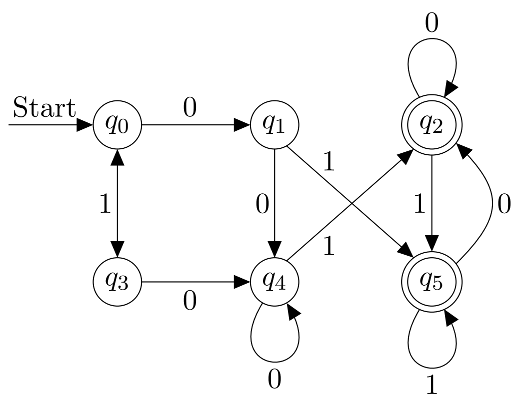
Master Degree in Computer Engineering

Automata, Languages and Computation
Final Exam

September 19th, 2022

1. **[5 points]** Consider the DFA A whose transition function is graphically represented below, where arcs with double direction represent two arcs in opposite directions.



   (a) Provide the definition of equivalent pair of states for a DFA.

   (b) Apply to A the tabular algorithm for detecting pairs of equivalent states, **reporting all the intermediate steps**.

   (c) Specify the minimal DFA equivalent to A.

2. **[8 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b\}$

$$\begin{aligned} L_1 &= \{w \mid w = a^p b a^q,\ p, q \geq 0,\ 1 \leq p + q\} ; \\ L_2 &= \{w \mid w = a^p b a^q,\ p, q \geq 0,\ 1 \leq p - q\} ; \\ L_3 &= \{w \mid w = a^p b a^q,\ p, q \geq 0,\ p = q^3 \text{ or } q = p^3\} . \end{aligned}$$

   For each of the above languages, state whether it belongs to the class REG. Provide a mathematical proof for all of your answers.

3. **[5 points]** Let G be some CFG in CNF. Let T be a parse tree for a string $w \in L(G)$. Using structural induction, prove that if the longest path in T has n arcs then $|w| \leq 2^{n-1}$

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4. **[5 points]** Consider the following language, defined over the alphabet $\Sigma = \{a, b\}$

$$L \quad = \quad \{w \mid w \in \Sigma^*,\ w = a^n b^{2n},\ n \geq 1\}.$$

Define a Turing machine M such that $L(M) = L$ and M stops for every possible input in Σ^*. Graphically represent the transition function of M and provide an informal discussion of the computation associated with each state of M.

5. **[8 points]** Let A be some fixed DFA with input alphabet $\Sigma = \{0, 1\}$ such that $L(A)$ is not finite and $L(A) \neq \Sigma^*$. Define the following property of the RE languages over Σ

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{RE},\ L \subseteq \Sigma^*,\ L \cap L(A) = \emptyset\}\ .$$

   (a) Apply Rice's theorem to prove that $L_{\mathcal{P}}$ is not in REC.

   (b) Prove that $L_{\overline{\mathcal{P}}}$ is in RE but not in REC, where $\overline{\mathcal{P}}$ is the complement of $\mathcal{P}$ with respect to all languages over Σ that are in RE.

   (c) Prove that $L_{\mathcal{P}}$ is not in RE.

6. **[2 points]** Let $\mathcal{NP}$ be the class of languages that can be recognised in polynomial time by a nondeterministic TM. State whether the relation $\mathcal{NP} \subseteq \text{REC}$ holds, and motivate your answer.