \\ x_n\end{pmatrix} = x_1\bar{u}_1 + \dots + x_n\bar{u}_n$$

$$u_i \in \mathbb{R}^m \ , \ x_i \in \mathbb{R}$$

$$(x_1 \cdots x_n)\begin{pmatrix}\bar{r}_1\\ \vdots \\ \bar{r}_n\end{pmatrix} = x_1\bar{r}_1 + \cdots + x_n\bar{r}_n$$

$$\bar{r}_i \in M_{1\times m}(\mathbb{R})$$

$r = \bar{r}_1$

$$\underbrace{\begin{pmatrix}2&1\\-1&0\end{pmatrix}\begin{pmatrix}1&4&2&0\\0&-1&2&1\end{pmatrix}}_{=\begin{pmatrix}2\bar{r}_1+1\bar{r}_2\\-1\bar{r}_1+0\bar{r}_2\end{pmatrix}}\begin{pmatrix}3&-2&1\\0&0&2\\1&-1&4\\5&1&-3\end{pmatrix} = \begin{pmatrix}2&7&6&1\\-1&-4&-2&0\end{pmatrix}\begin{pmatrix}3&-2&1\\0&0&2\\1&-1&4\\5&1&-3\end{pmatrix} =$$

$\bar{r}_2$

$$= \begin{pmatrix}17&-9&37\\-5&4&-17\end{pmatrix}$$

F7/E2

$(a\,b\,c\,d) \quad (1\ 4\ 2\ 0\ 1$

F7/E2

risolvere

$$\underbrace{\begin{pmatrix}2&1\\-1&0\end{pmatrix}}_{A}\underbrace{\begin{pmatrix}a&b&c&d\\x&y&z&w\end{pmatrix}}_{X}=\underbrace{\begin{pmatrix}1&4&2&0\\0&-1&2&1\end{pmatrix}}_{B}$$

$$AX=B \Leftrightarrow X=A^{-1}B \quad \text{se } \det A\neq 0$$

$$\|$$

$$\cancel{A^{-1}A}X$$

$$\det(A)=-(-1)=1\neq 0 \Rightarrow \exists A^{-1}$$

$$\left(\underbrace{\begin{matrix}2&1\\-1&0\end{matrix}}_{A}\middle|\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{\mathbb{1}}\right)\xrightarrow[\mathrm{I}+2\mathrm{II}]{-\mathrm{II}}\left(\underbrace{\begin{matrix}1&0\\0&1\end{matrix}}_{\mathbb{1}}\middle|\underbrace{\begin{matrix}0&-1\\1&2\end{matrix}}_{A^{-1}}\right)$$

$$\Rightarrow X=\begin{pmatrix}0&-1\\1&2\end{pmatrix}\begin{pmatrix}1&4&2&0\\0&-1&2&1\end{pmatrix}=\begin{pmatrix}0&1&-2&-1\\1&2&6&2\end{pmatrix}=\begin{pmatrix}a&b&c&d\\x&y&z&w\end{pmatrix}$$

F7/E3

$$A^{-1}=?\qquad A=\begin{pmatrix}1&0&1\\2&-1&0\\-1&-1&-2\end{pmatrix}$$

$$\left(\begin{matrix}1&0&1\\2&-1&0\\-1&-1&-2\end{matrix}\middle|\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\right)\xrightarrow[\mathrm{III}+\mathrm{I}]{\mathrm{II}-2\mathrm{I}}\left(\begin{matrix}1&0&1\\0&-1&-2\\0&-1&-1\end{matrix}\middle|\begin{matrix}1&0&0\\-2&1&0\\1&0&1\end{matrix}\right)$$

$$\begin{matrix}\\-\mathrm{II}\\\mathrm{III}-\mathrm{II}\end{matrix}\left(\begin{matrix}1&0&1\\0&1&2\\0&0&1\end{matrix}\middle|\begin{matrix}1&0&0\\2&-1&0\\3&-1&1\end{matrix}\right)\longrightarrow\begin{matrix}\mathrm{I}-\mathrm{III}\\\mathrm{II}-2\mathrm{III}\\\mathrm{III}\end{matrix}\left(\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}\middle|\underbrace{\begin{matrix}-2&1&-1\\-4&1&-2\\3&-1&1\end{matrix}}_{A^{-1}}\right)$$

$$\Downarrow$$

F7/E4

F7/E4

trovare $A\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$ con $A=\begin{pmatrix}1&0&1\\2&-1&0\\-1&-1&-2\end{pmatrix}$

$$\Downarrow$$

$\det(A)\neq 0$

$$\Updownarrow$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = A^{-1}\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}-2&1&-1\\-4&1&-2\\3&-1&1\end{pmatrix}\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}-6\\-11\\8\end{pmatrix}\begin{matrix}=x\\=y\\=z\end{matrix}$$

F7/E7

$K=?$ in $\mathbb{R}$ t.c. $\begin{cases}-x+Ky+2z=1\\-Kx+y+(1+K)z=2\\y+z=2\end{cases}$ non abbia sol.

$$\Downarrow$$

$\nexists\, x\in\mathbb{R}^3$ t.c. $Ax=b$

$$\Downarrow$$

$$\boxed{b\notin \operatorname{Im}A}$$

$$\left(\begin{array}{ccc|c}-1&K&2&1\\-K&1&1+K&2\\0&1&1&2\end{array}\right) \longleftrightarrow \boxed{Ax=b}$$

$\underbrace{}_{A}$ $\underbrace{}_{b}$

$$A:\mathbb{R}^3\longrightarrow\mathbb{R}^3$$
$$v\longmapsto Av$$

$$\boxed{\det A=0} \iff \dim(\operatorname{Ker}A)\geq 1 \iff \dim(\operatorname{Im}A)=\dim(\mathbb{R}^3)-\dim(\operatorname{Ker}A)\leq$$
$$\leq 3-1=2$$

$$0=|A|=-1\cdot\begin{vmatrix}1&1+K\\1&1\end{vmatrix}-(-K)\begin{vmatrix}K&2\\1&1\end{vmatrix}+0\begin{vmatrix}K&2\\1&1+K\end{vmatrix}$$

$$=-(\cancel{1}-\cancel{1}-K)+K(K-2)=K^2-2K+K=K(K-1)$$

$K(K-1)=0$

• Se $K=0 \Rightarrow A=\begin{pmatrix}-1&0&2\\0&1&1\\0&1&1\end{pmatrix} \Rightarrow \operatorname{Im} A=\left\langle \begin{array}{ccc} v_1 & v_2 & v_3 \\ \begin{pmatrix}-1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\1\end{pmatrix} & \begin{pmatrix}2\\1\\1\end{pmatrix}\end{array}\right\rangle$ $v_3=v_2-2v_1$

$=\left\langle \begin{array}{cc} \begin{pmatrix}1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\1\end{pmatrix} \\ u_1 & v_2 \end{array}\right\rangle$

$b\in \operatorname{Im}A$? $b=\begin{pmatrix}1\\2\\2\end{pmatrix}$

SI   infatti: $b=u_1+2v_2$

K=0 non fa il lavoro che vogliamo

• Se $K=1 \Rightarrow A=\begin{pmatrix}-1&1&2\\-1&1&2\\0&1&1\end{pmatrix} \Rightarrow \operatorname{Im} A=\left\langle \begin{array}{ccc} v_1 & v_2 & v_3 \\ \begin{pmatrix}-1\\-1\\0\end{pmatrix} & \begin{pmatrix}1\\1\\1\end{pmatrix} & \begin{pmatrix}2\\2\\1\end{pmatrix}\end{array}\right\rangle$ $v_3=v_2-v_1$

$=\left\langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\rangle$

$=\left\{ \begin{pmatrix}x\\x\\y\end{pmatrix} \,\middle|\, x,y\in\mathbb{R} \right\}$

$b\in \operatorname{Im}A$?

$b\notin \operatorname{Im}A$

$\begin{pmatrix}1\\2\\2\end{pmatrix}\in \operatorname{Im}A$

$\Updownarrow$

$1=2$ ↯

$\Rightarrow Ax=b$ non ha sol. $\Leftrightarrow K=1$

F7/E8

per quali $K\in\mathbb{R}$ $\begin{cases} 2x+Ky-z=1 \\ x+y=2 \\ (1+K)x+y-Kz=2 \end{cases}$ ∃ infinite soluzioni?

per tali K trovarle

$\left(\begin{array}{ccc|c} 2 & K & -1 & 1 \end{array}\right)$ $Ax=b$

$$\underbrace{\begin{pmatrix} 2 & k & -1 \\ 1 & 1 & 0 \\ 1+k & 1 & -k \end{pmatrix}}_{A}\underbrace{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}}_{b}$$

$Ax=b$

Sol: $\begin{array}{c} y_0+\{\text{sol di } Ax=0\} \\ \parallel \\ y_0+\ker A \end{array}$

$\Rightarrow$ vogliamo $\ker A \neq \langle 0 \rangle$

$\Rightarrow \dim(\ker A) \geq 1 \Rightarrow$ vogliamo $\det(A)=0$

$$\boxed{\det(A) \neq 0 \iff rk(A) = \text{massimo}}$$

$\downarrow$

$\det(A)=0 \iff rk(A) < n$ $\mathbb{R}^n$

$\parallel$

$\dim(\mathbb{R}^n)-\dim(\ker A) \iff \dim(\ker A) > \dim(\mathbb{R}^n)-n = n-n=0$

$\Updownarrow$

$\dim(\ker A) \geq 1$

- $$0=\begin{vmatrix} 2 & k & -1 \\ 1 & 1 & 0 \\ 1+k & 1 & -k \end{vmatrix} = -\begin{vmatrix} -1 & k & 2 \\ 0 & 1 & 1 \\ -k & 1 & 1+k \end{vmatrix} = \overset{+1}{\overbrace{-(-1)}}\cdot\begin{vmatrix} -1 & k & 2 \\ -k & 1 & 1+k \\ 0 & 1 & 1 \end{vmatrix} =$$

$$= k(k-1)$$

- se $k=1$ $\operatorname{Im} A \left\langle \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \right\rangle = \left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle = \left\{ \begin{pmatrix} x \\ y \\ x \end{pmatrix} \middle| x,y \in \mathbb{R} \right\}$

$b \in \operatorname{Im} A$? $b=\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $b \notin \operatorname{Im} A \Rightarrow \nexists$ soluzioni

NO

NO

- Se $K=0 \Rightarrow \operatorname{Im} A = \left\langle \begin{pmatrix}2\\1\\1\end{pmatrix} \begin{pmatrix}0\\1\\1\end{pmatrix} \begin{pmatrix}-1\\0\\0\end{pmatrix} \right\rangle = \left\langle \begin{pmatrix}0\\1\\1\end{pmatrix} \begin{pmatrix}1\\0\\0\end{pmatrix} \right\rangle$

$b \in \operatorname{Im} A \Rightarrow \exists$ sol. e sono infinite perché $\operatorname{Ker} A \neq \langle 0 \rangle$ ✓

$$\left(\begin{array}{ccc|c} 2 & 0 & -1 & 1 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 0 & 2 \end{array}\right)$$

$\operatorname{Ker} A = \left\langle \begin{pmatrix}1\\-1\\-2\end{pmatrix} \right\rangle$

$$\begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix}0\\2\\-1\end{pmatrix} = \begin{pmatrix}1\\2\\2\end{pmatrix}$$

$\Rightarrow$ le soluzioni sono $\boxed{\begin{pmatrix}0\\2\\-1\end{pmatrix} + \left\langle \begin{pmatrix}1\\-1\\-2\end{pmatrix} \right\rangle}$

F8/Ez 2

calc. determinante e se possibile l'inversa

- $A = \begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix}$ $\quad |A| = 6 - 2 \cdot 3 = 0 \Rightarrow \nexists A^{-1}$

- $A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \Rightarrow \det(A) = 2 \cdot 2 - 1 \cdot 3 = 1 \Rightarrow \exists A^{-1}$

$$\left(\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \longrightarrow \begin{array}{c} II \\ I-2II \end{array} \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -1 & 1 & -2 \end{array}\right) \longrightarrow \begin{array}{c} I+2II \\ -II \end{array} \left(\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -1 & 2 \end{array}\right)$$

$$\underbrace{\begin{matrix} 2 & -3 \\ -1 & 2 \end{matrix}}_{A^{-1}}$$

- $\begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix} = A$ $\quad |A| = 1 \cdot 2 - 1 \cdot 4 = -2 \quad \exists A^{-1}$

$$A^{-1}=\begin{pmatrix}2&-4\\-1&1\end{pmatrix}$$

- $\begin{pmatrix}1&2&3\\2&1&2\\1&-1&-1\end{pmatrix}=A$

$$|A|=1\cdot\begin{vmatrix}1&2\\-1&-1\end{vmatrix}-2\begin{vmatrix}2&3\\-1&-1\end{vmatrix}+\begin{vmatrix}2&3\\1&2\end{vmatrix}$$

$$=\cancel{-1}+2-2(-2+3)+\cancel{4}-\cancel{3}=2+4-6=0$$

$$\nexists A^{-1}$$

- $\begin{pmatrix}3&2&1\\1&3&2\\2&1&3\end{pmatrix}=A$

$$|A|=3\begin{vmatrix}3&2\\1&3\end{vmatrix}-\begin{vmatrix}2&1\\1&3\end{vmatrix}+2\begin{vmatrix}2&1\\3&2\end{vmatrix}$$

$$=3(9-2)-(6-1)+2(4-3)$$

$$=21-5+2=18\neq0\Rightarrow\exists A^{-1}$$

$$\left(\begin{array}{ccc|ccc}3&2&1&1&0&0\\1&3&2&0&1&0\\2&1&3&0&0&1\end{array}\right)\longrightarrow\begin{array}{c}II\\I-3II\\III-2II\end{array}\left(\begin{array}{ccc|ccc}1&3&2&0&1&0\\0&-7&-5&1&-3&0\\0&-5&-1&0&-2&1\end{array}\right)$$

$$\begin{array}{c}\\II-III\\-III\end{array}\left(\begin{array}{ccc|ccc}1&3&2&0&1&0\\0&-2&-4&1&-1&0\\0&5&1&0&2&-1\end{array}\right)$$

$$\longrightarrow\begin{array}{c}\\III+2II\\-II\end{array}\left(\begin{array}{ccc|ccc}1&3&2&0&1&0\\0&1&-7&2&0&-1\\0&2&4&-1&1&0\end{array}\right)$$

$$\left(\begin{array}{ccc|ccc}1&3&2&0&1&0\end{array}\right.$$

$$\xrightarrow{} \begin{array}{c} \\ \\ \text{III}-2\text{II} \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 0 & 1 & 0 \\ 0 & 1 & -7 & 2 & 0 & -1 \\ 0 & 0 & 18 & -5 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{} \begin{array}{c} \text{I}-\frac{2}{18}\text{III} \\ \text{II}+\frac{7}{18}\text{III} \\ \frac{\text{III}}{18} \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & \frac{10}{18} & 1-\frac{2}{18} & -\frac{4}{18} \\ 0 & 1 & 0 & 2-\frac{35}{18} & \frac{7}{18} & -1+\frac{14}{18} \\ 0 & 0 & 1 & -\frac{5}{18} & \frac{1}{18} & \frac{2}{18} \end{array}\right) \xrightarrow{} \text{I}-3\text{II} \left(\quad \right)$$

• $\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} = A$

$$\det(A) = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \end{vmatrix} - \begin{vmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} - \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -2(-1-1) = 4 \neq 0$$

$$\exists A^{-1}$$

$$\left(\begin{array}{cccc|c} 0 & 0 & 1 & 1 & \\ 0 & 0 & 1 & -1 & \mathbb{1}_4 \\ 1 & 1 & 0 & 0 & \\ 1 & -1 & 0 & 0 & \end{array}\right) \xrightarrow{} \begin{array}{c} \frac{\text{III}+\text{IV}}{2} \\ \frac{\text{III}-\text{IV}}{2} \\ \frac{\text{I}+\text{II}}{2} \\ \frac{\text{I}-\text{II}}{2} \end{array} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 & 0 \end{array}\right)$$

$$A^{-1}$$

ⓐ $rk(A_h) = 2$ se $h = 0$ se $rk(A_h) = 3$

ⓑ $$Im(A_h) = \underbrace{\left\langle \cancel{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}} \begin{pmatrix} 2 \\ h-1 \\ 3 \end{pmatrix} \begin{pmatrix} h \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} -1 \\ 1-h \\ -h \end{pmatrix} \right\rangle}$$

$$A_h \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \boxed{\left(A_h \,\middle|\, \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \right)}$$