NUMERICAL AND MATHEMATICAL METHODS

FINAL EXAM TEMPLATE

Please, write scripts in python to address the following exercises. When you are done, please upload the scripts, the plots and any additional relevant material to the google drive LINK TO BE ADDED FOR THE REAL EXAM. It will not be permitted to upload material after the end of the written exam.

EXERCISE 1. Download the file data_BBHs_0.0002.txt from the above google drive.
This ASCII file contains the properties of a simulated population of binary black hole mergers.
Line 0 has two columns: ignore line 0.
Lines from 1 to the end of the file have 13 columns each. Each line corresponds to a different binary black hole. Columns 3 and 4 (**numeration starting from zero**) are the masses of the two components of each binary in solar masses.

Write a python script that
A. reads columns 3 and 4 of this file (the first column is column zero) into two numpy arrays M1 and M2, respectively;

B. plots M1 versus M2 with a scatter plot;

C. plots M1 versus M2 with a two dimensional histogram (the colours indicate the number of systems with a given combination of M1 and M2);

D. calculates the mean values of M1 and M2 and their standard deviations.

EXERCISE 2. Write a python script that addresses the following points.
A. Using the inverse random sampling, draw 1e6 random number from the distribution $P(s)=\frac{C}{s}$, where C is a normalization constant that you have to calculate and s goes from s_{min}=1AU to s_{max}=1000 AU. This is the distribution of orbital periods of binary stars in the solar neighborhood (Duquennoy & Mayor 1991).

B. Plot an histogram of your random numbers and compare it with the expected distribution *P(s)*.

EXERCISE 3. Write a python script that addresses the following points.
A. Plot the following function

$$f(x)=\sin(x)(1-x)^2$$

in the interval *x*=[0., np.pi) at steps of 0.01 in *x*.

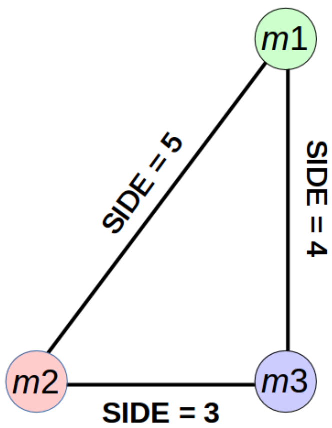
B. Integrate the above function with the mean value method using N = 1e3, 1e4, 1e5 and 1e6 random points.

C. Plot the result of the integral as a function of the number of random points.

EXERCISE 4. Three point masses attract each other according to the Newtonian law of gravitation:

$$\vec{a}_i=-G\sum_{j!=i} m_j \frac{\vec{x}_i-\vec{x}_j}{|\vec{x}_i-\vec{x}_j|^3}$$

The masses of the particles are m_1=3, m_2=4, and m_3=5; they are initially located at the apexes of a right triangle with sides 3, 4, and 5, as shown in the figure. The particles are free to move in the xy plane of the triangle and are at rest initially. We assume G = 1. This configuration is called Pythagorean 3-body problem.



A. Write a script to integrate the position and the velocity of the three particles in two dimensions with the midpoint method between time t = 0 and time tf = 5 with timesteps of h = 1e-5.

B. Plot the orbits of the three particles in the xy plane.

C. Calculate and plot the relative energy variation $\frac{E(t+h)-E(t)}{E(t)}$ of the system between two timesteps and plot it as a function of time.