

Consider the data 'data.npy'. Assume  $X$  is distributed according to a normal,

$$f_X(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{(x-\mu)^2}{\sigma^2}}$$

where

$$f_{prior}(\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \frac{(\mu_i - m)^2}{s^2}}$$

with

$$m = 4 \quad s = 2$$

and

$$f_{prior}(\sigma) = \frac{\beta^\alpha}{\Gamma(\alpha)} (1/x)^{\alpha+1} e^{-\beta/x}$$

with

$$\alpha = 2, \beta = 1$$

Estimate  $\mu, \sigma$  as posterior averages (with errors given by posterior standard deviations) from the data, using Metropolis algorithm to sample. In other words, sample

$$f(\mu, \sigma|x) \propto f_X(x|\mu, \sigma) f_{prior}(\mu) f_{prior}(\sigma)$$

using Metropolis algorithm. The proposal step  $T(\mu', \sigma'|\mu, \sigma)$  is normal with step size  $\epsilon$ ,

$$\mu \rightarrow \mu' \sim \mathcal{N}(\mu, \epsilon^2)$$

$$\sigma \rightarrow \sigma \sim \mathcal{N}(\mu, \epsilon^2)$$

Try different step sizes,  $\epsilon = [10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 0.5]$  and observe qualitatively the behavior of the chain and the convergence speed.