1) CONVERGENCE OF GENERALIZED INTEGRALS

2) CONVERGENCE OF SERIES

5) $\displaystyle\int_0^{1/2} \frac{(x-\sin x)\cos^2 x}{x^{|4-\alpha|}\log^2 x \log(1+x)}\,dx \qquad \alpha \in \mathbb{R}$

$\mathbb{D}$ $\begin{cases} x^{|4-\alpha|} \neq 0 \\ x \geq 0 \\ x > 0 \\ \log x \neq 0 \\ 1+x > 0 \\ \log(1+x) \neq 0 \end{cases} \to \begin{cases} x > 0 \\ x \neq -1 \\ x > -1 \\ x \neq 0 \end{cases}$ $\qquad \mathbb{D}: \mathbb{R}^+ =]0, +\infty[$

converge of $\int_0^a f(x)\,dx$

$f(x) \sim$?? for $x \to 0$

N: $\left[x - \left(x - \frac{x^3}{3!} + o(x^3)\right)\right]\left[1 - \frac{x^2}{2!} + o(x^2)\right]$

$\left(\frac{x^3}{3!} + o(x^3)\right)(- -)$

$\frac{x^3}{3!} - \cancel{\frac{x^5}{3!2!}} + o(x^3)$

D: $x^{|4-\alpha|}\log^2 x\,(x + o(x))$

$f(x) \sim \dfrac{x^3/3!}{x^{|4-\alpha|}\log^2 x\, x} = \dfrac{1}{6x^{-2+|4-\alpha|}\log^2 x}$

study $-2+|4-\alpha|$:

$\boxed{\alpha = 4}$: $f(x) = \dfrac{1}{6x^{-2}\log^2 x} = \dfrac{x^2}{6\log^2 x} \longrightarrow 0$ I converges for $\alpha = 4$

$\boxed{\alpha > 4}$ $f(x) = \dfrac{1}{6x^{-2-4+\alpha}\log^2 x}$ = ~~$\alpha > 5$~~ $\alpha \leq 7 \Rightarrow$ I converges $4 < \alpha \leq 7$

$x^{-6+\alpha}$

also = because $\log^2 x$

CORRECTION : General rule for integrals

$$\int_2^{+\infty} \frac{1}{x^\alpha \log^\beta x}\,dx \text{ converges iff } \alpha>1 \text{ or } \alpha=1 \text{ and } \beta>1$$

$$\int_0^{1/2} \frac{1}{x^\alpha \log^\beta x}\,dx \text{ converges iff } \alpha<1 \text{ or } \alpha=1 \text{ and } \beta>1$$

$\boxed{\alpha<4}$ $\quad f(x) = \dfrac{1}{6\, \underbrace{x^{-2+4-\alpha}}_{x^{2-\alpha}} \log^2 x} =$

$\alpha>1$ I converges

$\alpha=1$ I converges

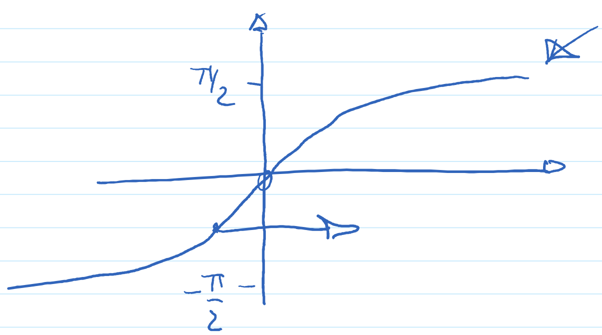
$\alpha<1$ I diverges

$\Rightarrow$ converges $1\leq\alpha<4$

I converges $\boxed{-1\leq\alpha\leq 7}$

8) $\displaystyle\int_0^{+\infty} \frac{\operatorname{arctg}(x^{-3})}{x^\alpha \log(1+\operatorname{arctg} x)}\,dx$

$$D: \begin{cases} x^\alpha \neq 0 \\ x>0 \\ 1+\operatorname{arctg} x>0 \\ 1+\operatorname{arctg} x \neq 1 \end{cases} \qquad \begin{cases} x>0 \\ \operatorname{arctg} x > -1 \\ \operatorname{arctg} x \neq 0 \end{cases} \qquad D = x>0$$



convergence must be study both at 0 and $+\infty$

$I1 : \int_0^a f(x)\,dx$

$$f(x) \sim \frac{\pi/2}{x^\alpha \log(1+x+o(x))} = \frac{\pi/2}{x^\alpha(x+o(x))} = \frac{\pi}{2\,\underbrace{x^{\alpha+1}}}$$

I1 converges $\alpha<0$

$I2 : \int_a^{+\infty} f(x)\,dx$

$I2: \int_{a}^{+\infty} f(x)\,dx$

$$f(x) \sim \frac{x^{-3}+o(x^{-3})}{x^{\alpha}\log(1+\pi/2)} = \frac{1}{x^{3+\alpha}\log(1+\pi/2)}$$

I2 converges $3+\alpha>1$ $\quad \alpha>-2$

I converges when both I1 and I2 converges $\Rightarrow$ $\boxed{-2<\alpha<0}$

3) $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n}$

necessary condition: $\lim_{n\to+\infty} \frac{\sqrt{n+2}-\sqrt{n-2}}{n} = \lim_{n\to+\infty} \frac{(\sqrt{n+2}-\sqrt{n-2})(\sqrt{n+2}+\sqrt{n-2})}{n(\sqrt{n+2}+\sqrt{n-2})}$

$$= \lim_{n\to+\infty} \frac{\cancel{n}+2-(\cancel{n}-2)}{n(\sqrt{n+2}+\sqrt{n-2})} = \frac{4}{+\infty} = 0$$

convergence: $\quad a_n \sim \frac{4}{n\cdot 2\sqrt{n}} = \frac{2}{n^{3/2}}$

$3/2 > 1 \Rightarrow S$ converges

14) $\sum_{n=2}^{\infty} \frac{n^{\alpha}}{\log^2 n}\left(\frac{1}{n}-\sin\frac{1}{n}\right)$

n.c. $\lim_{n\to+\infty} a_n = \lim_{n\to+\infty} \frac{n^{\alpha}}{\log^2 n}\left(\cancel{\frac{1}{n}}-\left(\cancel{\frac{1}{n}}-\frac{1}{n^3 3!}+o\left(\frac{1}{n^3}\right)\right)\right)$

$$= \lim_{n\to+\infty} \frac{1}{n^{3-\alpha}\,3!\,\log^2 n} \to 0 \qquad 3-\alpha>0 \quad \alpha<3$$

convergence: $\quad 3-\alpha>1$ S converges $\alpha<2$

$3-\alpha<1$ S deverges $\alpha>2$

$\boxed{\alpha=2}$

$a_n \to f(x) = \frac{1}{6x\log^2 x}$

$f(x)\geq 0 \Leftrightarrow \boxed{x>1}$

$f(x)$: decreasing

$f(x)$ is decreasing

$$f'(x) = \frac{1}{6}\left[\frac{-(1\cdot \cancel{\log^2 x} + \cancel{x}\cdot 2\cancel{\log x}\cdot \cancel{\tfrac{1}{x}})}{x^2 \log^{\cancel{4}\,3} x}\right] = \frac{1}{6}\,\frac{-2-\log x}{x^2\log^3 x}$$

$$\begin{cases} x^2 > 0 & \forall x\in D \\ \log^3 x > 0 & \forall x \in D \\ -2-\log x > 0 & \nexists x\in D \\ \quad \hookrightarrow -(2+\log x) > 0 & \end{cases}$$

$f'(x) < 0$

$\Downarrow$

$f(x)$ is decreasing

$$\int_2^{+\infty} \frac{1}{6x\log^2 x}\,dx = \frac{1}{6}\,\frac{\log^{-1} x}{-1}\Bigg|_2^{+\infty} = -\frac{1}{6\log x}\Bigg|_2^{+\infty}$$

$$= -\cancel{\frac{1}{6\log +\infty}}^{\to 0} + \frac{1}{6\log 2} = \frac{1}{6\log 2} \quad \square$$

∇

7) $\sum_{n=1}^{\infty} (-1)^n n^{\alpha}(1-e^{\frac{1}{n}})$ $\qquad |\cos n| \qquad \cos n\pi = (-1)^n$

n.c. $\lim_{n\to+\infty} n^{\alpha}(1-e \qquad \lim_{n\to+\infty} n^{\alpha}(\cancel{1}-\cancel{1}-\frac{1}{n}+o(\frac{1}{n})))$

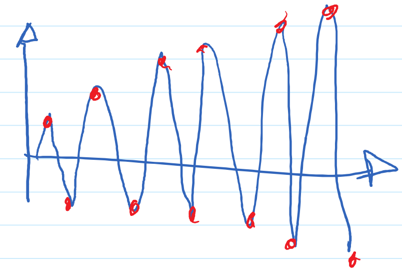
$1+x+o(x)$

$\sim -\frac{1}{n^{1-\alpha}} \to 0 \qquad \alpha < 1$

convergence $\quad \alpha < 0 \quad$ S. absolutly converges

$\frac{1}{n^{1-\alpha}}$ is decreasing $\qquad \alpha < 1 \quad$ converges

$\boxed{\alpha \geq 1}$ S. is indeterminate



15) $\sum^{\infty} (n!)^{3\alpha} \ldots (e^n \ldots)$ $\qquad \alpha^n \ldots \sqrt[n]{\ldots}$

15) $\sum_{n=2}^{\infty} (n!)^{3\alpha} \sinh\left(\frac{e^n}{(2n+1)!}\right)$

$a_n \geq 0$

$\hookrightarrow \frac{e^n}{n!} \to 0$

$\underline{\alpha}^n \to$ root $\sqrt[n]{\ }$

$n!, \binom{n}{2} \to$ ratio $\frac{a_{n+1}}{a_n}$

$\log x \ll x^\alpha \ll \alpha^x \ll x! \ll x^x$

$$a_n \sim (n!)^{3\alpha} \frac{e^n}{(2n+1)!}$$

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to+\infty} \frac{((n+1)!)^{3\alpha} e^{n+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{(n!)^{3\alpha} e^n}$$

$$= \lim_{n\to+\infty} \frac{(n+1)^{3\alpha} \cancel{(n!)^{3\alpha}} e \cdot \cancel{e^n}}{(2n+3)(2n+2)\cancel{(2n+1)!}} \frac{\cancel{(2n+1)!}}{\cancel{(n!)^{3\alpha}} \cancel{e^n}}$$

$$= \lim_{n\to+\infty} \frac{(n+1)^{3\alpha} e}{(2n+3)(2n+2)} \sim \frac{e}{4} \frac{n^{3\alpha}}{n^2} = \frac{e}{4} n^{3\alpha - 2}$$

RATIO if $\lim = A < 1$

$3\alpha - 2 < 0$ $\lim \to 0$ S converges

$3\alpha - 2 > 0$ $\lim \to +\infty$ S diverges

$3\alpha - 2 = 0$ $\lim \to \frac{e}{4} < 1$ S converges

S converges $\alpha \leq \frac{2}{3}$

5) $\int_{4/3}^{+\infty} \frac{\left|\sin \frac{1}{x}\right|^{\alpha-1}}{(3+4x)^\alpha \sqrt{1+x^2}} dx$ calculate for $\alpha = 1$

$D: \begin{cases} 3+4x > 0 \\ 1+x^2 > 0 \\ x > 0 \end{cases}$ $\begin{matrix} x > -3/4 \\ x \in \mathbb{R} \\ x > 0 \end{matrix}$ $D:]0, +\infty[$

convergence $x \to +\infty$

$$f(x) \sim \frac{\left(\frac{1}{x}\right)^{\alpha-1}}{4^\alpha x^\alpha \cdot x} = \frac{1}{4^\alpha x^{\cancel{\alpha}+1} x^{-\cancel{\alpha}+1}} = \frac{1}{x^2}$$ converges $\forall \alpha$

calculate $\alpha = 1$ settsinh b ~~α~~ 1 1 calculate $\alpha = 1$

$$\lim_{b\to+\infty}\int_{4/3}^{b}\frac{1}{(3+4x)\sqrt{1+x^2}}\,dx \;\underset{|}{=}\; \int_{\log 3}^{\operatorname{settsinh} b}\frac{\cancel{\cosh t}\,dt}{(3+4\sinh t)\cancel{\sqrt{1+\sinh^2 t}}}$$

$x = \sinh t$

$dx = \cosh t$

$x \to \frac{4}{3}$ $\quad t \to \operatorname{settsinh}\frac{4}{3} = \log 3$

$x \to b$ $\quad$ ——— .

$$= \int_{\log 3}^{\operatorname{settsinh} b}\frac{1}{3+2(e^t - e^{-t})}\,dt = \int_{\log 3}^{\operatorname{settsinh} b}\frac{e^t\,dt}{2e^{2t}+3e^t-2}$$

$y = e^t$

$dy = e^t dt$

$t \to \log 3 \quad y \to 3$

$t \to \ldots \quad y \to \ldots$

$$= \int_3^{e^{\operatorname{settsinh} b}}\frac{dy}{\underbrace{2y^2+3y-2}_{\to (y+2)(2y-1)}} \longrightarrow \Delta = 9+16 > 0$$

$$y_{1\text{-}2} \underset{|}{=} \frac{-3\pm\sqrt{25}}{4} = -2,\ 1/2$$

$$\frac{A}{y+2} + \frac{B}{2y-1} \Rightarrow \begin{cases} 2A+B = 0 \\ 2B - A = 1 \end{cases}$$

$B = -2A$

$-5A = 1 \qquad A = -1/5$

$B = 2/5$

$$= \int -\frac{1}{5(y+2)}\,dy + \int\frac{2}{5(2y-1)}\,dy$$

$$= -\frac{1}{5}\log|y+2| + \frac{1}{5}\log|2y-1| \Big|_3^{e^{\operatorname{settsinh} b}}$$

$$= \frac{1}{5}\log\left|\frac{2y-1}{y+2}\right| \Big|_3^{\ldots}$$

$$\Rightarrow \lim_{b\to+\infty}\frac{1}{5}\log\left|\frac{2e^{\operatorname{settsinh} b}-1}{e^{\operatorname{settsinh} b}+2}\right| - \cancel{\frac{1}{5}\log\left|\frac{5}{5}\right|}$$

$\lim_{b\to+\infty}$ 5 ... $e^{\text{settsin}\dots}+2$ ...

$\sim 1+\text{settsinh}\, b \sim 1+b$

$$\lim_{b\to+\infty}\frac{1}{5}\log\left|2\,\frac{+\infty}{+\infty}\right| = \boxed{\frac{1}{5}\log 2}$$

EXERCISES ABOUT GENERALIZED INTEGRALS

1) $\int_0^1 \frac{\cos x}{\log x}\,dx$

2) $\int_0^{+\infty} \frac{x+e^{-x}}{2+2x+x^2}\,dx$

3) $\int_0^1 \frac{\sin x}{x^{3/2}(1-x)^{1/2}}\,dx$

4) $\int_0^{+\infty} e^{-1/x}\left(\frac{1+x}{2+x^4}\right)dx$

5) $\int_{-1}^1 \frac{\sin x}{x\sqrt[3]{1-x^2}}\,dx$

6) $\int_{-1}^1 \frac{\cos x}{x\sqrt[3]{1-x^2}}\,dx$

7) $\int_0^{+\infty} \frac{\sin x^2}{x\sqrt{1+x^2}}\,dx$

8) $\int_0^{2\pi} \frac{\left(\frac{\pi}{2}-x\right)^e}{\cos x}\,dx$

9) $\int_{-\infty}^{\infty} \frac{e^x}{x+e^{2x}}\,dx$

10) $\int_0^{2\pi} \frac{1+\sin x}{\sqrt{1-\cos x}}\,dx$

PARAMETRIC GENERALIZED INTEGRALS

$$\boxed{\int_0^1 \frac{1}{x^\alpha \log^\beta x}\,dx \qquad \int_1^{+\infty} \frac{1}{x^\alpha \log^\beta x}\,dx}$$

1) $\int_1^{+\infty} \frac{1}{x^{2\alpha}(\log x)^\alpha}\,dx$

2) $\int_0^{+\infty} \frac{e^{-1/x}}{x^\alpha(1+x^\alpha)}\,dx$

3) $\int_0^{+\infty} \frac{x^\alpha \log x}{1+x^2}\,dx$

4) $\int_5^{+\infty} \frac{(\sin e^{-x})^\alpha \log x}{x^{2/3}(e^{3x}-1)}\,dx$

5) $\int_0^{+\infty} \frac{e^{\frac{x}{4}}-1}{(\sinh x)^\alpha x^{3/2}}\,dx$

6) $\int_0^{+\infty} x^\alpha e^{-x}\,dx$

VERIFY CONVERGENCE AND CALCULATE

1) $\int_2^{+\infty} (x-1)e^{-x^2+2x}\,dx$

2) $\int_0^{+\infty} \frac{\sinh x\,\sin^2\sqrt{x}}{(e^{2x}-1)(\log(1+e^{x^3}))}\,dx$

3) $\int^{+\infty} 3\ \frac{\operatorname{arctg} x}{\ldots}\,dx$

$(e^{\cdots}-1)(\log(1+e^{\cdots}))$

3) $\int_1^{+\infty} 3\,\frac{\operatorname{arctg} x}{x^2}\,dx$

4) $\int_0^{+\infty} \frac{\log x}{(1+x)^\alpha}\,dx$ calculate for $\alpha=2$

5) $\int_{4/3}^{+\infty} \frac{\left|\sin\frac{1}{x}\right|^{\alpha-1}}{(3+4x)^\alpha\sqrt{1+x^2}}\,dx$ calculate for $\alpha=1$

6) $\int_0^{+\infty} \frac{\operatorname{arctg}\frac{1}{\sqrt{x}}}{x^\alpha}\,dx$ calculate for $\alpha=\frac{3}{4}$

7) $\int_{1/3}^{+\infty} \frac{1}{x^\alpha\sqrt{9x^2-1}}\,dx$ calculate for $\alpha=1$

8) $\int_0^{\pi^2/3} x^\alpha \sin\sqrt{3x}\,dx$ calculate for $\alpha=1/2$

9) $\int_0^{\sqrt{2}} x^{2\alpha}\arcsin\frac{x^2}{2}\,dx$ calculate for $\alpha=1/2$

10) $\int_{-2}^{0} x^\alpha \sin x^2\, e^{2x^2}\,dx$ calculate for $\alpha=1$

EXERCISES ABOUT CONVERGENCE OF SERIES

1) $\sum_{n=1}^{\infty} \frac{\log n}{n^4}$

2) $\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n^2}\right)$

3) $\sum_{n=1}^{\infty} \frac{\sqrt{n+6}-\sqrt{n-3}}{n}$

4) $\sum_{n=1}^{\infty} \frac{1}{\log\sqrt{n^3}}$

5) $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\,\log n^3}$

6) $\sum_{n=1}^{\infty} 3^{2n}\cos^n(n\pi)$

7) $\sum_{n=1}^{\infty} \frac{3^{n^2}}{(n!)^n}$

8) $\sum_{n=1}^{\infty} \frac{2}{\binom{3n+2}{3n}}$

9) $\sum_{n=1}^{\infty} \frac{1}{\binom{4n}{3n}}$

10) $\sum_{n=1}^{\infty} \left(\frac{1}{n+2}\right)^n$

11) $\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{\log n}}$

12) $\sum_{n=1}^{\infty} \frac{\sin(4n^3)}{n(n+1)}$

13) $\sum_{n=1}^{\infty} \frac{1}{5^n}\left(\frac{n+2}{n}\right)^{n^2}$

14) $\sum_{n=1}^{\infty} 3^n\left(\frac{n-2}{n}\right)^{n^2}$

15) $\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

16) $\sum_{n=1}^{\infty} n\sqrt{1+\frac{4}{n^3}}$

17) $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\sin\frac{1}{n}\right)$

18) $\sum_{n=1}^{\infty} \left[9n^3\left(\frac{1}{n}-\sin\frac{1}{n}\right)\right]^n$

19) $\sum_{n=1}^{\infty} \left(1-\frac{1}{2n}\right)^{5n^2}$

20) $\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}\left(\cosh\frac{1}{n^3}-1\right)}{\cdots}$

10) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+2}\right)^n$

20) $\displaystyle\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}\left(\cosh\frac{1}{n^3}-1\right)}{\sin\frac{1}{n^{4/3}}-\frac{1}{n^{4/3}}}$

VERIFY CONVERGENCE AND CALCULATE (EXTRA)

1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

2) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}$

3) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)!}$

5) $\displaystyle\sum_{n=2}^{\infty} \log\left(1-\frac{1}{n^2}\right)$

6) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$

7) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

ALTERNATING SIGN SERIES

1) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{\log(n+1)}$

2) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n^{3/2}}$

3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n^2+1}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos((n+1)\pi)}{\sqrt{n}+\log n^3}$

5) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\log(n+1)-\log(n)}$

6) $\displaystyle\sum_{n=1}^{\infty} \left(2\,\mathrm{arctg}(n+1)-\pi\right)\cos((n+1)\pi)$

7) $\displaystyle\sum_{n=1}^{\infty} \sin\left(\frac{n^2+n+1}{n+1}\pi\right)$

8) $\displaystyle\sum_{n=1}^{\infty} \cos(n\pi)\frac{\log n}{n+1}$

9) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n+(-1)^n}$

10) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\left(\frac{1}{n}+\frac{(-1)^n}{n^2}\right)$

11) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n+2}$

12) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[5]{n+1}}{n^6+n}(-1)^n$

PARAMETRIC SERIES

1) $\displaystyle\sum_{n=1}^{\infty} (\cos n+2)\left(\frac{\sqrt{1+\alpha}}{|1-\alpha|}\right)^n$

2) $\displaystyle\sum_{n=1}^{\infty} \frac{n2^n+5^n}{\alpha^n+3^n}$

3) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^\alpha}\,\mathrm{arctg}\frac{3}{\sqrt{n}}$

4) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n\alpha)}{n(n+1)}$

11) $\displaystyle\sum_{n=1}^{\infty} \frac{n^\alpha(n^{3/2}-1)}{(n^3+1)\log(n+1)}$

12) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^\alpha}{(2n)!}\alpha^{3n}$ for $\alpha > 0$

13) $\displaystyle\sum_{n=5}^{\infty} \left(\frac{\log(n+1)}{n^\alpha(\log n)^{\alpha+1}}-\frac{1}{n^\alpha(\log n)^\alpha}\right)$

14) $\displaystyle\sum_{n=2}^{\infty} \frac{n^\alpha}{\log^2 n}\left(\frac{1}{n}-\sin\frac{1}{n}\right)$

4) $\sum_{n=1} \frac{\quad}{n(n+1)}$

5) $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^{\alpha}}$

6) $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}-\operatorname{arctg} n}{(n+1)^{\alpha}}$

7) $\sum_{n=1}^{\infty} \alpha^n \log \alpha^n$

8) $\sum_{n=0}^{\infty} (-1)^n (\operatorname{tg}\alpha)^{2n}$

9) $\sum_{n=0}^{\infty} \frac{3^n}{\alpha^n (\alpha-6)^n}$

10) $\sum_{n=1}^{\infty} \left(\frac{1}{1-\log|\alpha|}\right)^n$

14) $\sum_{n=2} \frac{\quad}{\log^2 n} \left(\frac{\quad}{n} \quad \frac{\quad}{n} \right)$

15) $\sum_{n=2}^{\infty} (n!)^{3\alpha} \sinh\left(\frac{e^n}{(2n+1)!}\right)$

16) $\sum_{n=1}^{\infty} \frac{(n+4)!}{(n+7)^{\alpha} n!}$

17) $\sum_{n=2}^{\infty} \left(n^{3\beta}+\log n^7\right)\left(\frac{1}{n}-\sin\frac{1}{n}\right)$

18) $\sum_{n=1}^{\infty} \frac{\left(\operatorname{arctg}\frac{1}{n}\right)^{\alpha}}{n\sqrt{n}\left(1-\cos\left(n^{-\alpha/2}\right)\right)}$

19) $\sum_{n=1}^{\infty} \left(1-\cos\left(\sqrt{3+n^{2(\alpha-1)}}\right)-n^{\alpha-1}\right)$

20) $\sum_{n=1}^{\infty} \left(e^{(\alpha-7)n}\frac{\sqrt{n}}{n^2+1} + \frac{\log(n^n)}{n^{\alpha+2}(\log n)^3}\right)$

21) $\sum_{n=1}^{\infty} \left(1+e^{n^2\alpha}\right)^{-1}$

22) $\sum_{n=1}^{\infty} \frac{1}{n+1}\left(\frac{4\alpha}{1+\alpha^2}\right)^n$

23) $\sum_{n=1}^{+\infty} \cos(\pi n) \left(\frac{3-\alpha}{\alpha^2+1}\right)^n \sin\frac{1}{n}$

24) $\sum_{n=1}^{\infty} \frac{\alpha^{n+3}}{7^n \operatorname{arctg} n \log(n^3+1)}$

THERE ARE ADDITIONAL EXERCISES IN THE MOODLE PAGE :
- TYPED NOTES
- PREVIOUS SOLVED EXAM