Ricalcatura

Manufacturing Processes for Engineering Materials, 5th ed.
Kalpakjian • Schmid

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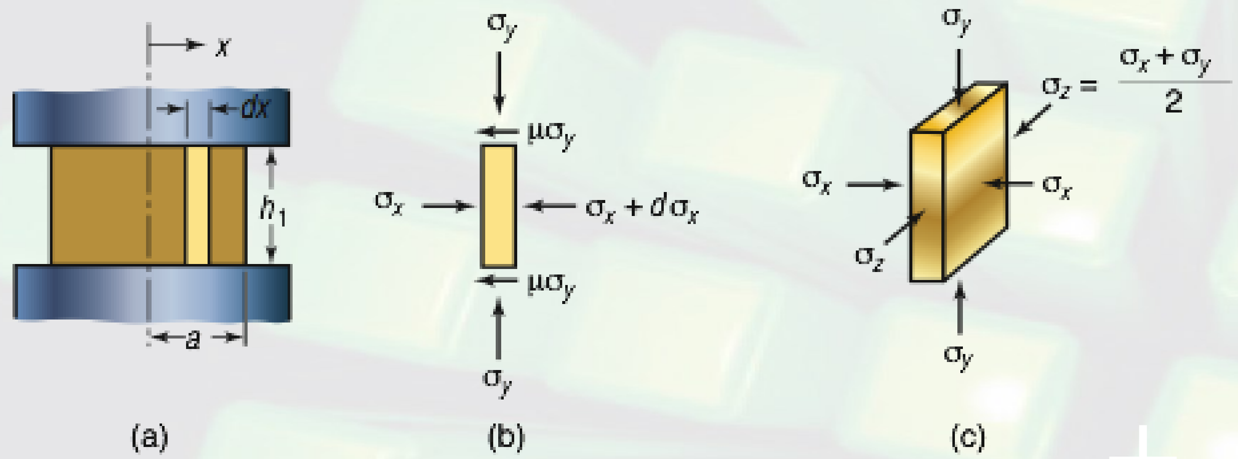
Analisi con la tecnica analitica (*Slab*) della Ricalcatura in condizioni di deformazione piana ($\varepsilon_z = 0$)



- Equilibrio forze in x

$$(\sigma_x + d\sigma_x)h + 2\mu\sigma_y dx - \sigma_x h = 0$$

$$h d\sigma_x + 2\mu\sigma_y dx = 0$$

- Criterio di von Mises per deformazione piana

  (σ_x e σ_y tensioni principali per μ piccolo)

$$\sigma_y - \sigma_x = \frac{2}{\sqrt{3}}Y = Y'$$

$d\sigma_x = d\sigma_y$

$$\frac{d\sigma_y}{\sigma_y} = -\frac{2\mu}{h}dx$$

$$\begin{cases} x = a \\ \sigma_x = 0 \qquad \sigma_y = Y' \end{cases}$$

$$ln\sigma_y = -\frac{2\mu x}{h} + lnC$$

$$ln\frac{\sigma_y}{C} = -\frac{2\mu x}{h}$$

$$\ln\left(\frac{Y'}{C}\right) = -\frac{2\mu\, a}{h}$$

$$Y' = C\, exp\left(\frac{-2\mu\, a}{h}\right)$$

$$C = Y' \exp\left(\frac{2\mu\, a}{h}\right)$$

$$\begin{cases} \sigma_y = p = Y' exp\left(\frac{2\mu}{h}(a - x)\right) \\ \sigma_x = \sigma_y - Y' = Y'\left(exp\left(\frac{2\mu}{h}(a - x)\right) - 1\right) \end{cases}$$

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$$F = 2b \cdot \int_0^a p dx = 2b \cdot Y' \cdot \int_0^a exp\left(2\mu\left(\frac{a-x}{h}\right)\right) dx$$

Sviluppando l'esponenziale in serie di Taylor cioè

$$\exp(x) = 1 + x + \frac{x^2}{2} + \cdots$$

$$F \cong 2b \cdot a \cdot Y' \cdot \left(1 + \frac{\mu a}{h}\right) \cong 2a \cdot b \cdot p_{aver}$$

dove

- b *è la profondità in direzione* z
- $p_{aver} = Y' \cdot \left(1 + \frac{\mu a}{h}\right)$ d

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Relazioni Levy-von Mises

$$d\varepsilon_1 = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_1 - \frac{1}{2}(\sigma_2 + \sigma_3)\right)$$
$$d\varepsilon_2 = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_2 - \frac{1}{2}(\sigma_1 + \sigma_3)\right)$$
$$d\varepsilon_3 = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_3 - \frac{1}{2}(\sigma_1 + \sigma_2)\right)$$

Nel caso di deformazione piana $\varepsilon_3 = \varepsilon_z = d\ \varepsilon_z = 0$

$$d\varepsilon_z = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_z - \frac{1}{2}(\sigma_x + \sigma_y)\right) = 0$$

Quindi

$$\sigma_z = \frac{1}{2}(\sigma_x + \sigma_y)$$

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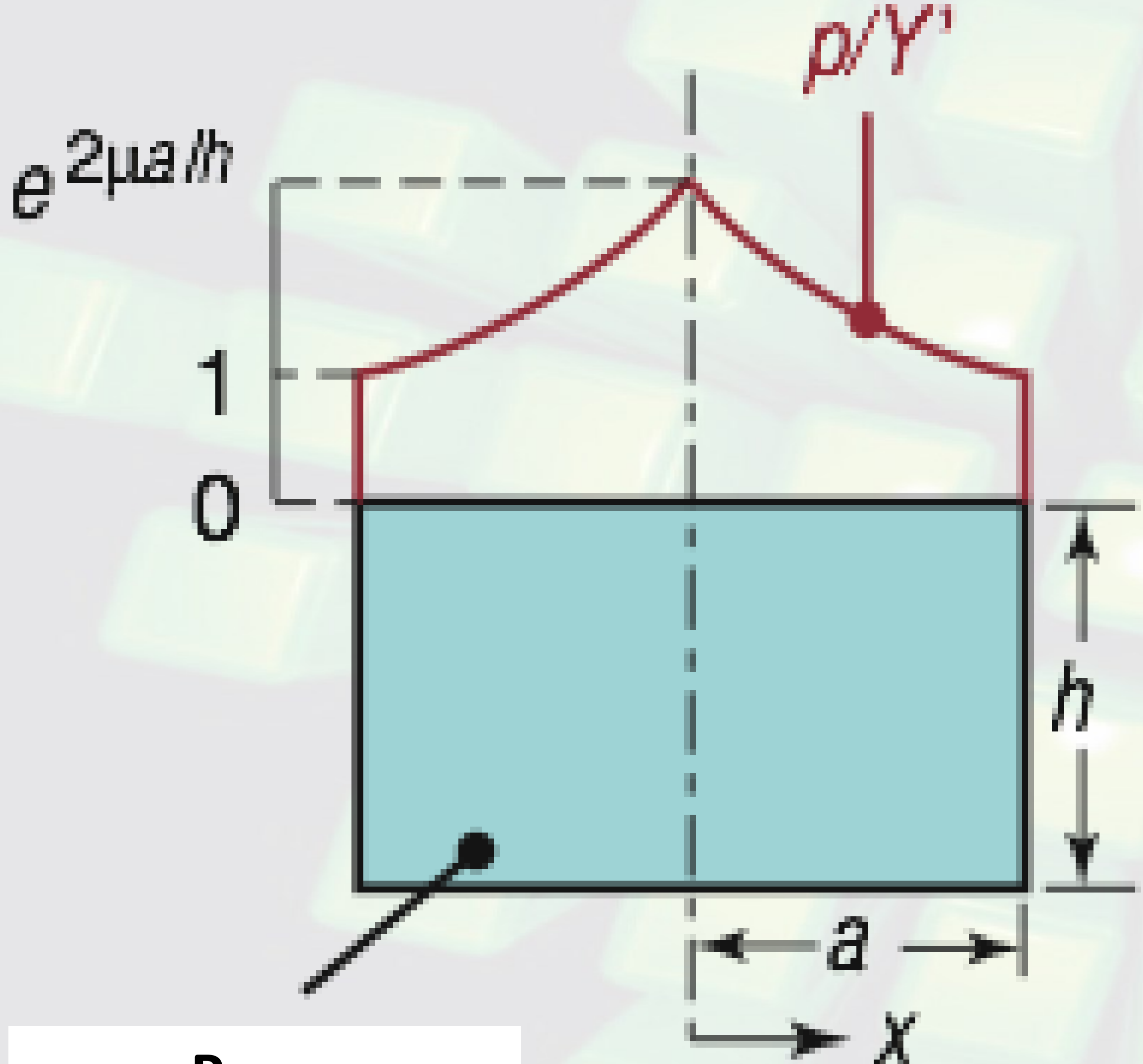
Pressione agente sullo stampo nelle condizioni di deformazione piana



Pezzo

Andamento della pressione in forma adimensionale (p/Y’) nelle condizioni di deformazione piana e in presenza di attrito proporzionale alla tensione normale

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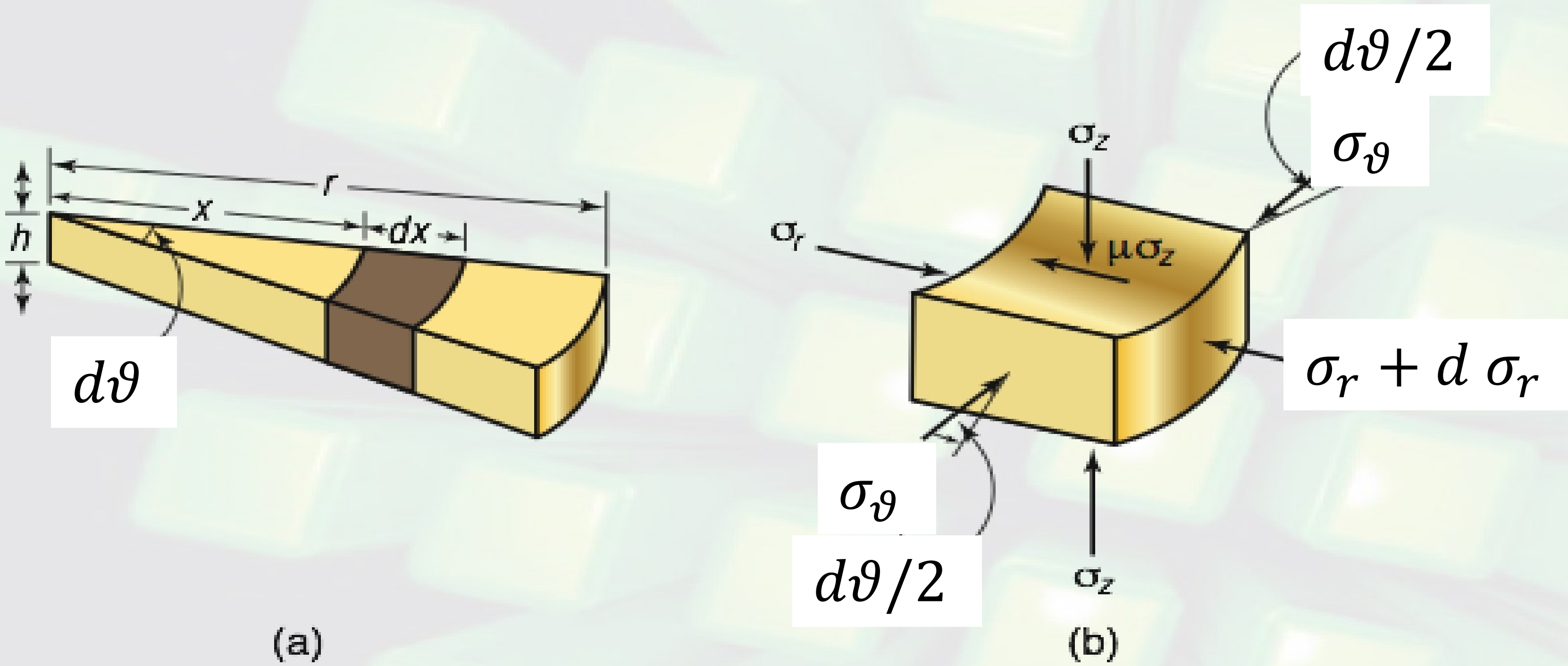
Analisi con la tecnica analitica (*Slab*) della Ricalcatura per un pezzo cilindrico



$$p = Ye^{2\mu(r-x)/h}.$$

Pressione media:

$$p_{av} \simeq Y\left(1+\frac{2\mu r}{3h}\right).$$

Forza di ricalcatura:

$$F = (p_{av})\left(\pi r^2\right)$$

Equilibrio radiale

$$\sigma_r(xd\theta)\mathrm{h} - (\sigma_r + d\sigma_r)(x+dx)d\theta\, h - 2\mu\sigma_z(xd\theta)dx + 2\sigma_\theta(hdx)\sin\frac{d\theta}{2} = 0$$

$$\sigma_r(xd\theta)\mathrm{h} - (x\sigma_r + xd\sigma_r + dx\sigma_r + dxd\sigma_r)d\theta\, h - 2\mu\sigma_z(xd\theta)dx + 2\sigma_\theta(hdx)\frac{d\theta}{2} = 0$$

$$\sigma_r\, xh\, d\theta + \sigma h\, dx\, d\theta - 2\mu\sigma_z x\, dx\, d\theta - (\sigma_r\, x + xd\sigma_r + \sigma_r\, dx + dx\, d\sigma_r)h\, d\theta = 0$$

$$\sin\left(\frac{d\theta}{2}\right) = \frac{d\theta}{2}$$

Infinitesimo d'ordine superiore

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Equilibrio radiale

$$dx d\theta(\sigma_\theta\, h - 2\mu\sigma_z x - h\sigma_r) = xhd\sigma_r d\theta$$

$$dx(\sigma_\theta\, h - 2\mu\sigma_z x - h\sigma_r) = xhd\sigma_r$$

$$\frac{d\sigma_r}{dx} + \frac{\sigma_r - \sigma_\theta}{x} = -\frac{2\mu\sigma_z}{h}$$

$$\frac{d\sigma_r}{dx} = -\frac{2\mu\sigma_z}{h}$$

- $d\varepsilon_\theta = \frac{2\pi(x+dx)-2\pi x}{2\pi x} = \frac{dx}{x} = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_\theta - \frac{1}{2}(\sigma_r + \sigma_z)\right)$
- $d\varepsilon_{\ r} = \frac{x+dx-x}{x} = \frac{dx}{x} = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_r - \frac{1}{2}(\sigma_\theta + \sigma_z)\right)$
- $d\varepsilon_\theta = d\varepsilon_r$
- $\frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_\theta - \frac{1}{2}(\sigma_r + \sigma_z)\right) = \frac{d\varepsilon_{eq}}{\sigma_{eq}}\left(\sigma_r - \frac{1}{2}(\sigma_\theta + \sigma_z)\right)$
- $\sigma_\theta = \sigma_r$

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Equilibrio radiale

- Criterio energetico

$$(\sigma_r - \sigma_\vartheta)^2 + (\sigma_\vartheta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 = 2Y^2$$
$$(0)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_r - \sigma_z)^2 = 2Y^2$$
$$2(\sigma_r - \sigma_z)^2 = 2Y^2$$
$$|\sigma_r - \sigma_z| = Y = \sigma_{max} - \sigma_{min} = \sigma_z - \sigma_r \Rightarrow (d\sigma_r - d\sigma_z) = 0 \Rightarrow d\sigma_r = d\sigma_z$$

Quindi sostituendo nell'equilibrio radiale si ottiene

$$\frac{d\sigma_z}{dx} = -\frac{2\mu\sigma_z}{h}$$

del tutto simile a quella della deformazione piana e quindi

$$\sigma_z = p = Y exp\left(\frac{2\mu}{h}(r - x)\right)$$

$$\sigma_r = -Y + \sigma_z = Y\left(-1 + exp\left(\frac{2\mu}{h}(r - x)\right)\right)$$

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Pressione media

$$p_{av} = \frac{\int_0^r 2\pi x p dx}{\pi r^2} = \frac{\int_0^r 2\pi x\, Y exp\left(\frac{2\mu}{h}(r-x)\right)dx}{\pi r^2} =$$

$$= \frac{2Y}{r^2}\int_0^r x\, exp\left(\frac{2\mu}{h}(r-x)\right)dx$$

poiché exp (z)=1+ z + $\frac{z^2}{2}$ $+\cdots\ dove\ z = \frac{2\mu}{h}(r-x)$

$$p_{av} = \frac{2Y}{r^2}\int_0^r x\left(1+\frac{2\mu}{h}(r-x)+\frac{1}{2}\,\frac{4\mu^2}{h^2}\,(r-x)^2\right)dx =$$

$$= \frac{2Y}{r^2}\int_0^r x dx + \frac{2Y}{r^2}\int_0^r x\frac{2\mu}{h}(r-x)dx + \frac{2Y}{r^2}\int_0^r x\frac{2\mu^2}{h^2}(\,(r-x)^2)dx =$$

$$= \frac{2Y}{r^2}\int_0^r (x+\frac{2\mu r x}{h}-\frac{2\mu x^2}{h})\,dx + \frac{2Y}{r^2}\int_0^r \frac{2\mu^2}{h^2}\,x\,\frac{r^2+x^2-2rx}{2}\,dx =$$

$$= \frac{2Y}{r^2}\left[\left\{1+\frac{2\mu r}{h}\right\}\frac{r^2}{2}-\frac{2\mu r^3}{3h}\right]+\frac{2Y}{r^2}\frac{2\mu^2}{2h^2}\left(\frac{\frac{r^4}{2}+\frac{r^4}{4}-\frac{2r^4}{3}}{2}\right)=$$

$$\sim Y\left(1+\frac{2\mu r}{3h}\right)$$

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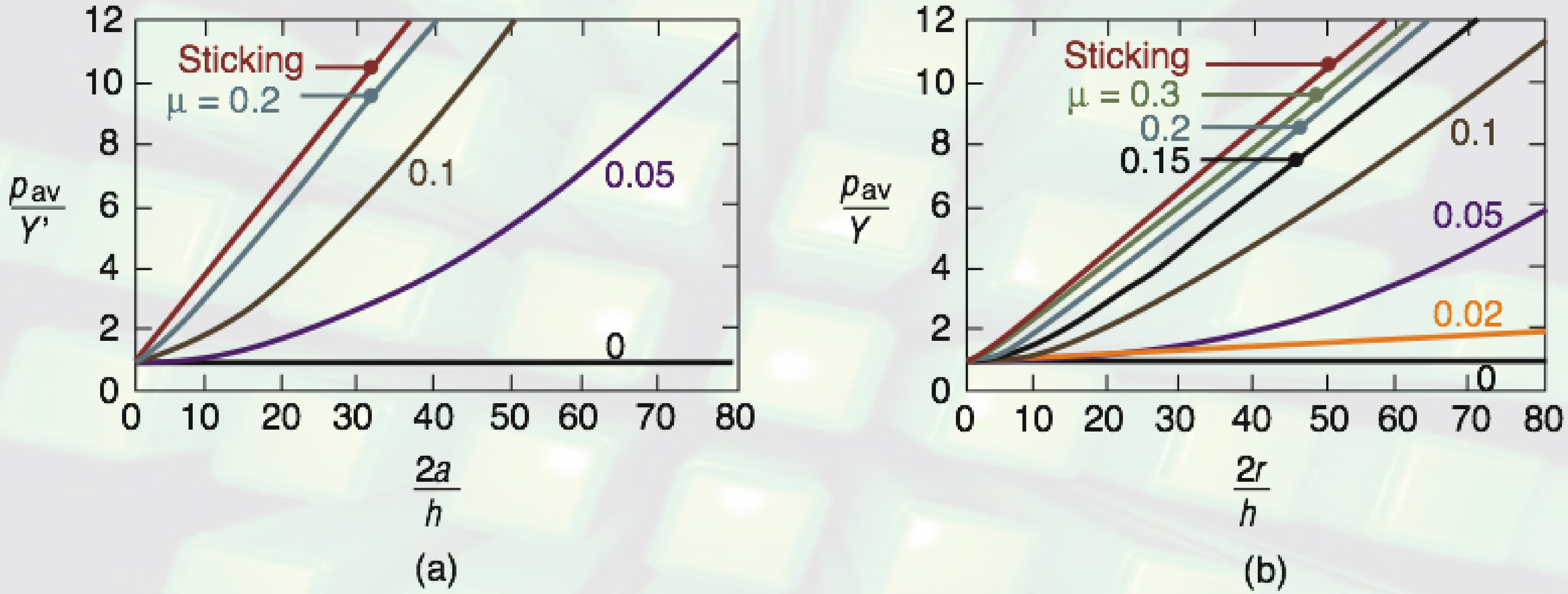
Pressione agente sullo stampo



Rapporto tra pressione media e tensione di snervamento in funzione dell'attrito e del aspetto di forma del campione (2a/h, 2r/h):

(a) Compressione in condizioni di deformazione piana
(b) Compressione di un cilindro pieno

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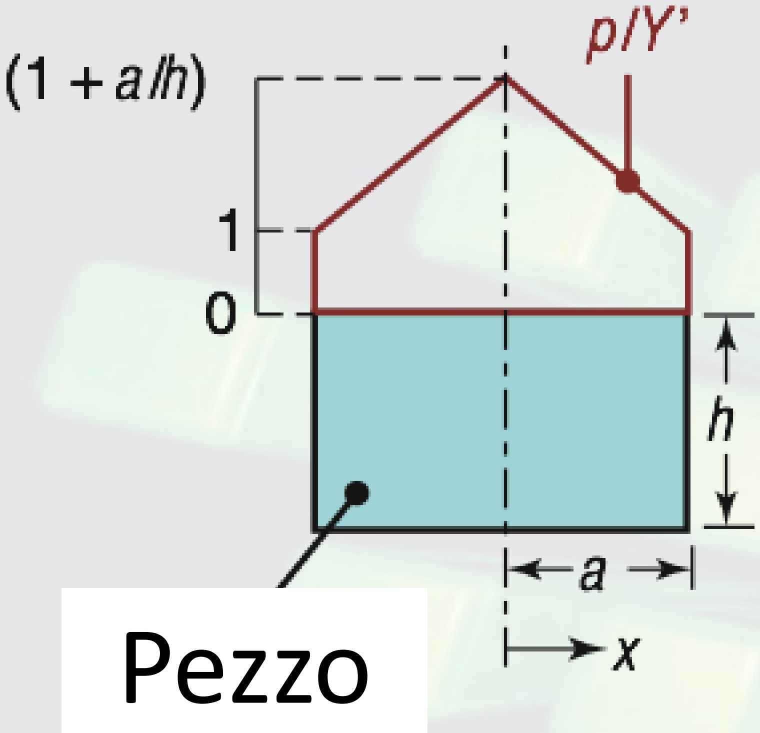
Pressione nella Ricalcatura in condizioni di adesione (Sticking) e deformazione piana

$$\tau = \mu\, p = k = tensione\ tangenziale\ di\ snervamento = \frac{\sigma_1 - \sigma_3}{2} = \frac{Y'}{2} = \frac{Y}{\sqrt{3}}$$



Andamento della pressione (*p/Y'*) in condizioni di deformazione piana e adesione (sticking)

In condizioni di deformazione piana ($\varepsilon_2 = 0$) dalle equazioni di Levi von Mises si ottiene $\sigma_2 = \frac{\sigma_1+\sigma_3}{2}$

quindi secondo il criterio energetico (di von Mises)

$$\left(\boldsymbol{\sigma_1} - \frac{\sigma_1+\sigma_3}{2}\right)^2 + \left(\frac{\sigma_1+\sigma_3}{2} - \boldsymbol{\sigma_3}\right)^2 + (\boldsymbol{\sigma_3} - \boldsymbol{\sigma_1})^2 = 2Y^2$$

$$\left(\frac{\sigma_1-\sigma_3}{2}\right)^2 + \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 + (\boldsymbol{\sigma_3} - \boldsymbol{\sigma_1})^2 = 2Y^2$$

$$\left(\frac{\sigma_1-\sigma_3}{2}\right)^2 (4+1+1) = 2Y^2$$

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 = \frac{2Y^2}{6} = \frac{Y^2}{3}$$

Equilibrio forze in x

$h d\sigma_x + 2\mu p dx = 0$

$h\, d\sigma_x + 2k dx = h d\sigma_x + 2\frac{Y'}{2} dx = 0$

$h\, d\sigma_x = -Y' dx$

$h\sigma_x = -Y'x + c$ *ma per x=a* $\sigma_x = 0$

quindi c= Y' a

e le tensioni risultano pari a

- $\sigma_x = \frac{Y'}{h}(a - x) = \sigma_{min} = \sigma_3$
- $\sigma_2 = \frac{\sigma_{max}+\sigma_{min}}{2}$
- $\sigma_y = Y' + \sigma_x = Y'\left(1 + \frac{a-x}{h}\right) = p = \sigma_{max} = \sigma_1 = \sigma_y$

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Pressione nella Ricalcatura in condizioni di adesione (Sticking) nel caso della deformazione assial - simmetrica

- Per **l'equilibrio radiale** $\frac{d\sigma_r}{dx} = -\frac{2\mu\sigma_z}{h}$

$$\tau = \mu\sigma_z = k$$

$$(\sigma_r - \sigma_\vartheta)^2 + (\sigma_\vartheta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 = 2Y^2$$

e poiché $\sigma_r = \sigma_\vartheta$ allora

$$(0)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_r)^2 = 2\,(\sigma_r - \sigma_z)^2 = 2Y^2$$

$|\sigma_r - \sigma_z| = Y = \sigma_{max} - \sigma_{min} = \sigma_z - \sigma_r$ ma poiché $\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{Y}{2} = k$ quindi

$$\frac{d\sigma_r}{dx} = -\frac{2\mu\sigma_z}{h} = -\frac{2\frac{Y}{2}}{h} = -\frac{Y}{h}$$

$$\sigma_r = -\frac{Y}{h}\,\mathrm{x} + c$$

Con $\sigma_r = 0 \ \ per \ \ x = r$ quindi $c = \frac{Y}{h} r$ quindi $\sigma_r = \frac{Y}{h}(r - x) = \sigma_{min}$

$$\sigma_z = p = \sigma_{max} = \sigma_{min} + Y = Y\left(1 + \frac{r - x}{h}\right)$$

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