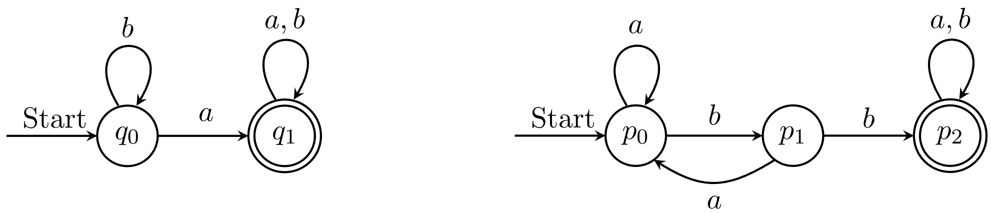
Master Degree in Computer Engineering

Final Exam for
Automata, Languages and Computation

July 18th, 2022

1. **[5 points]** Consider the two DFAs A_1 and A_2 whose transition functions are graphically represented as follows



Apply the construction in the textbook, using the cross-product of the state sets of A_1 and A_2, to produce a new DFA A that recognizes the intersection language $L(A_1) \cap L(A_2)$. For the transition function of A, provide either a tabular representation or else a graphical representation.

2. **[10 points]** Consider the following languages, defined over the alphabet $\Sigma = \{a, b\}$

$$\begin{aligned} L_1 &= \{w \mid w = a^n b^n b^m a^m,\ n, m \geq 1\}\ ; \\ L_2 &= \{w \mid w = a^n b^n b^n a^n,\ n \geq 1\}\ ; \\ L_3 &= \{w \mid w = a^n a^n a^n a^n,\ n \geq 1\}\ ;. \end{aligned}$$

For each of the above languages, state whether it belongs to one of the classes REG and CFL$\setminus$REG, or else if it is outside of CFL. Provide a mathematical proof for all of your answers.

3. **[5 points]** With reference to push-down automata (PDA), answer the following questions.

   (a) Provide the definition of language accepted by final state and language accepted by empty stack.

   (b) Prove that if P_N is a PDA accepting by empty stack, then there exists a PDA P_F accepting by final state such that $N(P_N) = L(P_F)$.

(please see next page)

4. **[5 points]** Provide the definition of a nondeterminstic TM M_N and of the recognized language $L(M_N)$. Prove that for each nondeterminstic TM M_N there exists a TM M_D such that $L(M_N) = L(M_D)$

5. **[6 points]** Let $\mathbf{N}$ be the set of natural numbers. Consider the following property of the RE languages defined over the alphabet $\Sigma = \{0, 1\}$

$$\mathcal{P} \quad = \quad \{L \mid L \in \text{RE}, \ \exists n \in \mathbf{N}, \ \Sigma^n \subseteq L\} \ .$$

   In words, a language L is in $\mathcal{P}$ if for some n, all strings of length n are in L.

   Assess whether the language $L_{\mathcal{P}}$ belongs to one of the classes REC and RE$\smallsetminus$REC, or else does not belong to RE, and provide a mathematical proof of your answer.

6. **[2 points]** Let $\mathcal{P}$ be the class of languages that can be recognised in polynomial time by a TM. Prove that if L_1 is in $\mathcal{P}$ and L_2 is in CFL, then $L_1 \cdot L_2$ is still in $\mathcal{P}$.