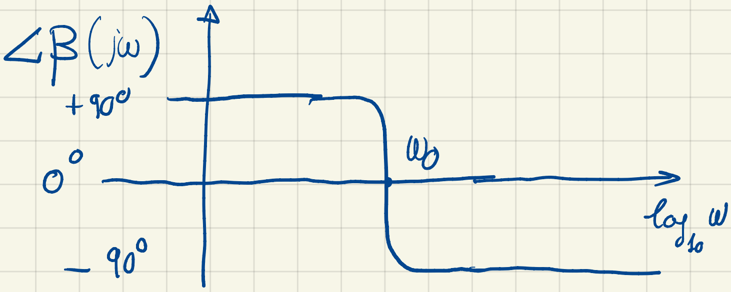
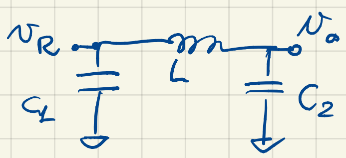
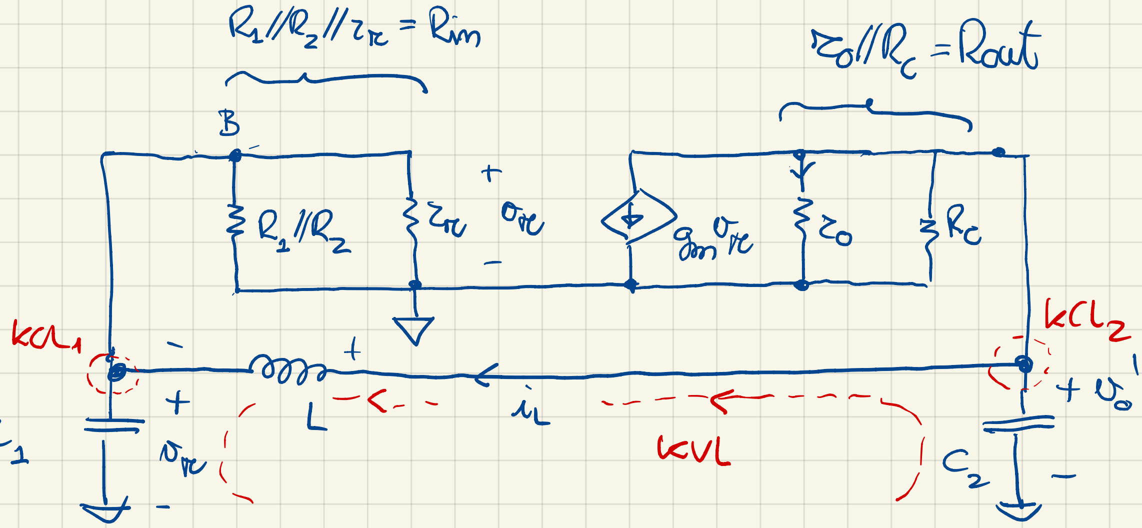


THE β-NETWORK



IS "SELECTIVE", ITS PHASE IS EQUAL TO $0°$ AT $\omega = \omega_0$ BUT PHASE VARIATION IS VERY "FAST". A SIGNAL WITH FREQUENCY DIFFERENT FROM ω_0 (EVEN BY A LITTLE AMOUNT) WILL NOT MEET BARKHAUSEN CONDITIONS AND WILL NOT PROPAGATE AROUND THE LOOP

SELECTIVITY ⟹ THE OSCILLATION FREQUENCY WILL BE VERY STABLE



$$\begin{cases} i_L = v_\pi \cdot sC_1 + v_\pi \cdot \dfrac{1}{R_{in}} = v_\pi \left(sC_1 + \dfrac{1}{R_{in}} \right) & \text{KCL}_1 \\[2ex] v_o' = sL\, i_L + v_\pi = v_\pi \cdot \left[1 + sL \left(sC_1 + \dfrac{1}{R_{in}} \right) \right] & \text{KVL} \\[2ex] v_o' = -\dfrac{1}{sC_2} \left(i_L + g_m v_\pi + \dfrac{v_o'}{R_{out}} \right) & \text{KCL}_2 \end{cases}$$

$$v_o' \cdot \left(1 + \frac{1}{sC_2 R_{out}} \right) = -\frac{v_\pi}{sC_2} \left(g_m + sC_1 + \frac{1}{R_{in}} \right)$$

$$v_o' = -\frac{v_\pi}{sC_2} \left(g_m + \frac{1}{R_{in}} + sC_1 \right) \cdot \frac{sC_2 R_{out}}{1 + sC_2 R_{out}}$$

Hyp: $\omega_0 \gg \dfrac{1}{R_{out} C_2} \Rightarrow \dfrac{sC_2 R_{out}}{1 + sC_2 R_{out}} \simeq 1$

$$v_{\pi}\left[1+sL\left(sC_1+\frac{1}{R_{in}}\right)\right] = -\frac{v_{\pi}}{sC_2}\left(g_m+\frac{1}{R_{in}}+sC_1\right)$$

$$v_{\pi}\left[1+s^2LC_1+s\frac{L}{R_{in}}+\frac{g_m}{sC_2}+\frac{1}{R_{in}sC_2}+\frac{C_1}{C_2}\right] = \emptyset \leftarrow$$

TWO SOLUTIONS

- $v_{\pi}=\emptyset \Rightarrow$ THE CIRCUIT IS "TURNED OFF" $\Rightarrow$ NOT INTERESTING

$v_{\pi}\neq 0$ THE EQUATION IS EQUIVALENT TO

$$sR_{in}C_2+s^3R_{in}LC_1C_2+s^2LC_2+g_mR_{in}+1+sC_1R_{in} = \emptyset$$

$s=j\omega_0 \Rightarrow$

$$\begin{cases} \mathrm{Re}(\ ) = -\omega_0^2LC_2+1+g_mR_{in} \overset{!}{=} \emptyset \Leftrightarrow |T(\omega_0)|=1 \\ \mathrm{Im}(\ ) = -\omega_0^3\cancel{R_{in}}LC_1C_2+\cancel{\omega_0}\cancel{R_{in}}(C_1+C_2) \overset{!}{=} \emptyset \Leftrightarrow \angle T(\omega_0)=2k\pi \end{cases}$$

FROM THE SECOND

$$\omega_0 = \sqrt{\frac{C_1+C_2}{L\,C_1C_2}} = \sqrt{\frac{1}{LC_{eq}}} \qquad C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

SERIES OF C_1 AND C_2

REPLACING IN THE FIRST

$$-\frac{C_1+C_2}{\cancel{L}C_1\cancel{C_2}}\cancel{L}\cancel{C_2}+1+g_mR_{in} = \emptyset \Leftrightarrow \boxed{g_mR_{in}=\frac{C_2}{C_1}}$$

ω_0 IS THE OSCILLATION FREQUENCY

THE OSCILLATION WILL START IF $g_mR_{in} > \frac{C_2}{C_1}$! $\leftarrow$

TO DESIGN THE CIRCUIT IT IS TYPICALLY EASIER TO CHOOSE

1. $C_1 \ll C_2 \Rightarrow \omega_0 \cong \sqrt{\frac{1}{LC_1}} \Rightarrow$ CAN BE CHOSEN $\ll \omega_T$

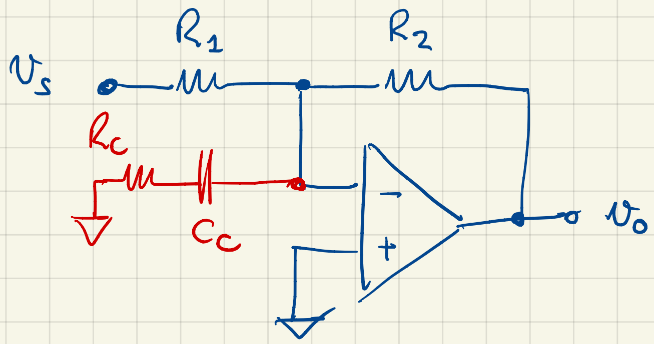
2. $\omega_0 \gg \frac{1}{R_{out} C_2} \Rightarrow$ WE CAN FIND C_2

10^7

$f_0 \cong 1 MHz$

$\propto 10^3 \Omega \Rightarrow C_2 \propto 10^{-9} F \Rightarrow C_1 \cong 100 pF \Rightarrow L \cong 100 \mu H$

EXERCIZE # 1

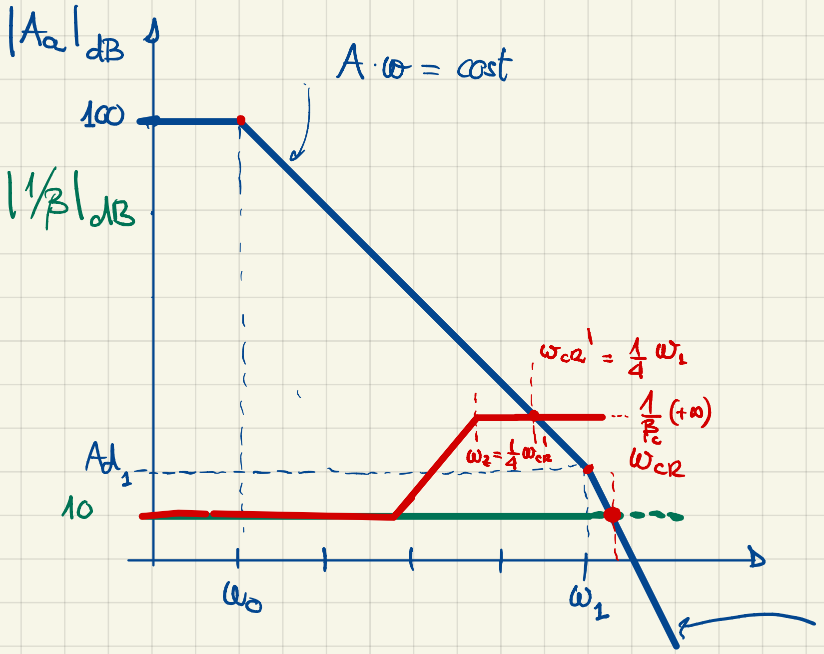


$$A_d(s) = \frac{10^5}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{10^5}\right)}$$

$R_1 = 5\,k\Omega$

$R_2 = 10\,k\Omega$

1. PHASE MARGIN BEFORE COMPENSATION
2. $R_c - C_c$ TO HAVE 60° MINIMUM PHASE MARGIN



$A_d(s) = A_d(s)$

$\omega_0 = 10$ rad/s

$\omega_1 = 10^5$ rad/s

$$\beta = \frac{R_1}{R_1+R_2} = \frac{1}{3}$$

1. PM BEFORE COMPENSATION

$$A_{d0} \cdot \omega_0 = A_{d_1} \cdot \omega_1 \Rightarrow A_{d_1} = \frac{A_{d0}\,\omega_0}{\omega_1}$$

$$\frac{1}{\beta} \cdot \omega_{cR}^2 = A_{d_1} \cdot \omega_1^2 \Rightarrow \omega_{cR} = \sqrt{\beta\, A_{d0}\,\omega_0\,\omega_1} = 1.82 \cdot 10^5 \text{ rad/s}$$

$$PM = 180^\circ - \operatorname{atan}\left(\frac{\omega_{cR}}{\omega_0}\right) - \operatorname{atan}\left(\frac{\omega_{cR}}{\omega_1}\right) \cong 28^\circ$$

2.

$$\beta_c(s) = \frac{R_1 /\!/ Z_c}{R_1 /\!/ Z_c + R_2} \xrightarrow{s \to +\infty} \frac{R_1 /\!/ R_c}{R_2 + R_1 /\!/ R_c}$$

$$\frac{1}{\beta_C}(+\infty) = 1 + \frac{R_2}{R_1 // R_C} = 40 \longrightarrow R_C$$

$$\frac{1}{\beta_C}(+\infty) \cdot \omega_{CR}' = A_{d\emptyset} \cdot \omega_0 \qquad \omega_{CR}' = \frac{\omega_1}{4} = 2.5 \cdot 10^4$$

$$\frac{1}{\beta_C}(+\infty) = \frac{A_{d\emptyset}\,\omega_0}{\omega_{CR}'} = \frac{10^6}{2.5 \cdot 10^4} = 40$$

TO FIND R_C

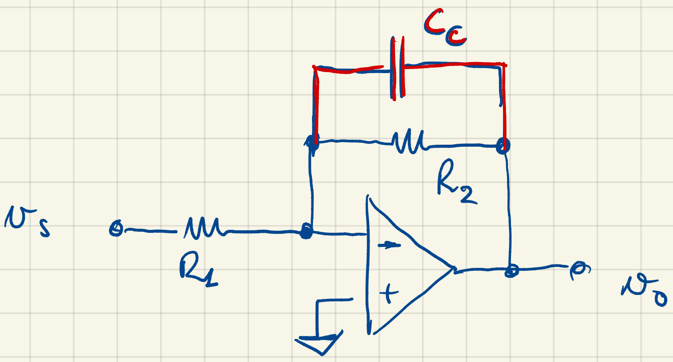
$$\frac{R_2}{R_1 // R_C} = 39 \longrightarrow R_1 // R_C = R_2 / 39 \longrightarrow$$

$$\rightarrow \frac{1}{R_1} + \frac{1}{R_C} = \frac{39}{R_2} \longrightarrow \frac{1}{R_C} = \frac{39}{R_2} - \frac{1}{R_1} \rightarrow R_C = 270\,\Omega,$$

THEN WE KNOW $\omega_z = \frac{1}{R_C C_C} = \frac{\omega_{CR}'}{4} \Rightarrow C_C = \frac{4}{R_C \cdot \omega_{CR}'} \cong 592\ pF$

$PM' \cong PM + 60^\circ = 88^\circ \Rightarrow$ LIKE A FIRST ORDER CLOSED LOOP SYSTEM

EXERCIZE # 2



$$\rightarrow A_d(s) = \frac{10^5}{\left(1+\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_1}\right)}$$

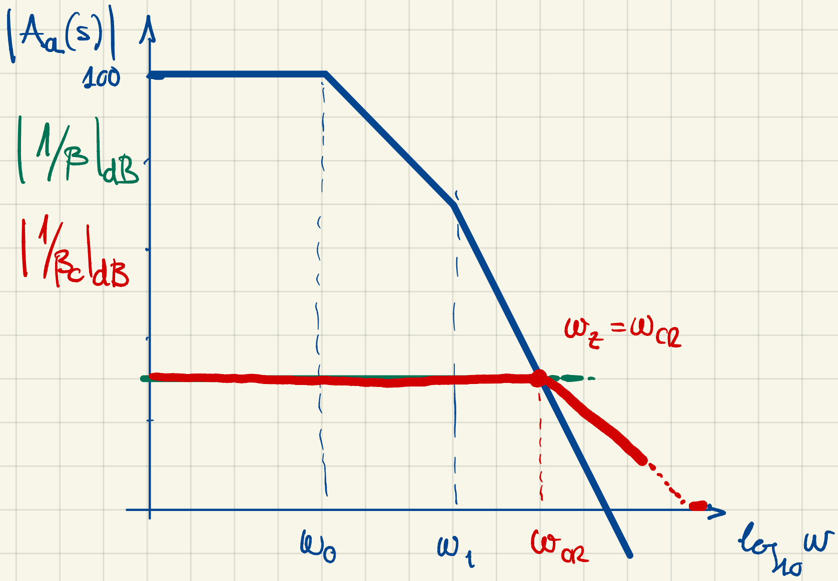
$\omega_0 = 10^2$ rad/s

$\omega_1 = 5 \cdot 10^3$ rad/s

1. PHASE MARGIN WITHOUT C_C
2. C_C TO HAVE THE SAME ω_{CR}
3. NEW PHASE MARGIN

$R_1 = 1\,k\Omega$

$R_2 = 33\,k\Omega$



$A_a(s) = A_d(s)$

$$1/\beta = 1 + \frac{R_2}{R_1} = 34$$

1. AS IN THE PREVIOUS CASE

$$\omega_{CR} = \sqrt{\beta A_{d0}\,\omega_0\,\omega_1} = 54 \cdot 10^3 \text{ rad/s}$$

$$PM = 180^\circ - \operatorname{atan}\left(\frac{\omega_{CR}}{\omega_0}\right) - \operatorname{atan}\left(\frac{\omega_{CR}}{\omega_1}\right) \simeq 7.6^\circ \ll 45^\circ$$

2.

$$\beta_C(s) = \frac{R_1}{R_1 + \frac{R_2}{1+sC_CR_2}} = \frac{R_1(1+sC_CR_2)}{R_1+R_2+sC_CR_1R_2} = \frac{R_1}{R_1+R_2}\cdot\frac{1+\overbrace{sC_CR_2}^{1/\omega_z}}{1+sC_C R_1/\!/R_2}$$

$$\omega_z = \frac{1}{R_2 C_c} = \omega_{CR} \Rightarrow C_c = \frac{1}{R_2 \cdot \omega_{CR}} \approx 0.8\ mF.$$

3. $$PM' \overset{!}{=} PM + 45^\circ = 52.6^\circ$$