Rules for the exam of Differential Equations (a.y. 2022-23)

The standard mode is an oral exam on the topics explained in class and reported in the PDF files of the lectures. The proofs in the notes [B] that were not done in class are optional; Sections 1.3 and 1.4 of [B] are not part of the program.

The student can also choose two partial alternatives. Both assume that the student study the program developed in class, but he/she can skip some details of the proofs and some technical issues. Questions on the main parts of the course can be asked at the exam.

a. Work out in detail all the exercises proposed during the course. The exam consists of solving a couple of them, chosen by the professor, with an explanation of the related material of the course. The student must inform the prof. of this option at least a week before the exam.

b. Focalize the study on one of the topics listed below. They are based on the material developed in the course, and the student must clarify all the connections. The references can be consulted to get an idea of the topic, but the material to be studied and presented at the exam must be discussed with the professor. It is advised to contact him at least ten days in advance. The same topic will not be assigned to two students taking the exam in the same day.

1. Infinite horizon optimal control via Dynamic Programming [BCD, cap. III.2].
1b. Infinite horizon Linear-Quadratic optimal control and vanishing discount [BCD, cap. III.2.1], [B, cap. 1.3 e 1.4].
2. Optimal control in minimal time [B-mT], [BCD, cap. IV.2].
3. Eikonal type equations [I-eik], [BCD cap. II.5].
4. Discrete approximation of HJB equations and construction of optimal controls [CD], [BCD, cap. VI.1 + prerequisites in V.1].
5. Pontryagin Maximum Principle and its connection with the value function (for those who already know the PMP) [Z], [BCD, cap. III.3 (in particular III.3.3)].
6. Critical value of a Hamiltonian and connections with homogenization or with the weak KAM theory [B-h], [LPV], [E].
7. Existence of solutions of H-J equations via Perron method [I-per], [BCD, cap. V.2]
8. Hopf formulas for non-convex Hamiltonians [BE], [BO].
9. Infinite horizon differential games (2-person, 0-sum) [BCD, cap. VIII.1-2].
10. Discrete approximation of differential games [BCD, cap. VIII.3 + prerequisites in V.1]
11. Representation of solutions of H-J equations and front propagation by the level-set method [ES, sect. 5-7].
12. Existence of solutions of the 1st order system for deterministic Mean Field Games [B, sect. 4.5] and [C, sect. 4.1-4.3], or [CP, sect. 1.3.4].
13. Static Mean Field Games [C, sect. 2].
14. Numerical solution of H-J-B equations and optimal control problems [BCD, cap. A.1].

Bibliografia

The articles with an asterisk * will be available on Moodle, the others can be got in the library or asked to the professor.

[B] M. Bardi, *Notes of the course Differential Equations*, 2022.*

[B-mT] M. Bardi: *A boundary value problem for the minimum-time function.* SIAM J. Control Optim. 27 (1989), 776–785.*

[B-h] M. Bardi, *Metodi di viscosità per l'omogeneizzazione...*, notes of PhD courses, 2011.*

[BCD] M. Bardi, I. Capuzzo Dolcetta, *Optimal control and Viscosity solutions of Hamilton-Jacobi-Bellman equations*, Birkhäuser, Boston, 1997; 2nd printing, Modern Birkhäuser Classics, 2008.

[BE] M. Bardi, L.C. Evans, *On Hopf's formulas for solutions of Hamilton-Jacobi equations.* Nonlinear Anal. 8 (1984), 1373–1381.*

[BO] M. Bardi, S. Osher, *The nonconvex multidimensional Riemann problem for Hamilton-Jacobi equations.* SIAM J. Math. Anal. 22 (1991), 344–351.*

[CD] I. Capuzzo Dolcetta, *On a discrete approximation of the Hamilton-Jacobi equation of dynamic programming.* Appl. Math. Optim. 10 (1983), 367–377.*

[C] P. Cardaliaguet, *Notes on Mean Field Games*, 2013, https://www.ceremade.dauphine.fr/ cardaliaguet/.*

[CP] P. Cardaliaguet, A. Porretta eds., *Mean Field Games*, Lecture Notes in Math. 2281, Springer, 2020.*

[E] Evans, L. C., *Weak KAM theory and partial differential equations.* in "Calculus of variations and nonlinear partial differential equations," 123154, Lecture Notes in Math. 1927, Springer, Berlin, 2008.*

[ES] L. C. Evans, P. Souganidis *Differential Games and representation formulas...*, Indiana Univ. Math. J. 1984.*

[I-eik] H. Ishii, *A simple, direct proof of uniqueness for solutions of the Hamilton-Jacobi equations of eikonal type.* Proc. Amer. Math. Soc. 100 (1987), 247–251.*

[I-per] H. Ishii, *Perron's method for Hamilton-Jacobi equations.* Duke Math. J. 55 (1987), 369–384.

[LPV] P.-L. Lions, G. Papanicolaou, S. Varadhan, *Homogenization of Hamilton-Jacobi equations*, unpublished 1986.*

[Z] X.Y. Zhou, *Maximum principle, dynamic programming, and their connection in deterministic control.* J. Optim. Theory Appl. 65 (1990), 363–373.*