$$\bar{z}\,\mathrm{Im}(z) - z|z| = 0$$

$$z = x + iy \qquad \bar{z} = x - iy$$

$$(x - iy)y - (x + iy)\sqrt{x^2 + y^2} = 0$$

$$\left(xy - x\sqrt{x^2+y^2}\right) + i\left(-y^2 - y\sqrt{x^2+y^2}\right) = 0$$

$$\Updownarrow \quad \begin{cases} x\left(y - \sqrt{x^2+y^2}\right) = 0 \\ -y\left(y + \sqrt{x^2+y^2}\right) = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ -y^2 - y|y| = 0 \end{cases} \quad \text{or} \quad \begin{cases} y = \sqrt{x^2+y^2} \\ -y^2 - y^2 = 0 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} x = 0 \\ y(y + |y|) = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases} \text{ or } \begin{cases} x = 0 \\ y + |y| = 0 \end{cases}$$

$$y = 0$$

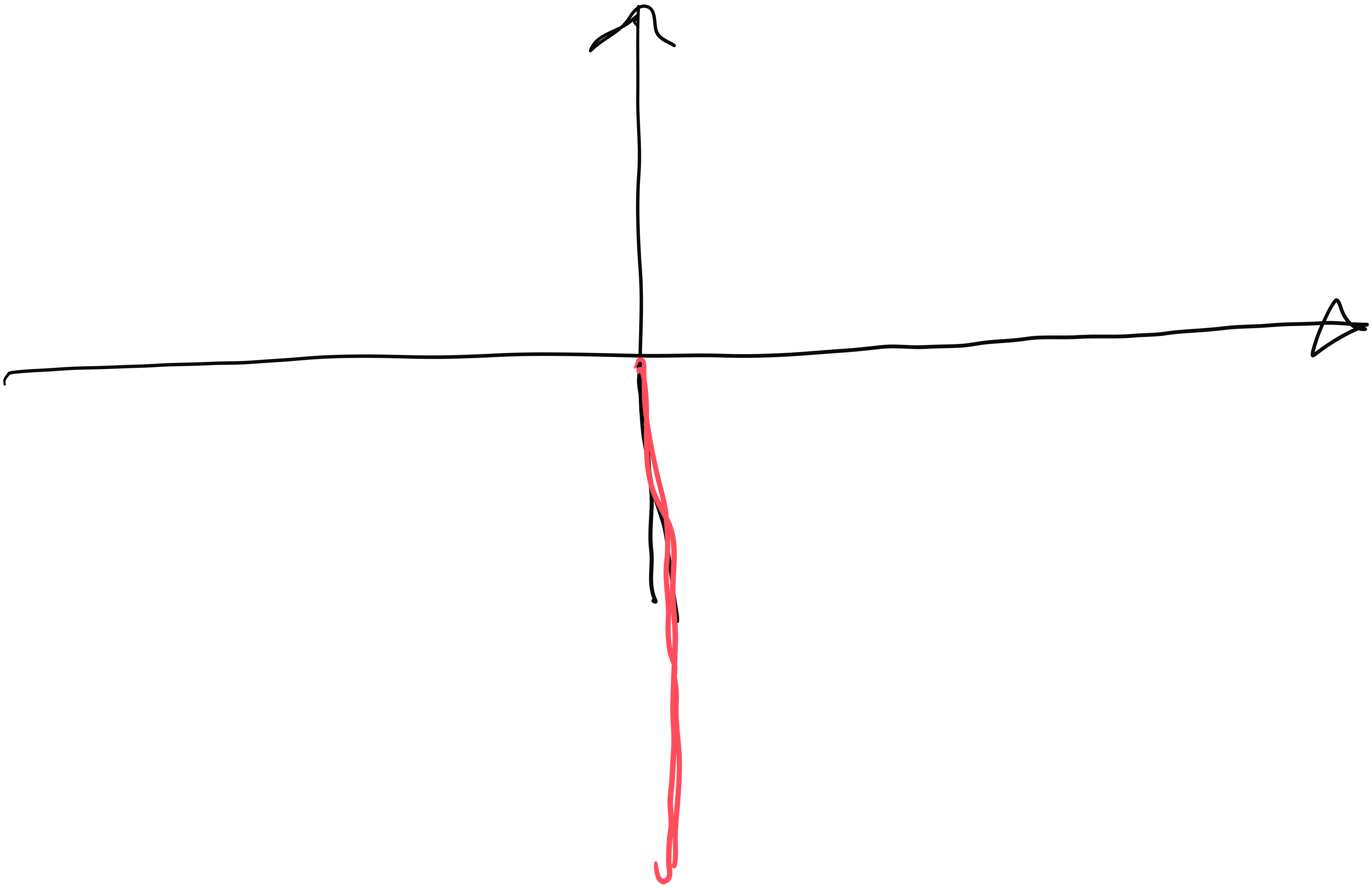
$$0 = \sqrt{x^2 + 0}$$

$$x = 0$$

$z_1 = 0$

$ri \quad \forall r \geq 0$

$\begin{cases} x = 0 \\ y = -|y| \end{cases}$



Plot $S = \left\{ z \in \mathbb{C} \;\middle|\; \left| \frac{z+1}{z} \right| \geq 1 \right\}$

$z \neq 0$

$$\frac{|z+1|}{|z|} \geq 1$$

$\boxed{\begin{array}{l} z = x+iy \\ z = \rho e^{i\theta} \end{array}}$

$|z+1| \geq |z|$

$|z+1|^2 \geq |z|^2$

$z = x + iy$

$$(x+1)^2 + y^2 \geq x^2 + y^2$$

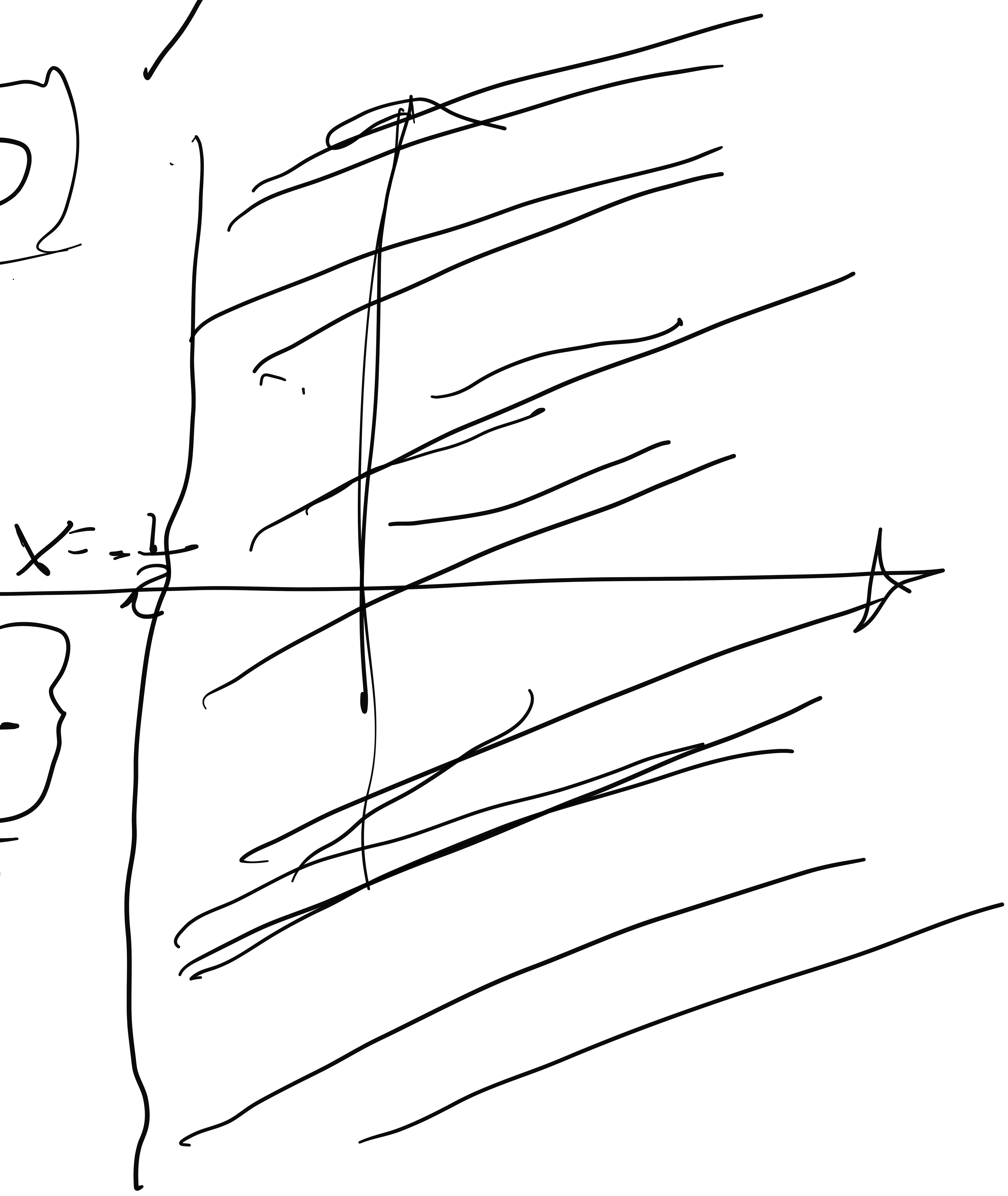
$$2x + 1 \geq 0$$

$$\boxed{x \geq -\frac{1}{2}}$$

$$S = \left\{ x + iy : x \geq -\frac{1}{2} \right\}$$

$$x + iy = \rho e^{i\theta}$$

$$(x, y)$$



Functions of this kind:

$$a : n \longmapsto a(n) = a_n$$

$$\mathbb{N} \longrightarrow \mathbb{R}$$

are called $\boxed{\text{sequences}}$

Examples:

1) $a_n = \boxed{\dfrac{n+1}{n-1}}$ $\quad \forall\, n \in \mathbb{N} \setminus \{1\}$

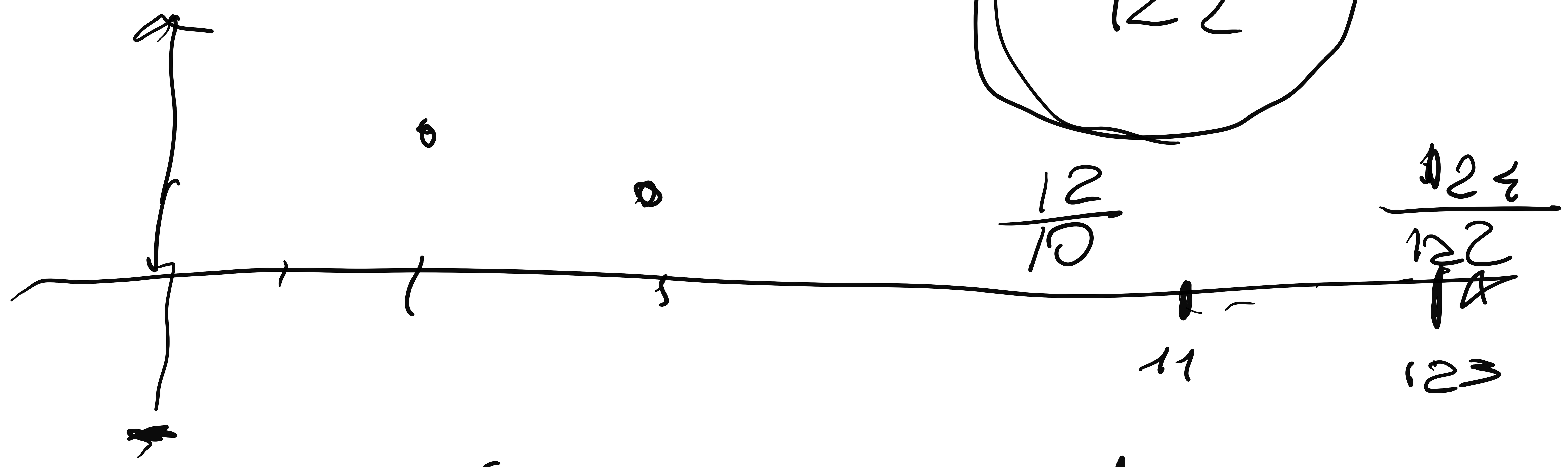
2) $a_n = \dfrac{n}{\sqrt{n}+5}$ $\quad \forall\, n \in \mathbb{N}$

3) $a_n = (-1)^n$ $\quad \forall\, n \in \mathbb{N}$

4) $a_n = \sin(n\pi)$ $\quad \forall\, n \in \mathbb{N}$.

1) $a_0 = -1$ $a_2 = 3$ $a_3 = \frac{4}{2} = 2$ $a_{1000000} = \frac{1000001}{999999}$

$a_{11} = \frac{12}{10}$ $a_{123} = \frac{124}{122}$



Is this sequence decreasing?

definitly "for $n \geq \bar{n}$ suitable $\bar{n}$"

Let us try for $n \geq 2$

$$\boxed{\text{if } n_1 < n_2 \quad a_{n_1} \geq a_{n_2}}$$

def of "decreasing"

It is sufficient to prove that $a_{n+1} \leq a_n$ for $n \geq \bar{n}$

$$a_{n+1} = \frac{n+1+1}{n+1-1} = \frac{n+2}{n} \overset{?}{\leq} \frac{n+1}{n-1}$$

$$\frac{n+2}{\underbrace{n}_{\geq 0}} \leq \frac{n+1}{\underbrace{(n-1)}_{\geq 0}} \qquad n \geq 2$$

$$\frac{(n+2)(n-1)}{n\cancel{(n-1)}} \leq \frac{(n+1)n}{n\cancel{(n-1)}}$$

$$\cancel{n^2} \cancel{+n} - 2 \leq \cancel{n^2} + \cancel{n}$$

o.k $\forall \; n \geq 2$

2) $a_n = \frac{n}{\sqrt{n}+5}$

$a_0 = 0 \quad a_1 = \frac{1}{6} \quad a_2 = \frac{2}{\sqrt{2}+5}$

$a_3 = \frac{3}{\sqrt{3}+5} \quad \ldots\ldots \quad a_{100} = \frac{100}{10+5}$

$6 \sim \frac{100}{15}$

$a_{1000} = \frac{1000}{\sqrt{1000}+5} \sim 30$

$1000 \sim a_{1000000} = \frac{1000.000}{1005}$

$$a_{n+1} \overset{?}{\geq} a_n \qquad \frac{n+1}{\sqrt{n+1}+5} \overset{?}{\geq} \frac{n}{\sqrt{n}+5}$$

$$(n+1)(\sqrt{n}+5) \overset{?}{\geq} n(\sqrt{n+1}+5)$$

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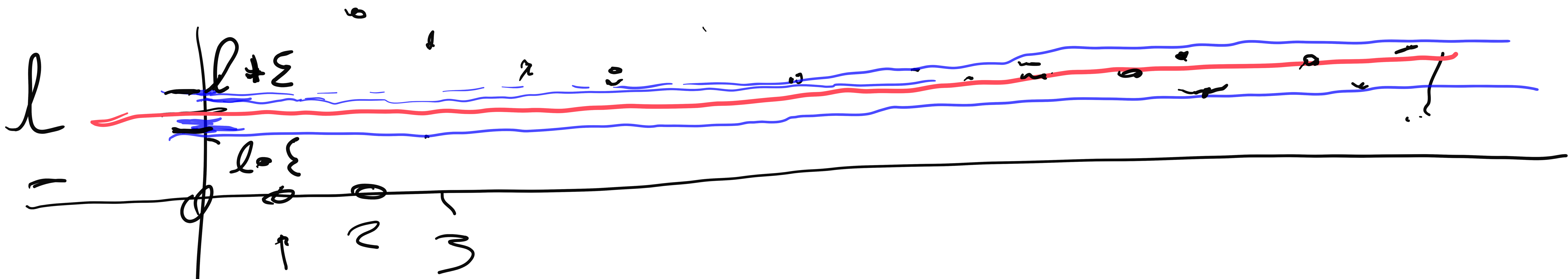
1) a_n "approaches" $\underline{1}$

2) a_n is "greater and greater"



$\boxed{\text{Definition}}$ Let $\underline{l \in \mathbb{R}}$. We say that "The limit of (a_n) is l"

$\Updownarrow$

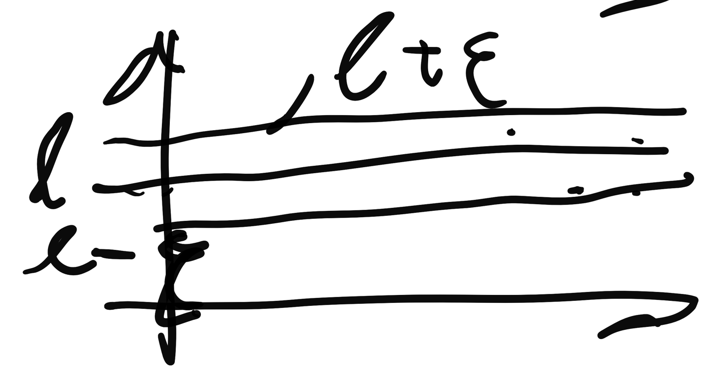
"a_n tends to l"

and we write $\boxed{\lim_{n\to\infty} a_n = l}$

or " " $a_n \to l$

if $\forall \varepsilon>0 \quad \exists\, N(\varepsilon) \in \mathbb{R} \quad \forall n \geq N_\varepsilon$

$$l-\varepsilon \leq a_n \leq l+\varepsilon$$



$$\lim_{n\to\infty} \frac{n+1}{n-1} \overset{?}{=} 1$$

$\forall \varepsilon>0 \quad \overset{?}{\exists}\, N(\varepsilon)$ s.t. $\forall n \geq N(\varepsilon)$

$$1-\varepsilon \leq \frac{n+1}{n-1} \leq 1+\varepsilon$$

$\forall n \in \mathbb{N}$
$n \geq 2$

if $n \geq 2$ denominator and numerator are >0

$$\boxed{\frac{n+1}{n-1} \leq 1+\varepsilon} \quad n \geq 2$$

$$n+1 \leq (1+\varepsilon)(n-1)$$

$$\cancel{n}+1 \leq \cancel{n}-1+\varepsilon n-\varepsilon$$

$$\varepsilon n \geq 2+\varepsilon$$

$$n \geq \frac{2+\varepsilon}{\varepsilon} = N(\varepsilon)$$

The idea of taking the square of both parts was silly. it is much simpler as it is

$$\tilde{N}(\varepsilon) = \underbrace{[N(\varepsilon)]}_{\text{integer part of } N(\varepsilon)} + 1$$

$$r > 0 \qquad \underbrace{[r]}_{\text{integer part of } r} = \max\{n : n \le r\}$$

$$[1.75] = 1 \qquad [36.3] = 36$$

$$[36] = 36$$

We can conclude that

$$\lim_{n \to +\infty} \frac{n+1}{n-1} = 1$$

<u>Definition</u> $\lim_{n \to +\infty} a_n = +\infty$ . . . .