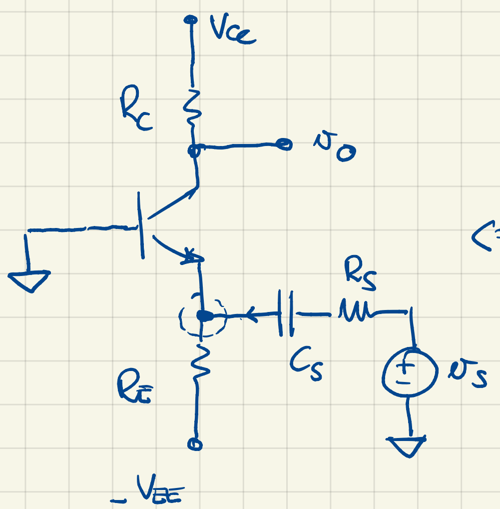
$$\begin{cases} -v_m + (R_L + R_{22}) i_m + R_o (i_m - A_i i_i) = 0 \\ i_i = -\beta i_m \cdot \alpha_i \end{cases}$$

$$-v_m + (R_L + R_{22}) i_m + R_o i_m + \beta A_i R_o \alpha_i i_m = 0$$

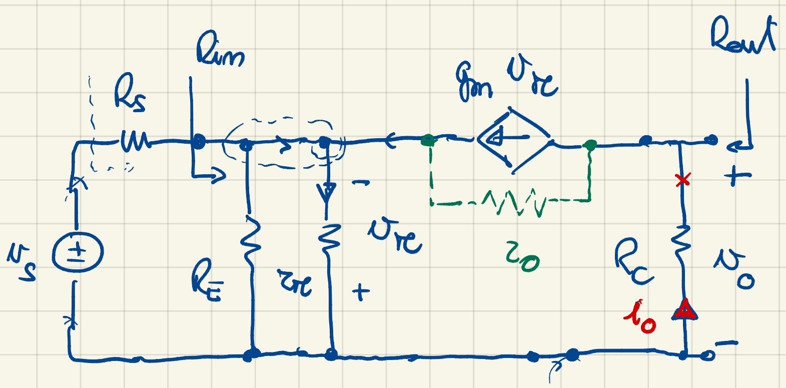
$$\frac{v_m}{i_m} = R_L + R_{22} + R_o + \alpha_i A_i \beta R_o = \underbrace{(R_L + R_{22} + R_o)}_{R^a_{out}} \big(1 + \underbrace{A_i \alpha_i \alpha_o \beta}_{\beta A_i^{a}}\big) \Rightarrow \ldots$$

WE HAVE THUS PROVEN ROW 2 AND 3 OF THE SUMMARY TABLE. PROVE ROW 1 AND 4 AS AN EXERCISE.

EXERCISE #1 : CB AMPLIFIER



$\Longleftrightarrow$



GOAL: APPLY FEEDBACK THEORY TO DERIVE:

$$A_v \triangleq \frac{v_o}{v_s}, \quad R_{in}, \quad R_{out}$$

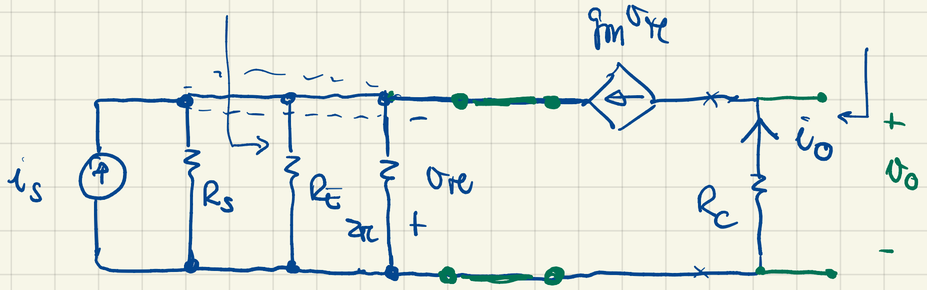
$$A_v = g_m R_C \cdot \frac{R_E \| r_\pi \| \frac{1}{g_m}}{R_S + R_E \| r_\pi \| \frac{1}{g_m}}$$

1. IDENTIFY THE AMPLIFIER'S TOPOLOGY
2. IDENTIFY β-NETWORK
3. BUILD THE 2-PORT MODEL OF β-NETWORK
4. REPLACE THE β-NETWORK WITH THE 2-PORT MODEL
5. SOLVE CIRCUIT IN OPEN LOOP CONDITIONS $\beta = 0$ $\delta \cong 0$
6. DERIVE CLOSED LOOP QUANTITIES APPLYING THEORY
7. "ADJUST" RESULTS TO FIND TARGET QUANTITIES

1. CIRCUIT INSPECTION SHOWS THE CIRCUIT IS PROBING CURRENT.

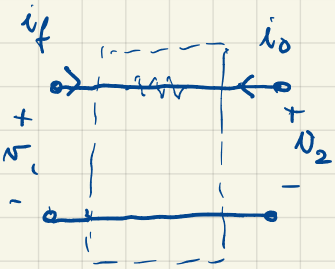
INPUT MIXING IS AGAIN CURRENT TYPE (SHORTING v_{π} KILLS FEEDBACK)

CONCLUSION: CB STAGE CAN BE CONSIDERED A CASE OF CURRENT AMPLIFIER.

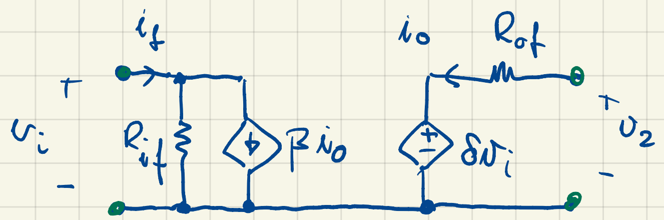


$i_s = \frac{v_s}{R_s}$ NORTON'S THEOREM

2. β-NETWORK IN THE CIRCUIT:



3. 2-PORT MODEL



$R_{11} = R_{if}$ $\qquad$ $R_{22} = R_{of}$

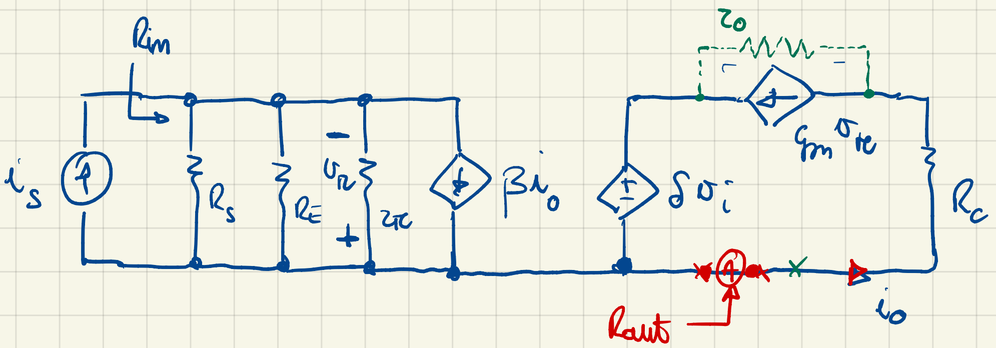
$$R_{if} \triangleq \left.\frac{v_i}{i_i}\right|_{i_o \equiv \emptyset} = +\infty$$

$$\beta \triangleq \left.\frac{i_f}{i_o}\right|_{v_i \equiv \emptyset} = -1$$

$$R_{of} \triangleq \left.\frac{v_2}{i_o}\right|_{v_i \equiv 0} = \emptyset$$

$$\delta \triangleq \left.\frac{v_2}{v_i}\right|_{i_o \equiv \emptyset} = +1$$

4.



OPTIONAL: DISCUSSION OF UNILATERALITY

OPEN CIRCUIT VOLTAGE

$$|g_m r_o \cancel{v_\pi}| >> |\delta \cancel{v_\pi}|$$

$$10^{-2} \cdot 10^{5} >> 1$$

AGAIN WE CAN MAKE A VERY SMALL ERROR NEGLECTING δ

5. STUDY OPEN LOOP AMPLIFIER $\beta \equiv \emptyset$ $\delta \cong \emptyset$

$$A_a \triangleq \frac{i_o}{i_s} = -g_m \cdot R_S // R_E // r_\pi = -g_m R_{in}$$

$$i_o = g_m v_\pi = -g_m (R_S // R_E // r_\pi) \cdot i_s$$

$$R_{in}^{OL} = R_S // R_E // r_\pi$$

$$R_{out}^{OL} = +\infty \qquad (\text{IN PRACTICE IT WILL BE } r_o + R_C)$$

6. APPLY FEEDBACK THEORY TO DERIVE CLOSED LOOP QUANTITIES

$$A_F = \frac{A_a}{1+\beta A_a} = \frac{-g_m R_{in}^{OL}}{1-1\cdot(-g_m R_{in}^{OL})} = -\frac{g_m R_{in}^{a}}{1+g_m R_{in}^{a}} = -g_m \cdot \left(\frac{1}{g_m} // R_{in}^{OL}\right)$$

$$R_{in}^{F} = \frac{R_{in}^{OL}}{1+\beta A_a} = \frac{R_{in}^{OL}}{1+g_m R_{in}^{a}}$$

$$R_{out}^{F} = R_{out}^{OL}\left(1+g_m R_{in}^{a}\right)$$

7. ADAPTION OF RESULTS TO FIND TARGET QUANTITIES

$$A_v = \frac{v_o}{v_s} = \underbrace{\frac{v_o}{i_o}}_{R_o} \cdot \underbrace{\frac{i_o}{i_s}}_{A_F} \cdot \underbrace{\frac{i_s}{v_s}}_{R_s} = -R_C A_F \cdot \frac{1}{R_s} = -g_m R_C \cdot \frac{\frac{1}{g_m} // R_S // R_E // r_\pi}{R_S} =$$

$$= -g_m R_C \frac{\frac{1}{g_m} // r_\pi // R_E}{R_S + \frac{1}{g_m} // r_\pi // R_E}$$

THIS IS THE SAME RESULT WE CAN GET BY DIRECTLY SOLVING THE CIRCUIT

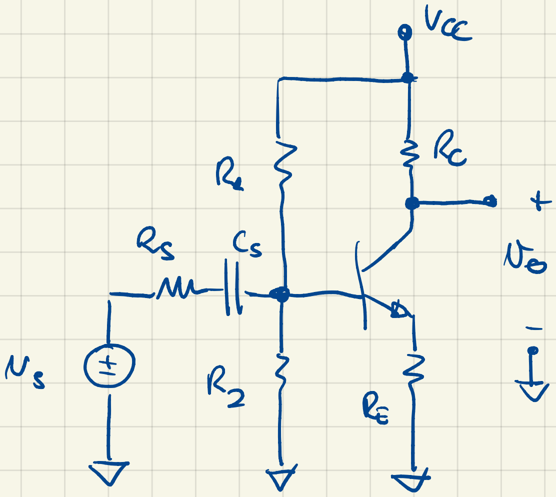
$$R_{in} = \frac{1}{\frac{1}{R_{in}^F} - \frac{1}{R_S}} = R_E // r_\pi // \frac{1}{g_m}$$

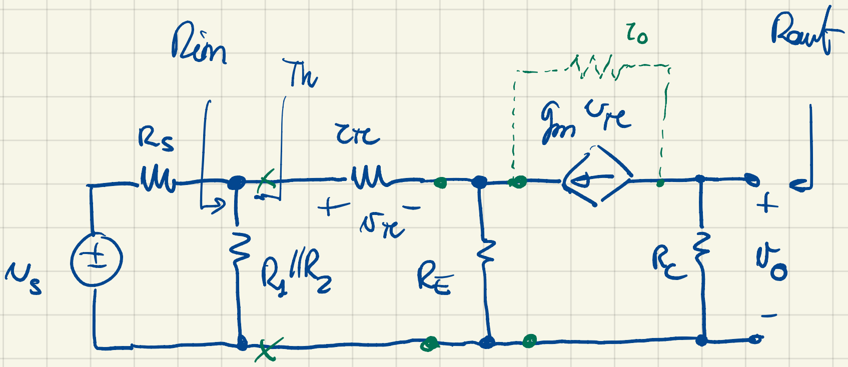
AGAIN, THIS IS WHAT WE FIND DIRECTLY ANALYSING THE CIRCUIT

$$R_{out} = \left(R_{out}^F - R_C\right) // R_C = R_C$$

SAME CONCLUSION HERE!

EXERCISE # 2 : CER AMPLIFIER





$$A_v = \frac{v_o}{v_s} = ? \qquad R_{in} = ? \qquad R_{out} = ?$$

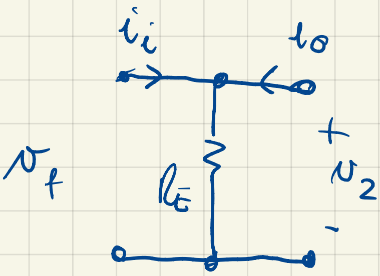
APPLYING FEEDBACK THEORY WE SEE THAT

INPUT MIXING IS VOLTAGE TYPE $\leftrightarrow$ SERIES $\leftrightarrow$ THEVENIN

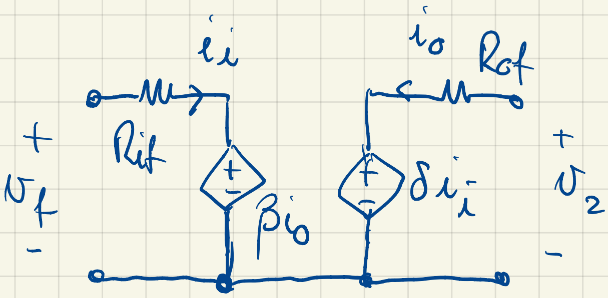
OUTPUT SENSING IS CURRENT TYPE $\leftrightarrow$ SERIES $\leftrightarrow$ THEVENIN

CER IS A CASE OF TRANSCONDUCTANCE AMPLIFIER !

β- NETWORK IS



THE MODEL IS



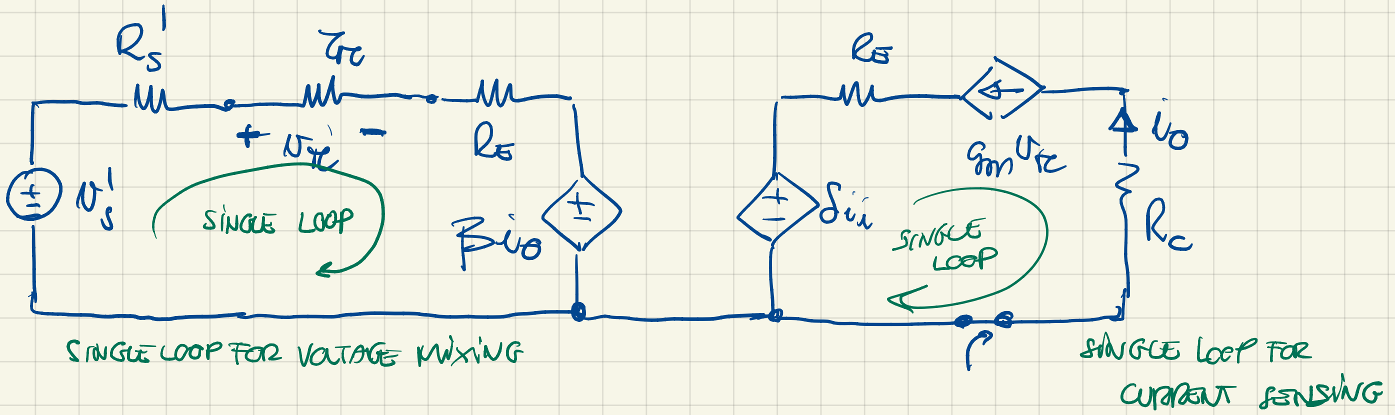
$$R_{if} \triangleq \left.\frac{v_f}{i_i}\right|_{i_o=0} = R_E$$

$$\beta \triangleq \left.\frac{v_f}{i_o}\right|_{i_i=0} = R_E$$

$$R_{of} \triangleq \left.\frac{v_2}{i_o}\right|_{i_i=0} = R_E$$

$$\delta \triangleq \left.\frac{v_2}{i_i}\right|_{i_o=0} = R_E$$

REPLACING THE ORIGINAL β-NETWORK WITH ITS 2-PORT MODEL WE FIND THE EQUIVALENT CIRCUIT OF THE FEEDBACK AMPLIFIER



$$v_s' = v_s \cdot \frac{R_1 // R_2}{R_s + R_1 // R_2}$$

$$R_s' = R_s // R_1 // R_2$$

$$\delta \cancel{i_i} \ll g_m r_o r_\pi \cdot \cancel{i_i}$$

$$\downarrow$$

$$R_E \ll \beta_o r_o$$

WELL VERIFIED IN PRACTICE !!

$$10^3 \ll 10^2 \cdot 10^5$$

UNILATERALITY ASSUMPTION IS WELL POSED !

$$G_a \triangleq \frac{i_o}{v_s'} = \frac{r_\pi}{r_\pi + R_E + R_s'} \cdot g_m = \frac{\beta_o}{r_\pi + R_E + R_s'}$$

$$R_{in}^a = R_s' + r_\pi + R_E$$

$$R_{out}^a = +\infty \qquad (R_c + R_E + r_o \text{ IN PRACTICAL CASES})$$

CLOSED LOOP PARAMETERS

$$G_F = \frac{G_a}{1 + \beta G_a} = \frac{\frac{\beta_o}{R_{in}}}{1 + \frac{R_E}{R_{in}} \cdot \beta_o} = \frac{\beta_o}{R_{in} + \beta_o R_E}.$$

$$R_{in}^F = R_{in}(1 + \beta G_a) = R_{in} \cdot \left(1 + R_E \frac{\beta_o}{R_{in}}\right) = R_{in} + \beta_o R_E =$$

$$= R_s' + r_\pi + (\beta_o + 1)$$

$$R_{out}^F = R_{out}(1 + \beta G_a) = +\infty$$

TARGET PARAMETERS

$$A_v = \frac{v_o}{v_s} = \underbrace{\frac{v_o}{i_o}}_{-R_C} \cdot \underbrace{\frac{i_o}{v_s'}}_{A_F} \cdot \underbrace{\frac{v_s'}{v_s}}_{\frac{R_1 // R_2}{R_s + R_1 // R_2}} = -\frac{\beta_0 R_C}{R_s' + r_\pi + (\beta_0+1)R_E} \cdot \frac{R_1 // R_2}{R_s + R_1 // R_2}$$

WHICH IS THE "USUAL" CER AMPLIFIER VOLTAGE GAIN

$$R_{in} = \left(R_{in}^F - R_s'\right) // R_1 // R_2 = \left[r_\pi + (\beta_0+1)R_E\right] // R_1 // R_2$$ AS USUAL!

$$R_{out} = \left(R_{out}^F - R_C\right) // R_C = R_C$$ AGAIN, AS USUAL.

ONCE AGAIN, THE APPLICATION OF FEEDBACK THEORY PROVIDES THE SAME RESULTS OF DIRECT CIRCUIT ANALYSIS.