2020/03/31

martedì 31 marzo 2020 14:38

es casa

$\mathbb{R}^6$

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 + 2x_5 + 3x_6 = a \\ \frac{1}{2}x_1 + \frac{1}{2}x_2 + 2x_3 - x_5 + 4x_6 = b \\ -2x_1 - 2x_2 - 3x_3 - 5x_4 - 6x_5 - 2x_6 = c \end{cases}$$

- soluzioni se $a=3, b=4, c=1$
- per quali a,b,c il sistema NON ha soluzione
- per quali a,b,c il sistema ha soluzione unica

$$\left(\begin{array}{cccccc|c} \multicolumn{6}{c|}{A} & b \\ 1 & 1 & 2 & 2 & 2 & 3 & a \\ \frac{1}{2} & \frac{1}{2} & 2 & 0 & -1 & 4 & b \\ -2 & -2 & -3 & -5 & -6 & -2 & c \end{array}\right)$$

$$\downarrow$$

$$\begin{array}{c} \\ 2II-I \\ III+2I \end{array} \left(\begin{array}{cccccc|c} 1 & 1 & 2 & 2 & 2 & 3 & a \\ 0 & 0 & 2 & -2 & -4 & 5 & 2b-a \\ 0 & 0 & 1 & -1 & -2 & 4 & c+2a \end{array}\right)$$

$$\downarrow$$

$$\begin{array}{c} \\ III \\ II-2III \end{array} \left(\begin{array}{cccccc|c} 1 & 1 & 2 & 2 & 2 & 3 & a \\ 0 & 0 & 1 & -1 & -2 & 4 & c+2a \\ 0 & 0 & 0 & 0 & 0 & -3 & \underbrace{2b-a-2(c+2a)}_{= -5a+2b-2c} \end{array}\right)$$

$$\underbrace{\qquad\qquad}_{\tilde{A}} \qquad \underbrace{\qquad}_{\tilde{b}}$$

$\Rightarrow$ da $\boxed{R-C}$ $\exists$ sempre soluzione

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 + 2x_5 + 3x_6 = a \\ x_3 - x_4 - 2x_5 + 4x_6 = c + 2a \\ -3x_6 = -5a + 2b - 2c \end{cases}$$

$$\begin{cases} x_3 = \underline{x_4} + 2\underline{x_5} - 4x_6 + c + 2a \\ x_6 = \dfrac{+5a - 2b + 2c}{3} \end{cases}$$

$\forall\, a, b, c \ \exists$ infinite soluzioni

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 + 2x_5 + 3x_6 = a \\ x_3 - x_4 - 2x_5 + 4x_6 = c + 2a \\ -3x_6 = -5a + 2b - 2c \end{cases}$$

$a = 3,\ b = 4,\ c = 1$

$$\begin{cases} x_1 + x_2 + 2x_3 + 2x_4 + 2x_5 + 3x_6 = 3 \\ x_3 - x_4 - 2x_5 + 4x_6 = 7 \\ -3x_6 = -9 \end{cases}$$

$$\begin{cases} x_1 = -x_2 - 2x_3 - 2x_4 - 2x_5 - 3x_6 + 3 = -x_2 - 2x_4 - 4x_5 + 10 - 2x_4 - 2x_5 - 9 + 3 \\ x_3 = x_4 + 2x_5 \underbrace{- 4x_6}_{-4\cdot 3} + 7 = x_4 + 2x_5 - 5 \\ x_6 = 3 \end{cases}$$

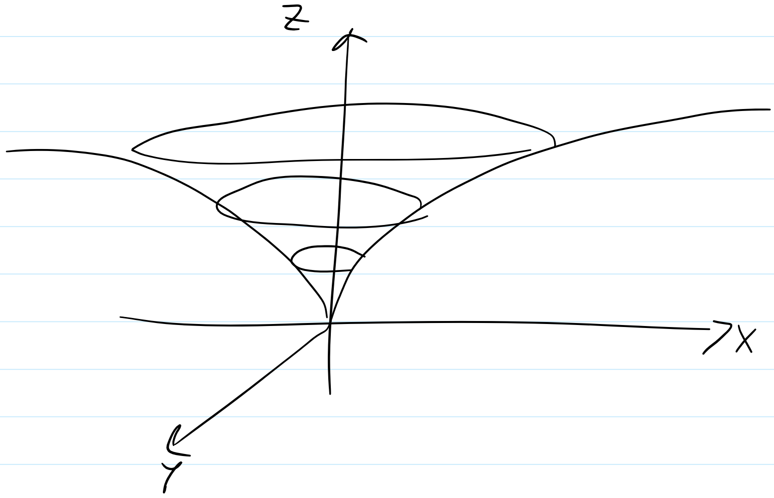
$$\Rightarrow \begin{cases} x_1 = -x_2 - 4x_4 - 6x_5 + 4 \\ x_3 = x_4 + 2x_5 - 5 \\ x_6 = 3 \end{cases}$$

$$S = \left\{ \begin{pmatrix} -x_2 - 4x_4 - 6x_5 + 4 \\ x_2 \\ x_4 + 2x_5 - 5 \\ x_4 \\ x_5 \\ 3 \end{pmatrix} \;\middle|\; x_2, x_4, x_5 \in \mathbb{R} \right\}$$

$\boxed{\text{Ez/2}}$

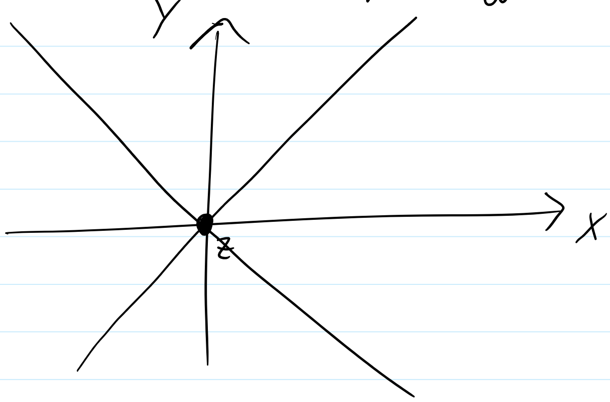
$\mathbb{R}^3$ sono sottosp. vett?

(a) $x^2 + y^2 = z$ NO $v = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, $w = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$ $v + w = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$ 4



(b) $|x| = |y|$ NO $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ $v + w = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$

v w



c) $x+y=z$

$v=\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ $w=\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ $\Rightarrow x_i+y_i=z_i$ $i=1,2$

$\lambda,\mu\in\mathbb{R}$

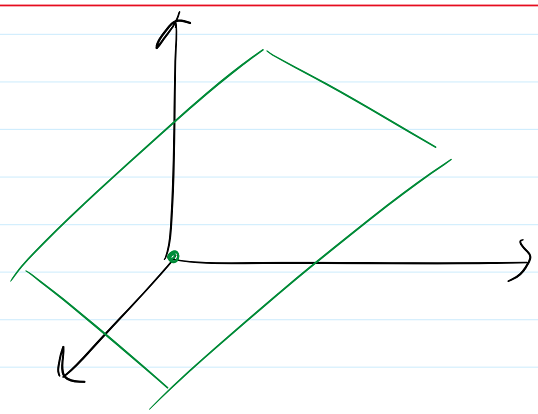
- $\boxed{\lambda v+\mu w}=\begin{pmatrix} \lambda x_1+\mu x_2 \\ \lambda y_1+\mu y_2 \\ \lambda z_1+\mu z_2 \end{pmatrix}$

$(\lambda x_1+\mu x_2)+(\lambda y_1+\mu y_2)=(\lambda z_1+\mu z_2)$

$\Updownarrow$

$\lambda(\underbrace{x_1+y_1-z_1}_{0})+\mu(\underbrace{x_2+y_2-z_2}_{0})=0$ ✓

- $\lambda v=\begin{pmatrix} \lambda x_1 \\ \lambda y_1 \\ \lambda z_1 \end{pmatrix}$ $\lambda x_1+\lambda y_1-\lambda z_1=\lambda(\underbrace{x_1+y_1-z_1}_{=0})=0$



d) $xy+zz=0$ NO

$\underset{v}{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}}$ $\underset{w}{\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}}$ $v+w=\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ↯

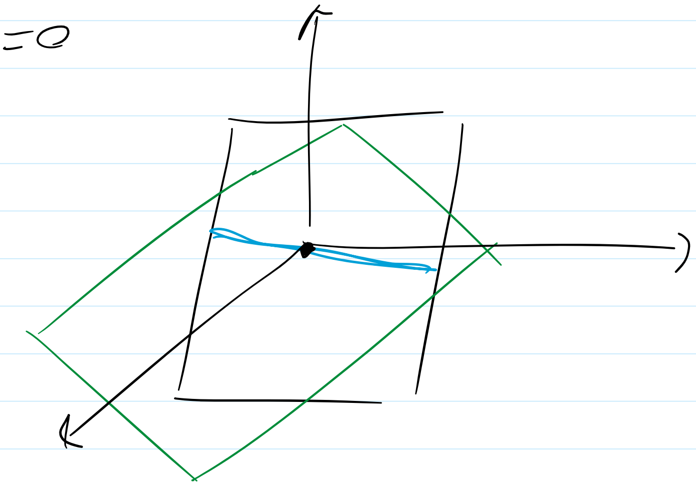
$\underset{\overset{\shortparallel}{1}}{1\cdot 1+1\cdot 0}\neq 0$

Criterio I: $\cdot\ \forall v,w\in V \Rightarrow v+w\in V$

$\cdot\ \forall \lambda\in K,\ v\in V\Rightarrow \lambda v\in V$

$xy+zz=0$

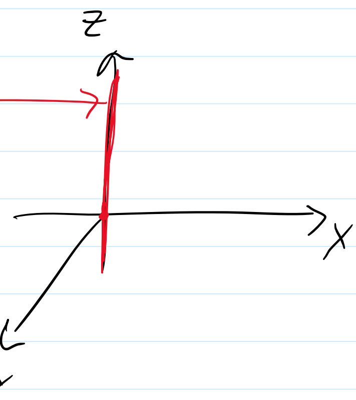
$y(x+z)=0$



$$F \quad \begin{cases} x-y^2=0 \\ x=0 \end{cases}$$

S1

$$\hookrightarrow \begin{cases} -y^2=0 \\ x=0 \end{cases} \Rightarrow \begin{cases} y=0 \\ x=0 \end{cases}$$



$$S=\left\{\begin{pmatrix}0\\0\\z\end{pmatrix} \middle| z\in\mathbb{R}\right\}=\left\{z\begin{pmatrix}0\\0\\1\end{pmatrix} \middle| z\in\mathbb{R}\right\}=\left\langle\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rangle$$

F2/4

$\mathbb{R}^3$ $\quad W_1=\left\{\begin{pmatrix}a\\b\\c\end{pmatrix} \middle| 2a+3b-c=0\right\}$ $\quad W_2=\left\{\begin{pmatrix}a\\b\\c\end{pmatrix} \middle| a+2b-c=0\right\}$

(a) W_1, W_2 sono sp. vet. ?

(b) trovare $v\in W_1\cap W_2$?

(c) $W_1\cap W_2=\{\alpha v \mid \alpha\in\mathbb{R}\}$

(d) $W_1+W_2=\mathbb{R}^3$ dimostrarlo

(a) $v \in W_1 \iff 2a+3b-c=0 \Rightarrow c = 2a+3b$

$v = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$W_1 = \left\{ \begin{pmatrix} a \\ b \\ 2a+3b \end{pmatrix} \middle| a,b \in \mathbb{R} \right\}$$

$$= \left\{ a\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \middle| a,b \in \mathbb{R} \right\}$$

$$= \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\rangle \Rightarrow V \text{ è sp. vett.}$$

$$\Rightarrow \dim W_1 = 2 \qquad W_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$$\Rightarrow \dim W_2 = 2$$

(b) $$W_1 \cap W_2 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| 2a+3b-c=0,\ a+2b-c=0 \right\}$$

$$= \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| \begin{cases} 2a+3b-c=0 \\ a+2b-c=0 \end{cases} \right\}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & -1 & 0 \\ 1 & 2 & -1 & 0 \end{array}\right) \longrightarrow \begin{array}{c} \mathrm{II} \\ \mathrm{I}-2\mathrm{II} \end{array}\left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right)$$

$$\Rightarrow \begin{cases} a+2b-c=0 \\ -b+c=0 \end{cases} \Rightarrow \begin{cases} a=-2b+c=-2c+c=-c \\ b=c \end{cases}$$

$$W_1 \cap W_2 = \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \middle| \begin{cases} a=-c \\ b=c \end{cases} \right\} = \left\{ \begin{pmatrix} -c \\ c \\ c \end{pmatrix} \middle| c \in \mathbb{R} \right\}$$

$$= \left\{ c\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \middle| c \in \mathbb{R} \right\}$$

$$= \left\{ c \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \middle| \, c \in \mathbb{R} \right\}$$

scegliamo $v = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ → **b** ✓

$$= \underbrace{\left\{ \alpha \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \middle| \, \alpha \in \mathbb{R} \right\}}_{\hookrightarrow\ c\ \checkmark} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

d) • modo 1: $\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 \cap W_1)$

$$= 2 + 2 - (+1) = 3$$

$$\left.\begin{array}{l} \dim(W_1 + W_2) = 3 \\ W_1 + W_2 \subseteq \mathbb{R}^3 \end{array}\right| \Rightarrow W_1 + W_2 = \mathbb{R}^3$$

• modo 2: $W_1 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \right\rangle \qquad W_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$

$$W_1 + W_2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

se $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \Rightarrow v \in W_1 + W_2$

$$\alpha \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} + \gamma \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \delta \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{cases} \alpha + \gamma = x \\ \beta + \delta = y \\ 2\alpha + 3\beta + \gamma + 2\delta = z \end{cases}$$

$$\begin{cases} \beta+\delta = y \\ 2\alpha+3\beta+\gamma+2\delta = z \end{cases}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & x \\ 0 & 1 & 0 & 1 & y \\ 2 & 3 & 1 & 2 & z \end{array}\right) \xrightarrow{III-2I} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & x \\ 0 & 1 & 0 & 1 & y \\ 0 & 3 & -1 & 2 & z-2x \end{array}\right)$$

$$\xrightarrow{III-3II} \left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & x \\ 0 & 1 & 0 & 1 & y \\ 0 & 0 & -1 & -1 & z-2x-3y \end{array}\right)$$

$\underbrace{\qquad}_{\tilde{A}}$ $\underbrace{\qquad}_{\tilde{b}}$

$\Rightarrow$ la sol. $\exists$ sempre per R-C

$$\boxed{W_1+W_2=\mathbb{R}^3}$$

$\boxed{\text{Ez/6}}$

$\mathbb{R}^4$ $v_1=\begin{pmatrix}1\\1\\0\\1\end{pmatrix}$ $v_2=\begin{pmatrix}1\\1\\0\\0\end{pmatrix}$ $v_3=\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ $v_4=\begin{pmatrix}1\\0\\1\\1\end{pmatrix}$ $v_5=\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$

a) sono generatori di $\mathbb{R}^4$?

b) dato $v\in\mathbb{R}^4$ quante comb. lin. $\exists$ per v?

a) dato $v=\begin{pmatrix}x\\y\\z\\t\end{pmatrix}\in\mathbb{R}^4$ se $\exists$ $\alpha,\beta,\gamma,\delta,\omega$ t.c.

$$\alpha v_1 + \beta v_2 + \gamma v_3 + \delta v_4 + \omega v_5 = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$$\begin{pmatrix} \alpha + \beta + \delta + \omega \\ \alpha + \beta \\ \delta \\ \alpha + \delta + \omega \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 1 & x \\ 1 & 1 & 0 & 0 & 0 & y \\ 0 & 0 & 0 & 1 & 0 & z \\ 1 & 0 & 0 & 1 & 1 & t \end{array}\right) \to \begin{array}{c} IV \\ II-IV \\ I-II \\ III \end{array} \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 1 & 1 & t \\ 0 & 1 & 0 & -1 & -1 & y-t \\ 0 & 0 & 0 & 1 & 1 & x-y \\ 0 & 0 & 0 & 1 & 0 & z \end{array}\right)$$

$$\to \begin{array}{c} I \\ II \\ IV \\ III-IV \end{array} \left(\underbrace{\begin{array}{ccccc} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}}_{\tilde{A}} \middle| \underbrace{\begin{array}{c} t \\ y-t \\ z \\ x-y-z \end{array}}_{\tilde{b}}\right) \to \text{per } \boxed{R\text{-}C} \text{ la sol. } \exists \text{ sempre}$$

$$\Rightarrow \mathbb{R}^4 = \langle v_1, v_2, v_3, v_4, v_5 \rangle$$

(b) $\exists$ infinite combinazioni lineari