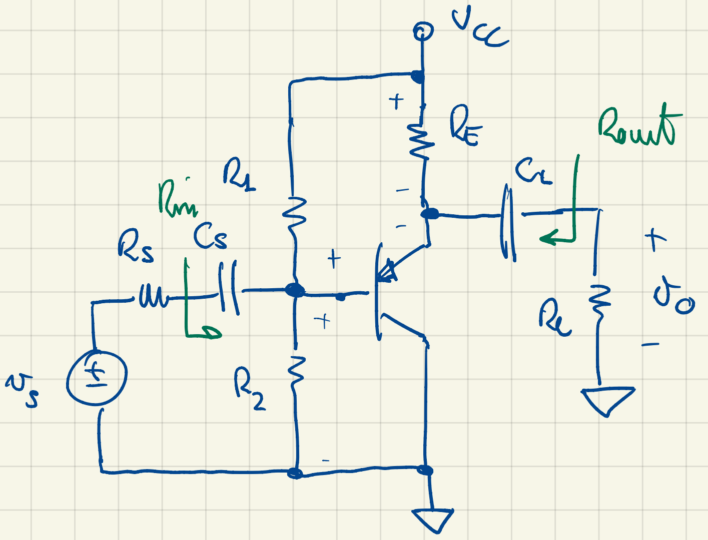
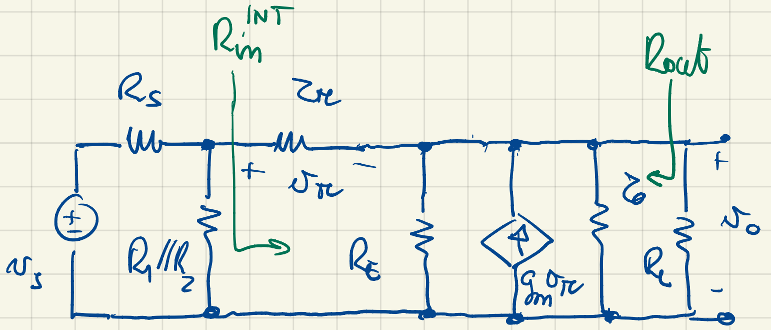
EXAMPLE: CC AMPLIFIER



SMALL SIGNAL EQUIVALENT CIRCUIT



THE SMALL SIGNAL EQUIVALENT CIRCUIT CAN BE "EASILY" SOLVED:

$$A_v \triangleq \frac{v_o}{v_s} = \frac{(\beta_o+1)\, R_L /\!/ R_E /\!/ r_o}{\underbrace{r_\pi + (\beta_o+1)\, R_L /\!/ R_E /\!/ r_o}_{R_{in}^{INT}}} \cdot \frac{R_1 /\!/ R_2 /\!/ R_{in}^{INT}}{R_s + R_1 /\!/ R_2 /\!/ R_{in}^{INT}} = \dots$$

$$\dots = \frac{(\beta_o+1)\, R_L /\!/ R_E /\!/ r_o}{R_s /\!/ R_1 /\!/ R_2 + R_{in}^{INT}} \cdot \frac{R_1 /\!/ R_2}{R_s + R_1 /\!/ R_2}$$

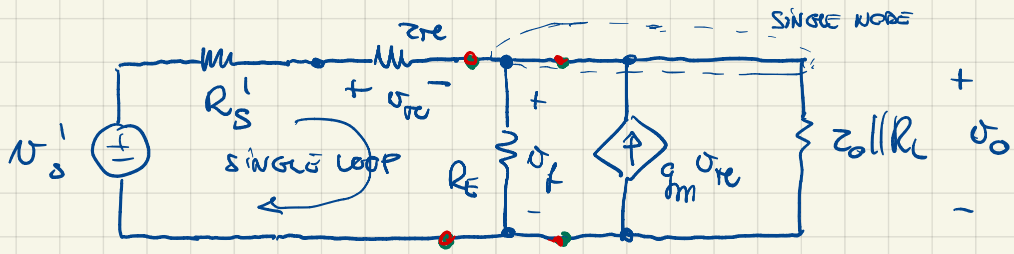
PROVE THIS AS AN EXERCISE !!

$$R_{in} = R_1 /\!/ R_2 /\!/ R_{in}^{INT}$$

$$R_{out} = r_o /\!/ R_E /\!/ \frac{r_\pi + R_1 /\!/ R_2 /\!/ R_s}{\beta_o + 1}$$

WE CAN DERIVE THE SAME RESULT APPLYING FEEDBACK THEORY

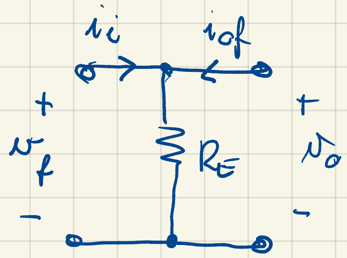
FIRST, WE CAN SIMPLIFY THE INPUT SIDE USING THEVENIN'S THEOREM:



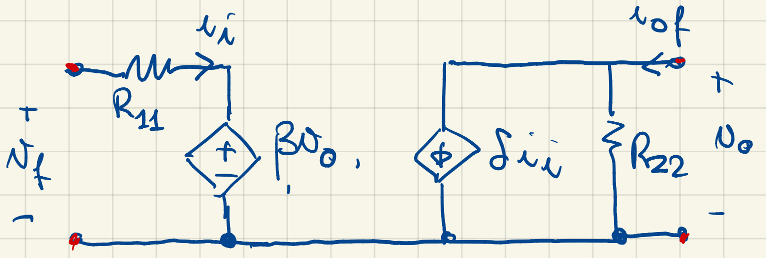
$$v_s' = \frac{R_1 /\!/ R_2}{R_s + R_1 /\!/ R_2}\, v_s$$

$$R_s' = R_1 /\!/ R_2 /\!/ R_s$$

β-NETWORK



THE TWO PORT EQUIVALENT MODEL OF THE β-NETWORK IS: THAT OF A VOLTAGE AMPLIFIER (SERIES-SHUNT):



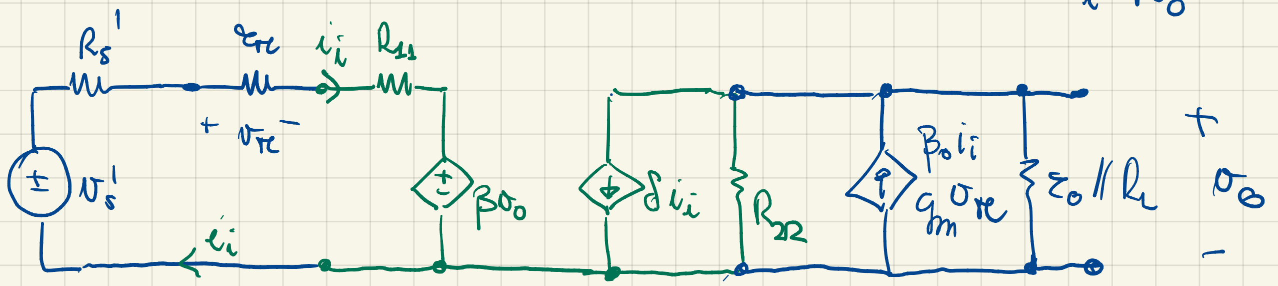
$$R_{11} \triangleq \left.\frac{v_f}{i_i}\right|_{v_o=0} = 0$$

$$\beta = \left.\frac{v_f}{v_o}\right|_{i_i=0} = 1$$

$$R_{22} = \left.\frac{v_o}{i_{of}}\right|_{i_i=0} = R_E$$

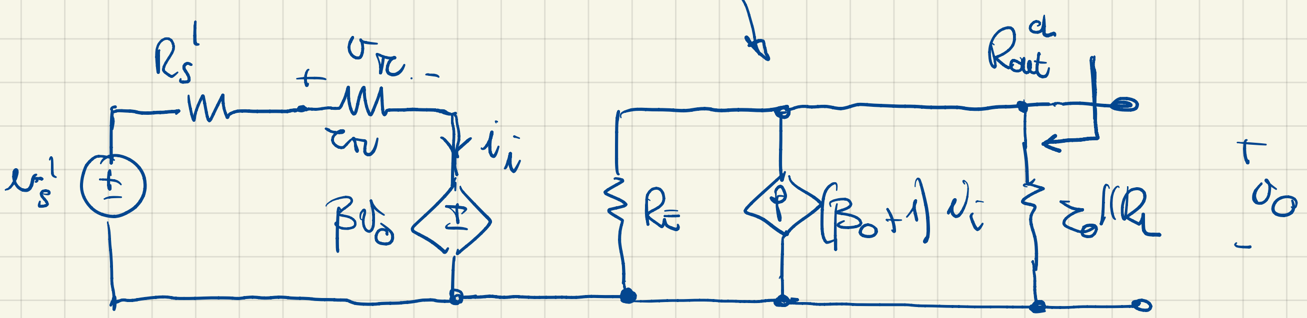
$$\delta = \left.\frac{i_{of}}{i_i}\right|_{v_o=0} = -1$$

LET'S REPLACE R_E WITH THE COHERENT TWO-PORT MODEL OF THE PHYSICAL β-NETWORK.



$$v_\pi = r_\pi \cdot i_i \Rightarrow g_m v_\pi = g_m r_\pi \cdot i_i = \beta_o i_i \Rightarrow$$ WE CAN MERGE THE TWO CONTROLLED CURRENT SOURCES

$$(\beta_o - \delta)\, i_i = (\beta_o + 1)\, i_i$$



FROM THIS CIRCUIT WE CAN FIND OPEN LOOP PARAMETERS BY SETTING $\beta = 0$

$$A_v^{ol} = \frac{v_o}{v_s'} = \frac{(\beta_o + 1)\, R_E \,\|\, r_o \,\|\, R_L}{R_s' + r_\pi}$$

$$R_{in}^{ol} = R_s' + r_\pi \qquad R_{out}^{ol} = r_o \,\|\, R_L \,\|\, R_E$$

THESE RESULTS ARE EXACT AS δ HAS BEEN TAKEN INTO ACCOUNT, NOT NEGLECTED.

WE CAN NOW DETERMINE THE CLOSED LOOP PARAMETERS

$$A_v^F \triangleq \frac{A_v^a}{1+\beta A_v^a} = \frac{(\beta_0+1)\, r_o /\!/ R_E /\!/ R_L}{\cancel{r_{\pi e}+R_s'}} \cdot \frac{\cancel{r_{\pi e}+R_s'}}{R_s' + r_{\pi e} + (\beta_0+1)\, r_o /\!/ R_E /\!/ R_L}$$

$$= \frac{(\beta_0+1)\, r_o /\!/ R_E /\!/ R_L}{R_s' + r_{\pi e} + (\beta_0+1)\, r_o /\!/ R_E /\!/ R_L} \triangleq \frac{v_o}{v_s'}$$

BUT WE ARE INTERESTED IN FINDING v_o/v_s →

$$A_v = \frac{v_o}{v_s} = \underbrace{\frac{v_o}{v_s'}}_{A_v^F} \cdot \frac{v_s'}{v_s} = A_v^F \cdot \underbrace{\frac{R_1 /\!/ R_2}{R_s + R_1 /\!/ R_2}}$$

k_s : INPUT SCALING FACTOR

WHICH IS EXACTLY EQUAL TO THE ONE WE FOUND DIRECTLY ON THE CIRCUIT (VERIFY AS AN EXERCISE). SIMILARLY:

$$R_{in}^F = R_{in}^{OL}\left(1+\beta A_v^a\right) = \left(R_s' + r_{\pi e}\right)\cdot\left(1 + \frac{(\beta_0+1)\, r_o /\!/ R_E /\!/ R_L}{r_{\pi e}+R_s'}\right) =$$

$$= R_s' + r_{\pi e} + (\beta_0+1)\, r_o /\!/ R_E /\!/ R_L$$

WE HAVE A SERIES MIXING SO WE CAN DISASSEMBLE R_{in}^F AND FIND

$$R_{in} = \underbrace{\left(R_{in}^F - R_s'\right)} /\!/ R_1 /\!/ R_2 = R_{in}^{INT} /\!/ R_1 /\!/ R_2$$

CLOSED LOOP INPUT RESISTANCE MEASURED AT THE BASE SECTION

WHICH MATCHES THE ONE FOUND IN THE ORIGINAL CIRCUIT

LET'S CALCULATE THE OUTPUT RESISTANCE

$$R_{out}^F = \frac{R_{out}^a}{1 + \frac{(\beta_0+1)\, r_o /\!/ R_E /\!/ R_L}{r_{\pi e}+R_s'}} = \frac{R_{out}^a\,(r_{\pi e}+R_s')}{R_s' + r_{\pi e} + (\beta_0+1)\, R_{out}^a} = R_{out}^a /\!/ \frac{R_s' + r_{\pi e}}{\beta_0 + 1}$$

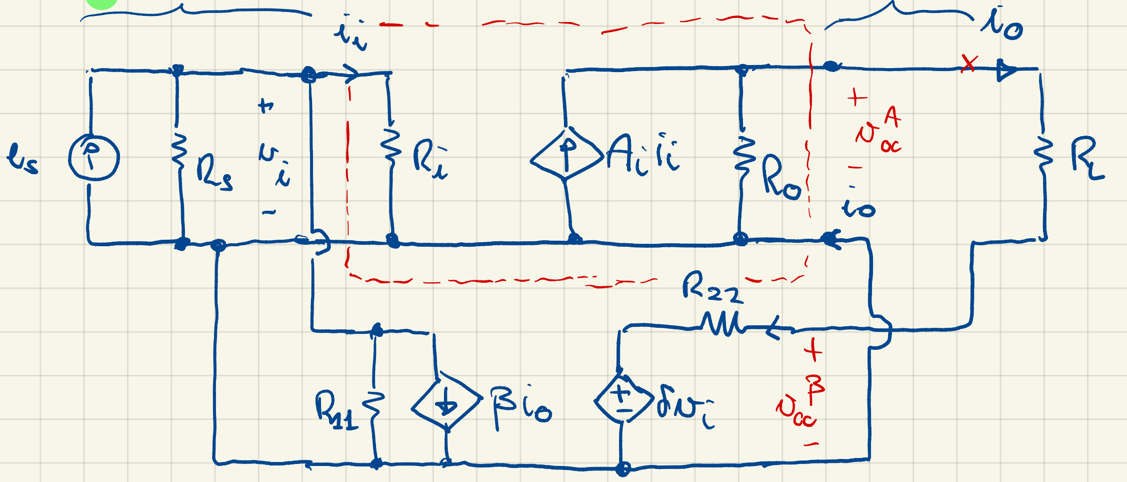
AT THE OUTPUT, INSTEAD, WE HAVE A SHUNT STRUCTURE. THEREFORE

$$R_{out} = \frac{1}{\frac{1}{R_{out}^F} - \frac{1}{R_L}} = r_o /\!/ R_E /\!/ \frac{R_s' + r_{\pi e}}{\beta_0 + 1}$$

WHICH MATCHES THE ONE FOUND ON THE ORIGINAL CIRCUIT (VERIFY AS AN EXERCISE)

◊ LET'S CONSIDER NOW THE CURRENT AMPLIFIER

SINGLE NODE ⟷ SHUNT → NORTON   SINGLE LOOP ⟷ SERIES → THEVENIN



TWO PORT MODEL OF β-NETWORK

LET'S DISCUSS THE UNILATERALITY ISSUE BY COMPARING OPEN CIRCUIT VOLTAGES

$$v_{oc}^A = R_o \cdot A_i \cdot i_i = R_o A_i \cdot i_s \cdot \underbrace{\frac{R_s \| R_{11}}{R_i + R_s \| R_{11}}}_{\alpha_i} = R_o A_i \alpha_i i_s$$

$$v_{oc}^\beta = \delta v_i = \delta \cdot R_s \| R_{11} \| R_i \cdot i_s$$

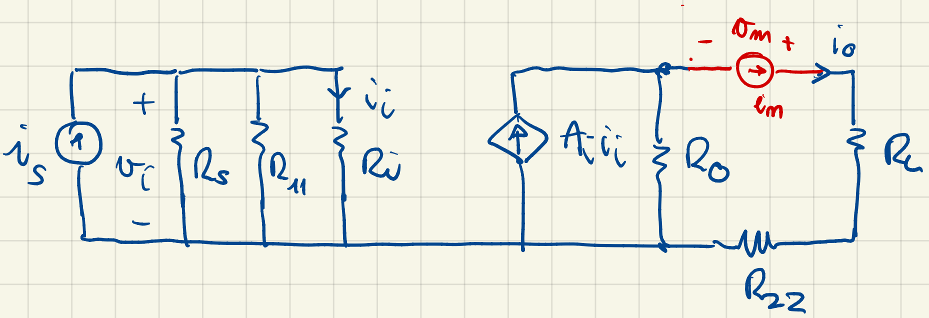
$$\frac{v_{oc}^A}{v_{oc}^\beta} = R_o A_i \frac{\cancel{R_s \| R_{11} \| R_i}}{R_i} \cdot \frac{1}{\delta \, \cancel{R_s \| R_{11} \| R_i}} = \frac{R_o}{R_i} \cdot \frac{A_i}{\delta} \gg 1$$

IDEALLY WE WOULD LIKE TO HAVE $\delta \ll A_i \cdot \frac{R_o}{R_i}$ — LARGE, LOW, $\gg 1$

β-NET IS PASSIVE $\Rightarrow |\delta| \leq 1$

THIS INEQUALITY IS MOST LIKELY WELL VERIFIED IN PRACTICAL CASES.

NOW WE CAN FIND THE AMPLIFIER PARAMETERS IN OPEN LOOP CONDITIONS THAT IS FOR $\beta \equiv \emptyset$ AND $\delta \cong \emptyset$.



$$A_i^{a} = \frac{i_o}{i_s} = \underbrace{\frac{R_s /\!/ R_{11}}{R_s /\!/ R_{11} + R_i}}_{\alpha_i} \cdot A_i \cdot \underbrace{\frac{R_o}{R_o + R_{22} + R_L}}_{\alpha_o} = \alpha_i A_i \alpha_o$$

$$R_{in}^{a} = R_s /\!/ R_{11} /\!/ R_i$$

$$R_{out}^{a} = R_L + R_{22} + R_o$$

$$i_s = \emptyset \Rightarrow i_i = \emptyset \Rightarrow A_i i_i = \emptyset$$

LET'S CONSIDER THE CLOSED LOOP CONFIGURATION $\beta \neq 0$ $\delta = \emptyset$

$$A_i^{F} \triangleq \frac{i_o}{i_s} = \frac{A_i^{a}}{1 + \beta A_i^{a}}$$ ■ AS INDICATED IN SUMMARY TABLE

$$\begin{cases} i_i = (i_s - \beta i_o)\alpha_i \\ i_o = \alpha_o \cdot A_i i_i \end{cases} \Leftrightarrow i_o = \alpha_o A_i (i_s - \beta i_o)\alpha_i \Leftrightarrow$$

$$\Leftrightarrow i_o(1 + \beta A_i^{a}) = A_i^{a} \cdot i_s \Rightarrow \frac{i_o}{i_s} = \ldots$$

$$R_{in}^{F} = \frac{R_{in}^{a}}{1 + \beta A_i^{a}}$$ ■ AS INDICATED IN SUMMARY TABLE

$$v_i = (i_s - \beta i_o) R_{in}^{a} = i_s \left(1 - \frac{\beta A_i^{a}}{1 + \beta A_i^{a}}\right) \cdot R_{in}^{a} \Rightarrow \frac{v_i}{i_s} = \ldots$$

$$R_{out}^{F} = R_{out}^{a} \cdot (1 + \beta A_i^{a})$$ ■ AS IN SUMMARY TABLE

$$\begin{cases} -v_m + (R_1 + R_{22})\, i_m + R_o (i_m - A_i\, i_i) = 0 \\ i_i = -\beta\, i_m \cdot \alpha_i \end{cases}$$

$$-v_m + (R_1 + R_{22})\, i_m + R_o i_m + \beta A_i R_o \alpha_i\, i_m = 0$$

$$\frac{v_m}{i_m} = R_1 + R_{22} + R_o + \alpha_i A_i \beta R_o = \underbrace{(R_1 + R_{22} + R_o)}_{R_{out}^{a}}\,(1 + \underbrace{A_i \alpha_i \alpha_o \beta}_{\beta A_i^{\alpha}}) \Rightarrow \ldots$$

WE HAVE THUS PROVEN ROW 2 AND 3 OF THE SUMMARY TABLE. PROVE ROW 1 AND 4 AS AN EXERCISE.