

TAYLOR & MCLAURIN SERIES

Polynomial approximations for $x \rightarrow 0$

$$\begin{aligned}e^x &= 1 + x + \frac{x^2}{2} + \cdots + \frac{x^n}{n!} + o(x^n) \\ \log(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} + \cdots + (-1)^{n+1} \frac{x^n}{n} + o(x^n) \\ (1+x)^a &= 1 + ax + \frac{a(a-1)}{2}x^2 + \cdots + \binom{a}{n}x^n + o(x^n) \quad \text{dove } \binom{a}{n} = \frac{a(a-1)\dots(a-n+1)}{n!} \\ \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\ \tan x &= x + \frac{x^3}{3} + \frac{2}{15}x^5 + \cdots + o(x^6) \\ \sinh x &= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) \\ \cosh x &= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) \\ \tanh x &= x - \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^6) \\ \arcsin x &= x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots + \frac{(2n)!}{4^n(n!)^2(2n+1)}x^{2n+1} + o(x^{2n+2}) \\ \arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + o(x^{2n+2}) \\ \operatorname{arsinh} x &= x - \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots + \frac{(-1)^n(2n)!}{4^n(n!)^2(2n+1)}x^{2n+1} + o(x^{2n+2}) \\ \operatorname{artanh} x &= x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})\end{aligned}$$