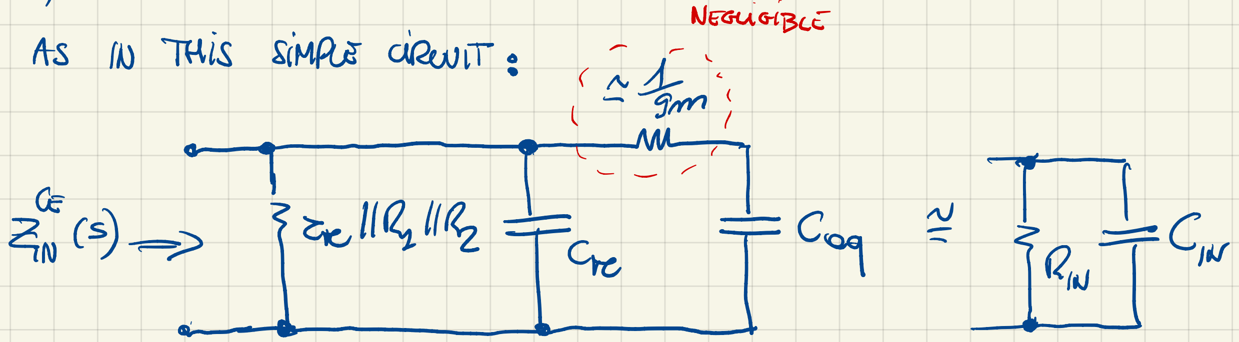
$$Z_x(s) = \frac{1}{\underbrace{sC_\mu(1+g_mR_L')}_{C_{eq}}} + \frac{\cancel{sC_\mu}R_L'}{\cancel{sC_\mu}(1+g_mR_L')} = \frac{1}{sC_{eq}} + \underbrace{R_L' \parallel \frac{1}{g_m}}_{\simeq \frac{1}{g_m}}$$

AS A RESULT, WE FOUND THAT $Z_{IN}(s)$ IS AS IN THIS SIMPLE CIRCUIT:



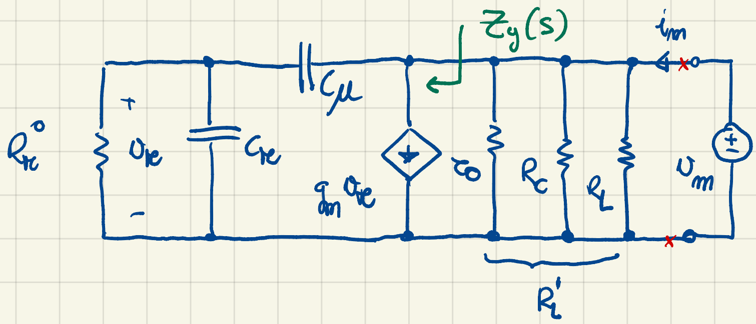
$R_{IN} = r_\pi \parallel R_1 \parallel R_2$

CE INPUT RESISTANCE

$$C_{IN} \simeq C_\pi + C_\mu \underbrace{(1+g_mR_L')}_{\text{MILLER MULTIPLIER}} \longleftarrow \text{MILLER EFFECT}$$

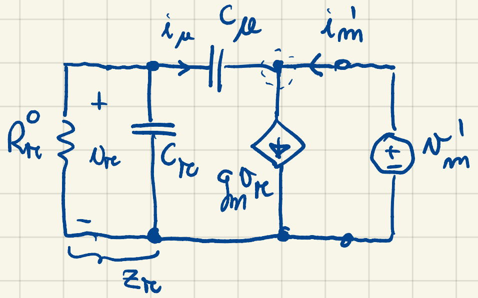
CONCLUSION: THE CE INPUT IMPEDANCE HAS A STRONG CAPACITIVE COMPONENT THAT IS DOMINATED BY C_μ, THROUGH MILLER EFFECT

◊ LET'S NOW DETERMINE $Z_o(s)$



$$Z_o(s) \triangleq \frac{V_m(s)}{I_m(s)}$$

$$Z_o(s) = R_L' \parallel Z_y(s)$$



$$Z_y(s) \triangleq \frac{V_m'(s)}{I_m'(s)}$$

$$\begin{cases} i_m' = g_m v_\pi - i_\mu = g_m v_\pi - sC_\mu(v_\pi - v_m') = (g_m - sC_\mu)v_\pi + sC_\mu v_m' \\ v_\pi = v_m' \cdot \dfrac{Z_\pi}{Z_\pi + \frac{1}{sC_\mu}} \end{cases}$$

$$v_\pi = v_m' \, \frac{\dfrac{R_\pi^0}{1+sC_\pi R_\pi^0}}{\dfrac{R_\pi^0}{1+sC_\pi R_\pi^0}+\dfrac{1}{sC_\mu}} = v_m' \cdot \frac{sC_\mu\left(1+sC_\pi R_\pi^0\right)R_\pi^0}{\left(1+sC_\pi R_\pi^0\right)\left[1+sR_\pi^0(C_\mu+C_\pi)\right]}$$

REPLACING INTO THE 1ST EQUATION

$$i_m' = v_m' \, sC_\mu - \left(sC_\mu - g_m\right)\frac{sC_\mu R_\pi^0\,\cancel{\left(1+sC_\pi R_\pi^0\right)}}{\cancel{\left(1+sC_\pi R_\pi^0\right)}\left[1+sR_\pi^0(C_\pi+C_\mu)\right]}\, v_m'$$

$$Z_y(s) = \frac{v_m'}{i_m'} = \frac{1+sR_\pi^0\left(C_\mu+C_\pi\right)}{sC_\mu\left(\underbrace{1+g_m R_\pi^0}+sC_\pi R_\pi^0\right)} = \frac{1}{sC_\mu\left(1+g_m R_\pi^0\right)} \cdot \frac{1+sR_\pi^0\left(C_\pi+C_\mu\right)}{1+sC_\pi\dfrac{R_\pi^0}{1+g_m R_\pi^0}}$$

$$i_m' = v_m' \cdot \frac{sC_\mu\cancel{\left(1+sC_\pi R_\pi^0\right)}\left[1+sR_\pi^0(C_\pi+C_\mu)\right]+\left(g_m-sC_\mu\right)sC_\mu R_\pi^0\cancel{\left(1+sC_\pi R_\pi^0\right)}}{\cancel{\left(1+sC_\pi R_\pi^0\right)}\left[1+sR_\pi^0\left(C_\pi+C_\mu\right)\right]} =$$

$$= v_m' \cdot \frac{sC_\mu + s^2C_\mu R_\pi^0\left(C_\pi+\cancel{C_\mu}\right)+sC_\mu R_\pi^0 g_m - \cancel{s^2C_\mu^2 R_\pi^0}}{1+sR_\pi^0\left(C_\pi+C_\mu\right)} =$$

$$= v_m' \cdot \frac{sC_\mu\left(1+sC_\pi R_\pi^0+g_m R_\pi^0\right)}{1+sR_\pi^0\left(C_\pi+C_\mu\right)}$$

IN CONCLUSION

CAPACITIVE COMPONENT (SMALLER THAN THE INPUT'S)

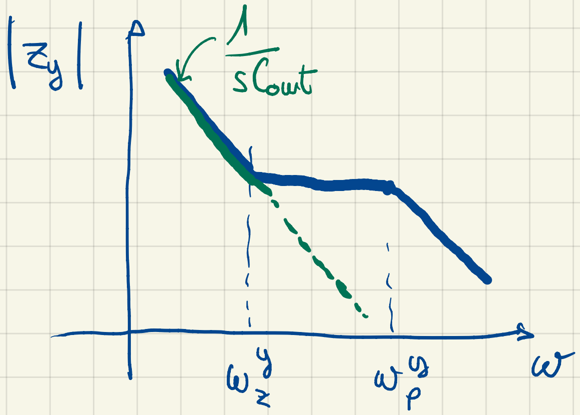
$$Z_o(s) = R_L' \,/\!/\, Z_y(s) \qquad \text{WHERE} \qquad Z_y(s) = \frac{1}{sC_{out}} \cdot \underbrace{\frac{1+s/\omega_z^y}{1+s/\omega_p^y}}_{\text{ZERO-POLE COUPLE}}$$

SAME SIZE OF C_π APPROXIMATELY!

$$C_{out} = C_\mu\left(1+g_m R_\pi^0\right)$$

$$\omega_z^y = \frac{1}{R_\pi^0\left(C_\pi+C_\mu\right)} \ll \omega_p^y$$

$$\omega_p^y = \frac{1}{C_\pi}\cdot\frac{1+g_m R_\pi^0}{R_\pi^0} \underset{g_m R_\pi^0 \gg 1}{\cong} \frac{1}{C_\pi}\cdot g_m \cong \omega_T$$



CONCLUSION: $Z_o(s)$ HAS AGAIN CAPACITIVE STRUCTURE, WITH LOWER EQUIVALENT CAPACITANCE COMPARED TO $Z_{IN}(s)$

GAIN · BANDWIDTH PRODUCT FOR CE AMPLIFIERS

$$GBP = |A_v^{MB}| \cdot \omega_H$$

$$|A_v^{MB}| = \frac{R_\pi^o}{R_s} \cdot g_m R_L'$$

$$\omega_H \simeq \frac{1}{a_1} = \frac{1}{C_\pi R_\pi^o + C_\mu R_\mu^o} \simeq \frac{1}{R_\pi^o (C_\pi + C_\mu g_m R_L')}$$

$$R_\mu^o = R_L' + R_\pi^o (1 + g_m R_L') \simeq R_\pi^o (1 + g_m R_L')$$
$$\simeq g_m R_\pi^o R_L'$$

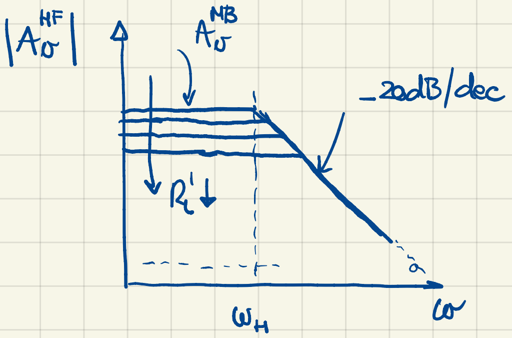
$$GBP \simeq \frac{R_L'}{R_s} \cdot \frac{1}{\frac{C_\pi}{g_m} + R_L' C_\mu}$$

>> **LIGHT LOAD OPERATION:** R_L IS RELATIVELY LARGE

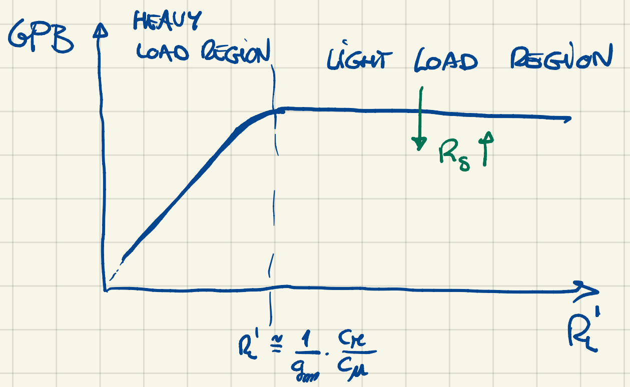
WE ARE ASSUMING THAT TRANSISTOR PARAMETERS $(g_m, C_\mu, C_\pi, r_o, \dots)$ DO NOT DEPEND ON R_L (I.E. THE PHYSICAL LOAD RESISTANCE)

IN LIGHT LOAD CONDITIONS $R_L' C_\mu \gg \frac{C_\pi}{g_m} \iff g_m R_L' \gg \frac{C_\pi}{C_\mu}$ $\quad (\omega \ll \omega_T)$
SO THAT

$$GBP \simeq \frac{1}{R_s C_\mu} \quad \text{CONSTANT WITH } R_L$$



$R_L' \downarrow \Rightarrow A_v^{MB} \downarrow \ \omega_H \uparrow \qquad \underbrace{A_v^{MB} \cdot \omega_H}_{GPB}$ IS CONSTANT



>> IN THE **HEAVY LOAD REGION:**

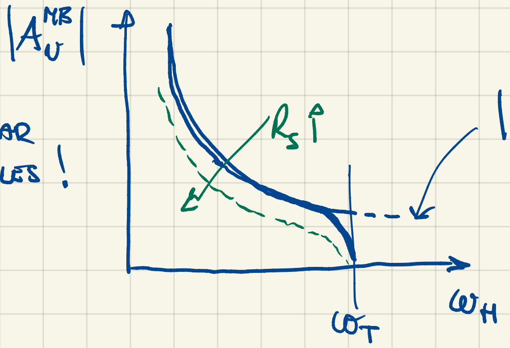
$$\frac{C_\pi}{g_m} \gg R_L' C_\mu$$

AND THEREFORE

$$GBP \simeq \frac{g_m R_L'}{R_s C_\pi} \propto R_L'$$

PUSHING R_L' DOWN, GBP DECREASES LINEARLY AS THE AMPLIFICATION DOES $\Rightarrow$ $\omega_H \simeq \omega_T$

LINEAR SCALES !

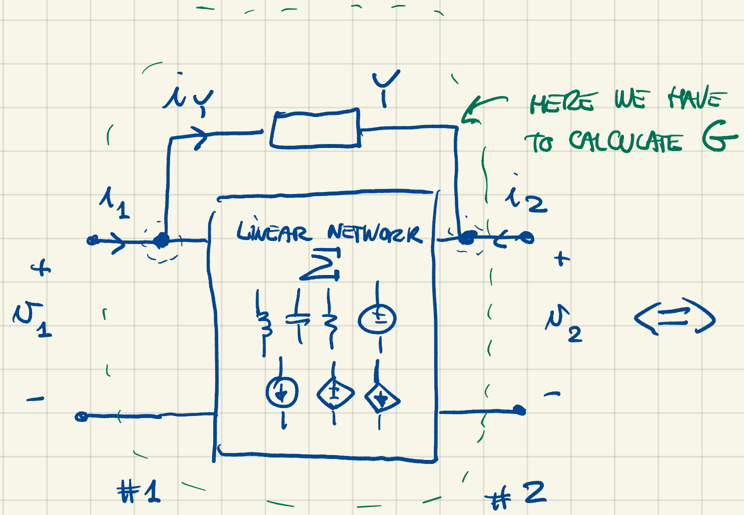


$|A_v^{MB}| \cdot \omega_H = \text{COST}$

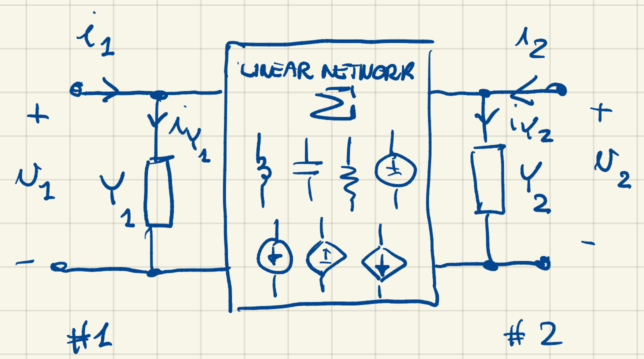
AT HEAVY LOADS ω_H STOPS GROWING AND TENDS TO ω_T AS THE MID-BAND GAIN GOES DOWN.

IMPORTANT CONCLUSION: IT IS NOT POSSIBLE TO HAVE HIGH GAIN AND LARGE BANDWIDTH AT THE SAME TIME FROM A SINGLE STAGE CE AMPLIFIER !!

◊ MILLER'S THEOREM



$\Longleftrightarrow$



MILLER'S THEOREM STATES THE TWO CIRCUITS ARE EQUIVALENT (I.E. THEY HAVE THE SAME CURRENTS AND VOLTAGES) IF AND ONLY IF

$$\begin{cases} i_{Y_1} = i_Y \\ i_{Y_2} = -i_Y \end{cases} \Longleftrightarrow \begin{cases} v_1 Y_1 = Y(v_1 - v_2) \\ v_2 Y_2 = -Y(v_1 - v_2) \end{cases} \Longleftrightarrow$$

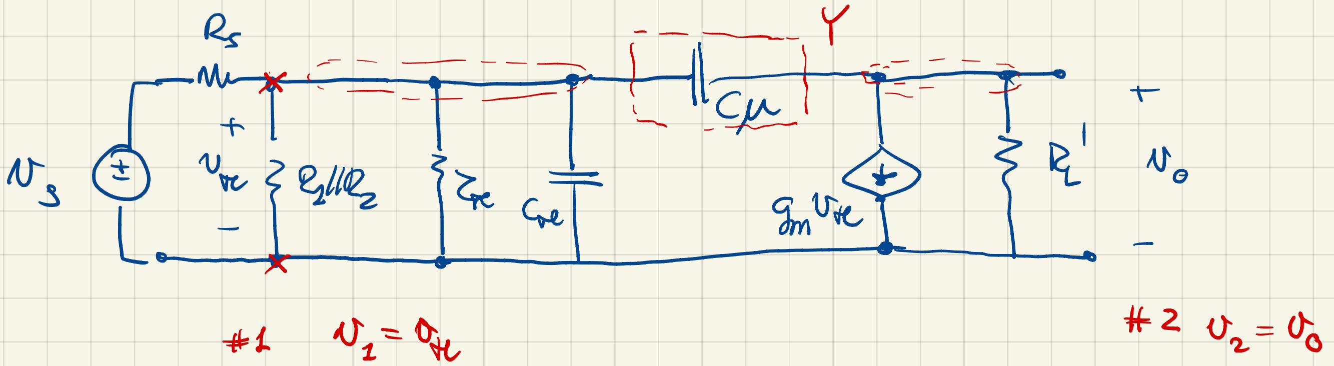
$$\Longleftrightarrow \begin{cases} Y_1 = Y\left(1 - \frac{v_2}{v_1}\right) = Y(1 - G) \\ Y_2 = -Y\left(\frac{v_1}{v_2} - 1\right) = Y\left(1 - \frac{1}{G}\right) \end{cases}$$

$$\boxed{G \triangleq \frac{v_2}{v_1}}$$

MUST BE KNOWN !!

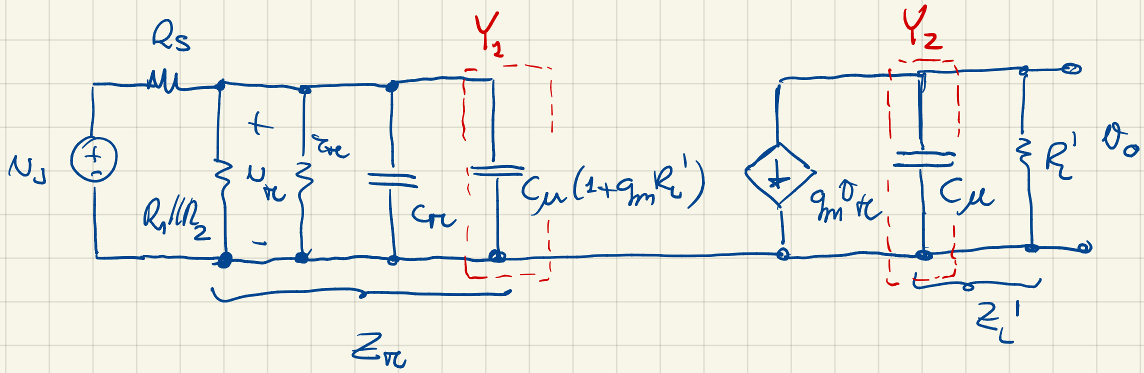
NOW IF $|G| \gg 1$ $\quad Y_1 \simeq -GY \quad Y_2 \simeq Y$

LET'S REVISE THE CE AMPLIFIER USING MILLER'S THEOREM



$$G \cong \left.\frac{v_o}{v_\pi}\right|_{MB} = -g_m R_L' \qquad |G| \gg 1$$

APPLYING THE THEOREM WE CAN SIMPLIFY THE CIRCUIT AS



$$A_v^{HF}(s) = -\frac{Z_\pi}{R_s + Z_\pi} \cdot g_m \cdot Z_L'$$