2020/03/19

martedì 10 marzo 2020 11:00

V $\quad K$ campo

$$+ : V \times V \longrightarrow V$$
$$(v,w) \longmapsto v+w$$

$+$
- $u+(v+w)=(u+v)+w$
- $u+v=v+u$
- $\exists\, 0\in V,\ v+0=v \quad \forall v\in V$
- $\forall v\in V\ \exists\ -v$ t.c. $v+(-v)=0$

$$\cdot : K \times V \longrightarrow V$$
$$(\lambda, v) \longmapsto \lambda\cdot v$$

$\cdot$
- $\exists\, 1\in K \quad 1\cdot v=v$
- $s,t\in K,\ v\in V \quad s(tv)=(st)v$
- $t(v+w)=tv+tw \quad t\in K$
- $(s+t)v=sv+tv \quad t,s\in K$

(I) $\cdot\ \forall v_1, v_2 \in V \Rightarrow v_1+v_2\in V$

$\cdot\ \forall \lambda\in K,\ v\in V \Rightarrow \lambda\cdot v\in V$

$\boxed{\text{(II) } \forall \lambda_1,\lambda_2\in K\ \ \forall v_1,v_2\in V \quad \lambda_1 v_1+\lambda_2 v_2\in V}$

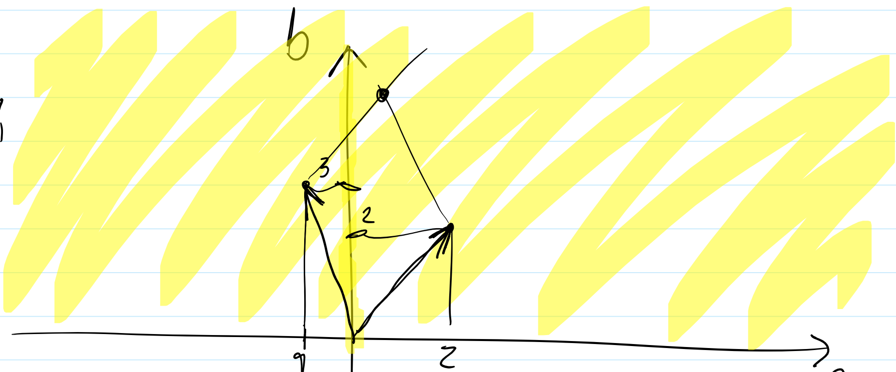
test $\quad 0\in V$

es 1

$\mathbb{R}^2 \quad V_1=\{(a,b)\in\mathbb{R}^2 \mid b\geq 0\}$ $\quad$ Sono sp. vett?

$V_2=\{(x,y)\in\mathbb{R}^2 \mid y=1\}$

V_1



$v=\begin{pmatrix}2\\2\end{pmatrix} \quad w=\begin{pmatrix}3\\1\end{pmatrix}$

$v+w=\begin{pmatrix}5\\ \ \end{pmatrix}^{a} \in V_1 \iff 3\geq 0$

$v+w=\begin{pmatrix}5\\3\end{pmatrix}\begin{matrix}a\\b\end{matrix} \in V_1 \iff 3\geq 0$

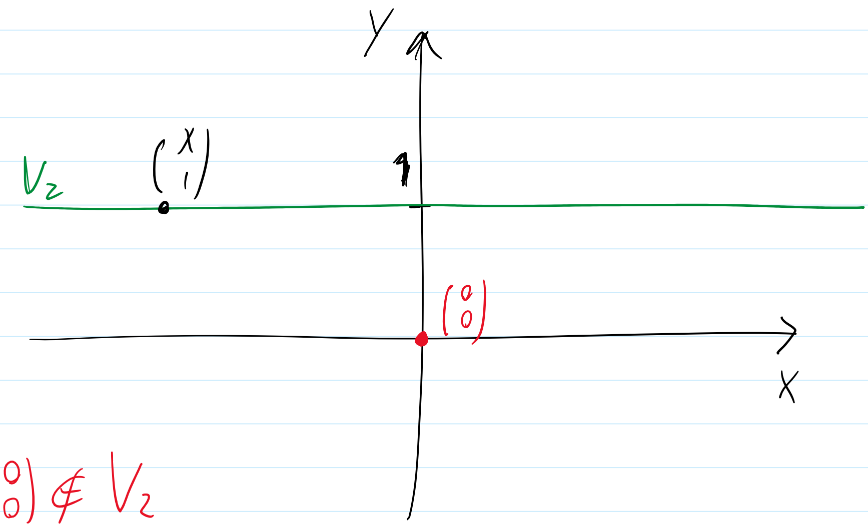
$\lambda=5$

$\lambda\cdot v = 5\begin{pmatrix}2\\2\end{pmatrix}=\underbrace{\begin{pmatrix}10\\10\end{pmatrix}}_{\in V_1}$ $\qquad 10\geq 0$

$\lambda=-1 \qquad \lambda\cdot v=(-1)\begin{pmatrix}2\\2\end{pmatrix}=\begin{pmatrix}-2\\-2\end{pmatrix}\notin V_1$

$\Rightarrow V_1$ NON è sp. vett.

V_2



$\begin{pmatrix}0\\0\end{pmatrix}\notin V_2$

$\underbrace{\begin{pmatrix}2\\1\end{pmatrix}}_{\in V_2}+\underbrace{\begin{pmatrix}0\\1\end{pmatrix}}_{\in V_2}=\begin{pmatrix}2\\ \underset{\substack{\neq\\1}}{2}\end{pmatrix}\notin V_2$ $\qquad V_2$ non è sp. V.

$\boxed{\text{es } 2}$

$V=\{(x,y)\in\mathbb{R}^2 \mid 2x-3y=0\}$ $\qquad K=\mathbb{R}$

(II) criterio $\quad \lambda_1,\lambda_2\in\mathbb{R} \quad v_1=(x_1,y_1) \quad v_2=(x_2,y_2)\in\mathbb{R}^2$

III) CT. VCT (i) $\lambda_1, \lambda_2 \in \mathbb{R}$ $v_1 = (x_1, y_1)$ $v_2 = (x_2, y_2) \in \mathbb{R}$

supponiamo $v_1, v_2 \in V \Rightarrow$ $2x_1 - 3y_1 = 0$

$2x_2 - 3y_2 = 0$

$V \overset{?}{\ni} \boxed{\lambda_1 v_1 + \lambda_2 v_2} = \lambda_1 (x_1, y_1) + \lambda_2 (x_2, y_2) = (\lambda_1 x_1 + \lambda_2 x_2, \lambda_1 y_1 + \lambda_2 y_2) \overset{?}{\in} V$

$2(\lambda_1 x_1 + \lambda_2 x_2) - 3(\lambda_1 y_1 + \lambda_2 y_2) = \lambda_1 \underbrace{(2x_1 - 3y_1)}_{= 0} + \lambda_2 \underbrace{(2x_2 - 3y_2)}_{= 0} = 0$

$0 + 0$

$\lambda_1 v_1 + \lambda_2 v_2 \in V$ $\boxed{V \text{ è sp. V.}}$

$+ : V \times V \longrightarrow V$

V' sp. vet?

$$\boxed{\begin{array}{l} + : V' \times V' \longrightarrow V' \\ \cdot : K \times V' \longrightarrow V' \end{array}}$$

es3

$V = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \overset{\text{sp. V.}}{M_{2\times 2}(K)} \;\middle|\; c = 0 \right\}$

$= \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \in M_{2\times 2}(K) \right\}$

$v_1, v_2 \in V$ • $v_1 + v_2 \in V$?

$v_1, v_2 \in V$ • $v_1 + v_2 \in V$?

$v_1 = \begin{pmatrix} a_1 & b_1 \\ 0 & d_1 \end{pmatrix}$ $\Rightarrow$ $v_1 + v_2 = \begin{pmatrix} a_1 + a_2 & b_1 + b_2 \\ 0 & d_1 + d_2 \end{pmatrix} \in V$

$v_2 = \begin{pmatrix} a_2 & b_2 \\ 0 & d_2 \end{pmatrix}$

$0 = 0$

• $\lambda \in K$ $\lambda v_1 \in V$?

$\lambda \begin{pmatrix} a_1 & b_1 \\ 0 & d_1 \end{pmatrix} = \begin{pmatrix} \lambda a_1 & \lambda b_1 \\ 0 & \lambda d_1 \end{pmatrix} \in V$

$0 = 0$

$\Rightarrow V$ è sp. vett. //

es 4

$K[x]$, $V = \left\{ \sum_{i=0}^{+\infty} a_{2i} x^{2i} \in K[x] \right\}$

$v \in V$ $v = \sum_{i=0}^{+\infty} a_{2i} X^{2i} = a_0 + a_2 X^2 + a_4 X^4 + a_6 X^6 + a_8 X^8 + \dots$

II v, w $v = \sum_{i=0}^{+\infty} \underset{\underset{K}{\cap}}{a_{2i}} X^{2i}$, $w = \sum_{i=0}^{+\infty} \underset{\underset{K}{\cap}}{b_{2i}} X^{2i}$

$\lambda, \mu \in K$

$$\lambda v + \mu w = \lambda \sum_{i=0}^{+\infty} a_{2i} X^{2i} + \mu \sum_{i=0}^{+\infty} b_{2i} X^{2i}$$

$$= \sum_{i=0}^{+\infty} \lambda a_{2i} X^{2i} + \sum_{i=0}^{+\infty} \mu b_{2i} X^{2i}$$

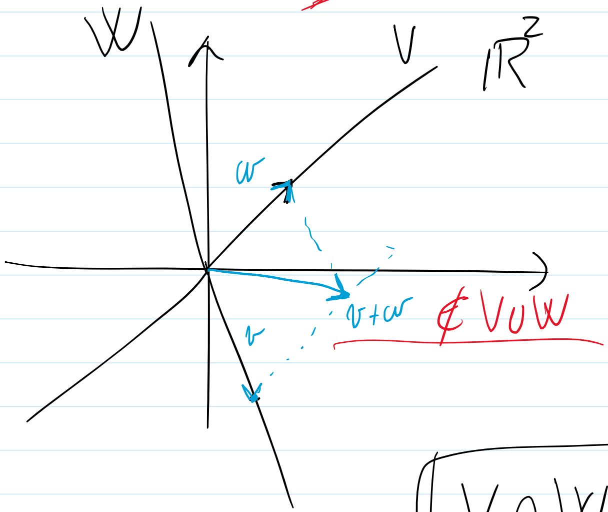
$$= \sum^{+\infty} \left(\lambda a_{2i} X^{2i} + \mu b_{2i} X^{2i} \right)$$

$$= \sum_{i=0}^{+\infty} \left(\lambda a_{2i} x^{2i} + \mu b_{2i} x^{2i} \right)$$

$$= \sum_{i=0}^{+\infty} \underbrace{(\lambda a_{2i} + \mu b_{2i})}_{\in K,\ = c_{2i}} x^{2i} = \sum_{i=0}^{+\infty} c_{2i} x^{2i} = c_0 + c_2 x^2 + c_4 x^4 + \dots$$

V è sp. vettoriale #

V, W sp. v. ~~$V \cup W$ è sp. V.?~~ NO



$$\boxed{V \cap W \text{ è sp. vett.?}}$$

es5

$\mathbb{R}^3$ $V_1 = \{ (x, y, z) \in \mathbb{R}^3 \mid x - y = 0 \} \rightarrow$ è sp. V.!

$V_2 = \{ (x, y, z) \in \mathbb{R}^3 \mid 2x - z = 0 \} \rightarrow$ è sp. V.!

$V_1 \cap V_2$ è sp. V.?

$V_1 \cap V_2 = \{ (x, y, z) \in \mathbb{R}^3 \mid x - y = 0 \quad 2x - z = 0 \}$

$V_1 \cap V_2$ è sp. v. ?

$$V_1 \cap V_2 = \{ (x,y,z) \in \mathbb{R}^3 \mid x-y=0,\ 2x-z=0 \}$$

$$\begin{cases} x-y=0 \\ 2x-z=0 \end{cases}$$

$$V_1 \cap V_2 = \left\{ (x,y,z) \in \mathbb{R}^3 \;\middle|\; \begin{cases} x-y=0 \\ 2x-z=0 \end{cases} \right\}$$

$v, w \in V_1 \cap V_2$

$\lambda, \mu \in \mathbb{R} = K$

$$\lambda v + \mu w = \lambda \overset{v}{\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}} + \mu \overset{w}{\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}}$$

$$= \begin{pmatrix} \lambda x_1 + \mu x_2 \\ \lambda y_1 + \mu y_2 \\ \lambda z_1 + \mu z_2 \end{pmatrix} \in V_1 \cap V_2 ?$$

$$\boxed{\begin{cases} (\lambda x_1 + \mu x_2) - (\lambda y_1 + \mu y_2) = 0 ? \\ 2(\lambda x_1 + \mu x_2) - (\lambda z_1 + \mu z_2) = 0 ? \end{cases}}$$

$$\begin{cases} \lambda \underbrace{(x_1 - y_1)}_{=0} + \mu \underbrace{(x_2 - y_2)}_{=0} = 0 \\ \lambda \underbrace{(2x_1 - z_1)}_{=0} + \mu \underbrace{(2x_2 - z_2)}_{=0} = 0 \end{cases}$$

Sì $V_1 \cap V_2$ è sp. V.

es 6

$V = \left\langle \underbrace{\begin{pmatrix}0\\0\\1\end{pmatrix}}_{e_3} \underbrace{\begin{pmatrix}0\\1\\0\end{pmatrix}}_{e_2} \right\rangle$ chi è V?

$e_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$ $e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$ $e_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$

$v \in V \Rightarrow v = \lambda e_3 + \mu e_2$

$= \begin{pmatrix}0\\0\\\lambda\end{pmatrix} + \begin{pmatrix}0\\\mu\\0\end{pmatrix} = \begin{pmatrix}0\\\mu\\\lambda\end{pmatrix} \in V$

$V = \{(x,y,z) \in \mathbb{R}^3 \mid x = 0\}$

$$\boxed{\begin{cases} x = 0 \\ y = \mu \\ z = \lambda \end{cases}}$$

es casa

$\mathbb{R}[x]$ $V_1 = \langle \underbrace{x}_{v_1}, \underbrace{x^3}_{w_1} \rangle$ $V_2 = \langle \underbrace{x+1}_{v_2}, \underbrace{x-1}_{w_2} \rangle$

(1) $v = x \in V_2$? ✓

(2) $w = x+1 \in V_1$? ✗

(1) $\exists \mu_2, \lambda_2 \in \mathbb{R}$ $v = \mu_2 v_2 + \lambda_2 w_2$

$x = \mu_2(x+1) + \lambda_2(x-1)$

$= (\mu_2 + \lambda_2)x + \mu_2 - \lambda_2$

$\underline{x = 1 \cdot x + 0}$

$$\begin{cases} \mu_2 + \lambda_2 = 1 \\ \mu_2 - \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} \mu_2 = 1/2 \\ \mu_2 = \lambda_2 = 1/2 \end{cases}$$

(2) $\exists\, \mu_1, \lambda_1 \in \mathbb{R}$ t.c. $1 \cdot x + 1 = \mu_1 x + \lambda_1 x^3 + 0$?

$$\begin{cases} \mu_1 = 1 & \text{1° grado} \\ \lambda_1 = 0 & \text{3° grado} \\ 0 = 0 & \text{2° grado} \\ 1 = 0 & \text{termine noto} \end{cases}$$

$w = x + 1 = 0 \cdot x^3 + 0 \cdot x^2 + 1 \cdot x + 1$