Answer the following questions, providing a motivation for each answer.
(In case of negative answer the "motivation" is a counterexample)

1. Is it true that a continuous function defined on the interval $]1,3]$ has an absolute maximum and an absolute minimum?

2. Is it true that a continuous function defined on the interval $]1,3]$ has a relative maximum and a relative minimum?

3. Consider a function $f :]-1,1[\to \mathbb{R}$, and assume that it is differentiable on $]-1,1[\setminus \{0\}$ and that the limit $\lim_{x\to 0} f'(x)$ does NOT exist. Can we conclude that the function f is not differentiable at $x=0$?

4. Suppose that a function $f : [0,1] \to \mathbb{R}$ is differentiable at all points of $]0,1[$ and that f is continuous at $x=0$ and $x=1$. Can we conclude that f is integrable?

5. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series which does not converge absolutely. Can we conclude that
$$\lim_{n\to\infty} (a_n)^4 = 0?$$

6. Let $\sum_{n=1}^{\infty} a_n$ be a convergent series. Is it true that the series $\sum_{n=1}^{\infty} b_n$, with $b_n = -a_n$ $\forall n \in \mathbb{N}$, is a convergent series as well?

7. Consider a complex polynomial $P(z) = b_5 z^5 + b_4 z^4 + b_3 z^3 + b_2 z^2 + b_1 Z + b_0$ $(b_0, \ldots, b_5 \in \mathbb{C})$. Is it true that the equation $P(z) = 0$ has at least one real solution?

8. Same as in the previous question but with $b_0, \ldots, b_5 \in \mathbb{R}$

9. Consider a function $f : [0, +\infty[\to \mathbb{R}$ and assume that the series $\sum_{n=1}^{\infty} a_n$, where $a_n := f(n)$ $\forall n \in \mathbb{N}$, converges. Can we conclude that $\lim_{x\to\infty} f(x) = 0$?