TUTORATO 01/06.

Ex. 1.

$$P = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$r : \begin{cases} x - 2y + 2z - 4 = 0 \\ 5x + 2y - 1 = 0 \end{cases}$$

$$\Pi = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} + \left\langle \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \right\rangle$$

(a) $\begin{cases} x = 2y - 2z + 4 \\ 10y - 10z + 20 + 2y - 1 = 0 \end{cases}$

$\begin{cases} x = 2y - 2z + 4 \\ 12y - 10z = -19 \end{cases} \rightarrow y = \frac{5}{6} z - \frac{19}{12}$

$x = \frac{5}{3} z - \frac{19}{6} - 2z + 4$

$= -\frac{1}{3} z + \frac{5}{6}$

$$r = \begin{pmatrix} 5/6 \\ -19/12 \\ 0 \end{pmatrix} + \Big\langle \underbrace{\begin{pmatrix} -2 \\ 5 \\ 6 \end{pmatrix}}_{= v_r} \Big\rangle$$

$$\rightarrow r : \begin{cases} x = \frac{5}{6} - 2t \\ y = -\frac{19}{12} + 5t \\ t = 6t \end{cases} , t \in \mathbb{R}$$

(b)

$$\Pi = \underbrace{\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}}_{= R} + \Big\langle \underbrace{\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}}_{= v_1} , \underbrace{\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}}_{= v_2} \Big\rangle$$

$$n_\Pi = v_1 \times v_2 = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & 1 & 0 \\ 3 & -1 & 2 \end{vmatrix} = (2, -4, -5)$$

$$\Pi : 2x - 4y - 5z = d$$

$$R \in \Pi \rightarrow 6 + 20 = d \Rightarrow d = 26$$

$$\Rightarrow \Pi : \boxed{2x - 4y - 5z = 26}$$

(c) $\pi' \perp r$, $P \in \pi'$ $\quad P = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

$\pi': -2x + 5y + 6z = d$

$P \in \pi': 5 - 6 = d \Rightarrow d = -1$

$\pi': -2x + 5y + 6z = -1$

$$\boxed{2x - 5y - 6z = 1}$$

$2x = 5y + 6z + 1$

$x = \frac{5}{2}y + 3z + \frac{1}{2}$

$$\pi': \begin{cases} x = \frac{1}{2} + \frac{5}{2}t + 3s \\ y = t \\ z = s \end{cases}, \quad t, s \in \mathbb{R}$$

(d) POS. RECIPROCA TRA π e π'.

$$rk\begin{pmatrix} 2 & -4 & -5 \\ 2 & -5 & -6 \end{pmatrix} = rk \begin{array}{c} \begin{pmatrix} 1 & -2 & -5/2 \\ 0 & -1 & -1 \end{pmatrix} \end{array} \begin{array}{l} \underline{\underline{I/2}} \\ II - I \end{array}$$

$$= 2 = rk\left(\begin{array}{ccc|c} 2 & -4 & -5 & 26 \\ 2 & -5 & -6 & 1 \end{array}\right) \Rightarrow \pi \text{ e } \pi'$$

INCIDENTI.

(e) $d(\pi, \pi') = 0$.

Ex. 3.

$r_1: \begin{cases} x+y-z+2=0 \\ 3x+y+z-8=0 \end{cases}$ $r_2: \begin{cases} 3x+2y-z-1=0 \\ 3x+y-z=0 \end{cases}$

$$\pi = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \left\langle \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

(a) r_1

$\begin{cases} z = x+y+2 = -x+5 \\ 3x+y+x+y+2-8=0, \; 4x+2y=6 \end{cases}$

$\hookrightarrow y = -2x+3$

$r_1: \begin{cases} x = t \\ y = 3-2t \\ z = 5-t \end{cases}$, $t \in \mathbb{R}$ $\quad r_1 = \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix} + \left\langle \underbrace{\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}}_{= v_{r_1}} \right\rangle$

r_2

$\begin{cases} z = 3x+y = 3x+1 \\ \cancel{3x}+2y-\cancel{3x}-y-1=0 \longrightarrow y=1 \end{cases}$

$$r_2 : \begin{cases} x = s \\ y = 1 \\ z = 1 + 3s \end{cases}, \quad s \in \mathbb{R} \qquad r_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \left\langle \underbrace{\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}}_{v_{r2}} \right\rangle$$

(b)

$$n_\pi = \begin{vmatrix} e_1 & e_2 & e_3 \\ 2 & -3 & 0 \\ 1 & -1 & 1 \end{vmatrix} = (-3, -2, 1)$$

$$\pi : -3x - 2y + z = d$$

$$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \in \pi : -6 - 2 + 5 = d \Rightarrow d = -3$$

$$\pi : \boxed{3x + 2y - 1z = 3} \qquad n_\pi = (3, 2, -1)$$

(c) $r \perp r_1,\ r \perp r_2,\ P = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \in r$.

$$v_r = v_{r1} \times v_{r2} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & -2 & -1 \\ 1 & 0 & 3 \end{vmatrix} = (-6, -4, 2)$$

$$r = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} -6 \\ -4 \\ 2 \end{pmatrix} \right\rangle = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \right\rangle.$$

(d) $s \perp \pi,\ P' = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} \in s$.

$$s = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \right\rangle.$$

$$s = P' + \langle v_s \rangle$$

(e) POS. REC. DI r e s.

$$r: \begin{cases} x = 2 - 3t \\ y = -1 - 2t \\ z = 2 + t \end{cases}, t \in \mathbb{R}$$

$t = z - 2$

$x = 2 - 3z + 6$

$y = -1 - 2z + 4$

$$r: \begin{cases} x + 3z = 8 \\ y + 2z = 3 \end{cases}$$

$$s: \begin{cases} x = 2 + 3t \\ y = 2t \\ z = 2 - t \end{cases}, t \in \mathbb{R}$$

$t = \frac{y}{2}$

$$\begin{cases} x = 2 + \frac{3}{2}y \\ z = 2 - \frac{1}{2}y \end{cases}$$

$$s: \begin{cases} 2x - 3y = 4 \\ y + 2z = 4 \end{cases}$$

$\Longrightarrow$ r e s PARALLELE.

$$d(r,s) = d(P,s) = d(P,H)$$

H ← proiezione ort. di P su s.

$P \in \pi, \quad s \perp \pi$

$$3a + 2b - c = 0$$

$$c = 3a + 2b$$

$$\pi = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \left\langle \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$$\pi : \begin{cases} x = 2 + t \\ y = -1 + s \\ z = 2 + 3t + 2s \end{cases}, \quad t, s \in \mathbb{R}$$

$$t = x - 2$$

$$s = y + 1$$

$$z = 2 + 3x - 6 + 2y + 2$$

$$\pi : \boxed{3x + 2y - z = 2}$$

$$\pi \cap s : \begin{cases} 3x + 2y - z = 2 \\ 2x - 3y = 4 \\ y + 2z = 4 \end{cases}$$

$$y = -2z+4 = -\frac{30}{7}+4 = -\frac{2}{7}$$

$$2x+6z-12 = 4 \longrightarrow x = -3z+8$$

$$-9z+24-4z+8-z = 2 \qquad = -\frac{45}{7}+8$$

$$-14z = -30 \qquad = \frac{11}{7}$$

$$z = \frac{15}{7}$$

$$H = \left(\frac{11}{7}, -\frac{2}{7}, \frac{15}{7}\right) \qquad P = (2,-1,2)$$

$$d(r,s) = \|PH\| = \sqrt{\left(2-\frac{11}{7}\right)^2 + \left(-1+\frac{2}{7}\right)^2 + \left(2-\frac{15}{7}\right)^2} =$$

$$= \sqrt{\frac{9}{49}+\frac{25}{49}+\frac{1}{49}} = \frac{\sqrt{35}}{7}$$

<u>EX.</u>

$$A = (0,-1,1), \; B = (-1,0,2), \; C = (1,-1,-4)$$

(a) $\pi: ax+by+cz = d$

$$\begin{cases} A \in \pi: & -b+c = d \\ B \in \pi: & -a+2c = d \\ C \in \pi: & a-b-4c = d \end{cases}$$

$$\begin{cases} d = -b+c \\ -a+2c = -b+c \to -5c+2c-c=-b \\ a \cancel{-b} -4c = \cancel{-b}+c \to a = 5c \end{cases}$$

$$\begin{cases} a = 5c \\ b = 4c \\ d = -3c \\ c \text{ qls.} \end{cases} \longrightarrow \begin{cases} c = 1 \\ a = 5 \\ b = 4 \\ d = -3 \end{cases}$$

$$\pi : \boxed{5x + 4y + z = -3} \qquad n_\pi = (5,4,1)$$

(b) $A \in r$, $M \in r$, M p.to medio di BC

r? $\qquad A = (0,-1,1)$

$$M = \frac{B+C}{2} = \frac{\begin{pmatrix} 0 \\ -1 \\ -2 \end{pmatrix}}{2} = \left(0, -\frac{1}{2}, -1\right)$$

$$v_r = M - A = \left(0, \frac{1}{2}, -2\right)$$

$$r = A + \langle v_r \rangle$$

$$r : \begin{cases} x = 0 \\ y = -1 + \frac{1}{2}t \\ z = 1 - 2t \end{cases}, \quad t \in \mathbb{R}$$

(c) $A \in s$, s contenuto in π, $s \perp r$

$$\Downarrow$$

$s = A + \langle v_s \rangle$ $\uparrow$ $s \perp n_\pi$

$$v_s = n_\pi \times v_r = \begin{vmatrix} e_1 & e_2 & e_3 \\ 5 & 4 & 1 \\ 0 & \frac{1}{2} & -2 \end{vmatrix} = \left(-\frac{17}{2}, 10, \frac{5}{2}\right)$$

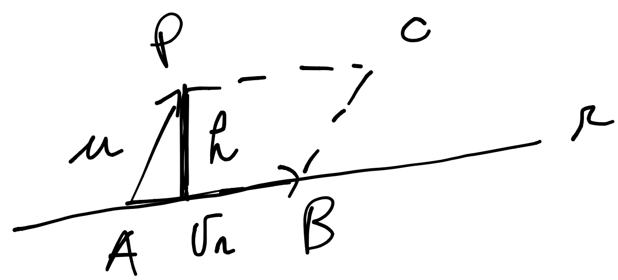
$$s = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \left\langle \begin{pmatrix} -17/2 \\ 10 \\ 5/2 \end{pmatrix} \right\rangle$$

$$s : \begin{cases} x = -\frac{17}{2} t \\ y = -1 + 10t \\ z = 1 + \frac{5}{2} t \end{cases}, \quad t \in \mathbb{R}$$

(d) $P = (1, 1, 1)$

$d(P, r) = ?$ $\quad d(P, \pi) = ?$

$$d(P, r) = \frac{\|u \times v_r\|}{\|v_r\|} \qquad u = P - A = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$



Area(ABCP) =

$= \|u \times v_r\| =$

$$= \|v_r\| \cdot h \implies h = \frac{\|u \times v_r\|}{\|v_r\|}.$$

$$d(P, r) = \frac{\|u \times v_r\|}{\|v_r\|} =$$

$$u \times v_r = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & 2 & 0 \\ 0 & \frac{1}{2} & -2 \end{vmatrix} = \left(-4, 2, \frac{1}{2}\right)$$

$$\|u \times v_r\| = \sqrt{16 + 4 + \frac{1}{4}} = \frac{9}{2}$$

$$\|v_r\| = \sqrt{\frac{1}{4} + 4} = \frac{\sqrt{17}}{2}$$

$$d(P, r) = \frac{\frac{9}{2}}{\frac{\sqrt{17}}{2}} = \frac{9}{\sqrt{17}} = \frac{9\sqrt{17}}{17}.$$

$$d(P, \pi) = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|5 + 4 + 1 + 3|}{\sqrt{25 + 16 + 1}} = \frac{13}{\sqrt{42}} = \frac{13\sqrt{42}}{42}.$$

$\pi: 5x + 4y + z + 3 = 0 \quad P = (1, 1, 1)$

Ex.

$\pi: 3x - y + z + 2 = 0 \quad n_\pi = (3, -1, 1) \equiv v_r$

$A = (0, 0, -2) \quad B = (0, 2, 0)$

(a) $A \in r$, $r \perp \pi$

$$r = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} + \left\langle \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

$$r: \begin{cases} x = 3t \\ y = -t \\ z = -2 + t \end{cases}, \ t \in \mathbb{R}$$

$$r: \begin{cases} x + 3y = 0 \\ y + z = -2 \end{cases}$$

(b) ? s

$B \in s$, $B = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \quad A = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

$s \in \pi$, $s \perp AB$

$$AB = B - A = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} \Longrightarrow w = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$v_s = w \times n_\pi = \begin{vmatrix} e_1 & e_2 & e_3 \\ 0 & 1 & 1 \\ 3 & -1 & 1 \end{vmatrix} = (2, 3, -3)$$

$$s: \begin{cases} x = 2t \\ y = 2 + 3t \\ z = -3t \end{cases}, \quad t \in \mathbb{R}$$

$$t = \frac{x}{2}$$

$$s: \begin{cases} y = 2 + \frac{3}{2}x \\ z = -\frac{3}{2}x \end{cases}$$

(c) $P = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$d(P, r) = ?$ $\quad d(P, \pi) = ?$ $\quad P' = ?$

$\uparrow$

PROIEZIONE ORT.
DI P SU π.

$$d(P, r) = \frac{\|u \times v_r\|}{\|v_r\|}, \quad u = P - A = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix},$$

$$u \times v_r = \begin{vmatrix} e_1 & e_2 & e_3 \\ 3 & 2 & 1 \\ 3 & -1 & 1 \end{vmatrix} = (3, 0, -9)$$

$$\|u \times v_r\| = \sqrt{9 + 81} = \sqrt{90}$$

$$\|v_r\| = \sqrt{9+1+1} = \sqrt{11}$$

$$d(P, r) = \frac{\sqrt{90}}{\sqrt{11}}$$

$$d(P,\pi) = \frac{|9 - \cancel{2} - 1 + \cancel{2}|}{\sqrt{9+1+1}} = \frac{8}{\sqrt{11}}$$

$\rightarrow$ RETTA PER P e $\perp$ π.

r'

$$r' : \begin{cases} x = 3+3t \\ y = 2-t \\ z = -1+t \end{cases}, \ t \in \mathbb{R}$$

$$r' \cap \pi : \begin{cases} x = 3+3t \\ y = 2-t \\ z = -1+t \\ 3x - y + z + 2 = 0 \end{cases}$$

$t = z+1$

$x = 3+3z+3 = 6+3z$

$y = 2-z-1 = 1-z$

$18+9z-1+z+z+2=0$

$$11z = -19 \longrightarrow z = -\frac{19}{11}$$

$$x = 6 - 3\cdot\frac{19}{11} = \frac{9}{11}$$

$$y = 1 + \frac{19}{11} = \frac{30}{11}$$

$$\Longrightarrow P' = \left(\frac{9}{11}, \frac{30}{11}, -\frac{19}{11}\right).$$