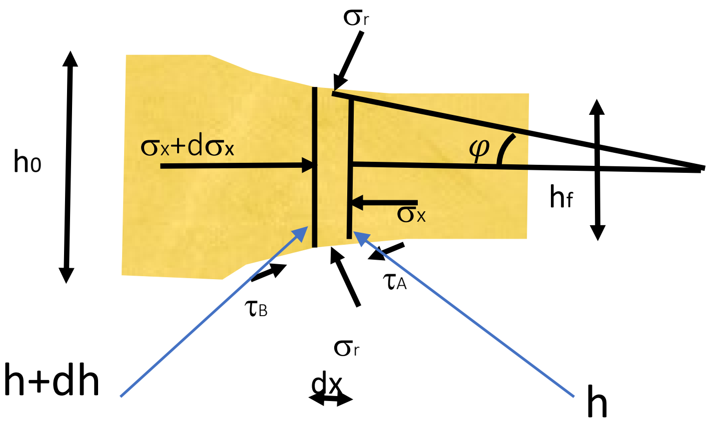
Metodo dello Slab per la Laminazione



Equilibrio forze in x

$$\sigma_x h\, b - (\sigma_x + d\sigma_x)(h + dh)\, b + 2\sigma_r R\, d\varphi\, b \sin\varphi = \begin{cases} 2\tau_{attrito} R d\varphi b \cos\varphi & \textbf{\textit{nella zona A}} \\ -2\tau_{attrito} R d\varphi b \cos\varphi & \textbf{\textit{nella zona B}} \end{cases}$$

Dividendo tutto per **b** ottengo

$$\sigma_x h - \sigma_x h - \sigma_x dh - d\sigma_x\, h - d\sigma_x\, dh = 2\sigma_r R d\varphi(-\sin\varphi \mp \mu\cos\varphi)$$

$$\mathrm{d}(\sigma_x h) = 2\sigma_r \mathrm{R} d\varphi(\sin\varphi \mp \mu\cos\varphi)$$

Infinitesimo di ordine superiore

Poiché le **tensioni principali** sono

$$\sigma_1 = \sigma_x$$

$$\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$$

$$\sigma_3 = \sigma_r$$

von Mises

$$(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_2)^2 = 2Y^2$$

$$\left(\sigma_x - \frac{1}{2}(\sigma_x + \sigma_r)\right)^2 + \left(\frac{1}{2}(\sigma_x + \sigma_r) - \sigma_r\right)^2 + (\sigma_r - \sigma_x)^2 = 2Y^2$$

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$$\left[\frac{1}{2}(\sigma_x - \sigma_r)\right]^2 + \left[\frac{1}{2}(\sigma_x - \sigma_r)\right]^2 + (\sigma_r - \sigma_x)^2 = (\sigma_r - \sigma_x)^2\left(\frac{1}{4} + \frac{1}{4} + 1\right)$$

$$\frac{6}{4}(\sigma_r - \sigma_x)^2 = 2\,Y^2 \quad \Rightarrow \quad |\sigma_r - \sigma_x| = \frac{2}{\sqrt{3}}\,Y = Y'$$

$$\sigma_x = \sigma_r - Y'$$

$\sin\varphi = \varphi$

$\cos\varphi = 1$

Per ϕ piccolo

$$\mathrm{d}(\sigma_x h) = \mathrm{d}\big((\sigma_r - Y')h\big) = d\left[hY'\left(\frac{\sigma_r}{Y\prime} - 1\right)\right] = 2\sigma_r \mathrm{Rd}\varphi(\sin\varphi \mp \mu\cos\varphi) = 2\sigma_r \mathrm{Rd}\varphi(\ \varphi \mp \mu)$$

$$d\left[hY'\left(\frac{\sigma_r}{Y\prime} - 1\right)\right] = hY'd\frac{\sigma_r}{Y\prime} + \left(\frac{\sigma_r}{Y\prime} - 1\right)\mathrm{d}(hY') = 2\sigma_r \mathrm{Rd}\varphi(\ \varphi \mp \mu)$$

Man mano che h diminuisce tanto più il materiale incrudisce cioè Y ed Y’ aumentano
Quindi h Y’ rimane quasi costante ⇒ $\mathrm{d}(hY') = 0$

- $d\left[hY'\left(\frac{\sigma_r}{Y\prime} - 1\right)\right] = hY'd\frac{\sigma_r}{Y\prime} = 2\sigma_r \mathrm{Rd}\varphi(\ \varphi \mp \mu)$

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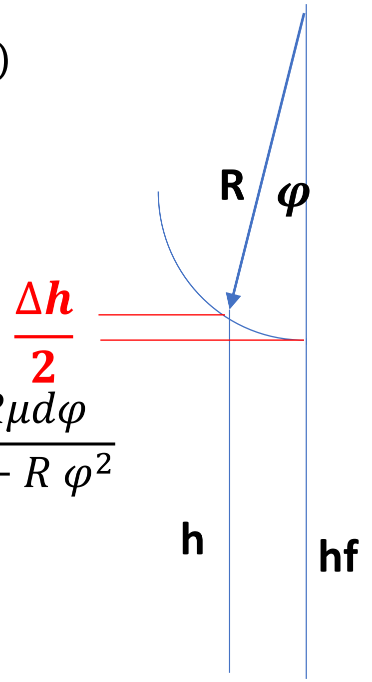
ma $h = h_f + 2\frac{\Delta h}{2} = h_f + 2(R - R\cos\varphi) = h_f + 2R(1-\cos\varphi)$

$\cos\varphi = 1 - \frac{\varphi^2}{2} + \cdots$ $\Rightarrow$ $h = h_f + R\,\varphi^2$



$$hY' d\frac{\sigma_r}{Y'} = 2\sigma_r \mathrm{R d}\varphi(\,\varphi \mp \mu)$$

$$\frac{d\frac{\sigma_r}{Y'}}{\frac{\sigma_r}{Y'}} = 2\frac{\mathrm{R}}{h}\mathrm{d}\varphi(\,\varphi \mp \mu) \Rightarrow \frac{d\left(\frac{\sigma_r}{Y'}\right)}{\left(\frac{\sigma_r}{Y'}\right)} = \frac{2R}{h_f + R\,\varphi^2}(\,\varphi \mp \mu)\mathrm{d}\varphi = \frac{2R\varphi d\varphi}{h_f + R\,\varphi^2} \mp \frac{2R\mu d\varphi}{h_f + R\,\varphi^2}$$

Introduco le nuove variabili

- $w = \varphi\sqrt{\frac{R}{h_f}}$ $\qquad dw = \sqrt{\frac{R}{h_f}}d\varphi$
- $z = h_f + R\,\varphi^2 = h$ $\qquad dz = 2R\varphi d\varphi$

$$\Rightarrow \frac{d\left(\frac{\sigma_r}{Y'}\right)}{\left(\frac{\sigma_r}{Y'}\right)} = \frac{dz}{z} \mp 2\mu\sqrt{\frac{R}{h_f}}\frac{dw}{1+\,w^2}$$

$$\Downarrow$$

$$ln\left(\frac{\sigma_r}{Y'}\right) = ln(z) \mp 2\mu\sqrt{\frac{R}{h_f}}arctan\left(\varphi\sqrt{\frac{R}{h_f}}\right) + ln(C)$$

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$$ln\left(\frac{\sigma_r}{Y'}\right) = ln(Cz) \mp ln\left(\exp\left(2\,\mu\sqrt{\frac{R}{h_f}}arctan\left(\varphi\sqrt{\frac{R}{h_f}}\right)\right)\right)$$

chiamo $K(\varphi) = 2\sqrt{\frac{R}{h_f}}arctan\left(\varphi\sqrt{\frac{R}{h_f}}\right)$

$$ln\left(\frac{\sigma_r}{Y'}\right) = ln(Cz) \mp ln\big(\exp\ (\mu K)\big) = ln\big(Cz\,exp(\mp\mu K)\big)$$

$$\left(\frac{\sigma_r}{Y'}\right) = C\big(h_f + R\varphi^2\big)exp\left(\mp 2\mu\sqrt{\frac{R}{h_f}}arctan\left(\varphi\sqrt{\frac{R}{h_f}}\right)\right)$$

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Ingresso – Zona A $\sigma_x = 0$ h=h_0 $\varphi = \alpha$ $King = 2\left(\frac{R}{h_f}\right)^{\frac{1}{2}} arctan\left(\sqrt{\frac{R}{h_f}}\ \alpha\right)$

$\sigma_r = Y'$

$$\frac{\boldsymbol{\sigma_r}}{\boldsymbol{Y\prime}} = \frac{\boldsymbol{h}}{\boldsymbol{h_0}}\ \mathbf{exp}[\boldsymbol{\mu}(\boldsymbol{K_{ing}} - \boldsymbol{K})]$$

Uscita – Zona B $\sigma_x = 0$ h=h_f $\varphi = 0$ $Kusc = 0$

$\sigma_r = Y'$

$$\frac{\boldsymbol{\sigma_r}}{\boldsymbol{Y\prime}} = \frac{\boldsymbol{h}}{\boldsymbol{h_f}}\ \mathbf{exp}[\boldsymbol{\mu}(\boldsymbol{K})]$$