ANALISI MATEMATICA 1
Area dell'Ingegneria dell'Informazione

Appello del 9.07.2018

TEMA 2

Esercizio 1 [**6 punti**] Si consideri la funzione

$$\boxed{f(x) = \log|2e^{2x} - 3|.}$$

domain

i) Si determini il dominio D e si studi il segno di f;
ii) si determininio i limiti di f agli estremi di D e gli eventuali asintoti;
iii) si calcoli la derivata e si studi la monotonia di f, determinandone gli eventuali punti di estremo relativo ed assoluto; non è richiesta la derivata seconda;
iv) si disegni un grafico qualitativo di f.

Esercizio 2 [**6 punti**] Risolvere la disequazione

COMPLEX

$$\operatorname{Im}\left(\frac{1}{z}\right) \geq \frac{\operatorname{Im}(z^2 - \bar{z}^2)}{|z|^2}$$

rappresentandone le soluzioni sul piano di Gauss.

Esercizio 3 [**6 punti**] Calcolare il limite

$$\lim_{x\to+\infty} \frac{\left(\cosh\frac{1}{x} - 1\right)^2 - e^{-x}}{\left(\log(2+x) - \log x + \frac{2\alpha}{x}\right)^2}$$

al variare di $\alpha \in \mathbb{R}$.

Esercizio 4 [**6 punti**] Studiare al variare di $\alpha \in \mathbb{R}$ la convergenza della serie

$$\sum_{n=1}^{\infty} n^2 \arctan\left(\frac{4^{\alpha n}}{n^2}\right).$$

Esercizio 5 [**8 punti**] a) Calcolare una primitiva di

$$f(x) = \frac{x^2}{(x^2+4)(x^2+1)}$$

(sugg.: cercare una decomposizione dell'integrando del tipo $\frac{A}{x^2+1} + \frac{B}{x^2+4}$).
b) Studiare la convergenza dell'integrale generalizzato

$$\int_0^{+\infty} \log\frac{x^\alpha + 1}{x^\alpha + 4}\, dx.$$

al variare di $\alpha > 0$.
c) Calcolarlo per $\alpha = 2$.

1

2

30 - 35 $\boxed{}$

18 - 30, 30 cum laude

1) DOMAIN
2) INDUCTION
3) COMPLEX NUMBER

FUNCTION DOMAIN

1) find a, b | $f(x) = \dfrac{x-1-\sqrt{2x^2+x+1}}{\sqrt{x^4+ax^3+bx^2}}$ $D[f(x)] =]-\infty, -\frac{1}{2}[\cup]2, +\infty[$

$$\begin{cases} 2x^2+x+1 \geq 0 \longrightarrow \Delta < 0 \\ x^4+ax^3+bx^2 > 0 \longrightarrow x^2(x^2+ax+b) > 0 \end{cases}$$

$x^2>0 \ \forall x \neq 0$

$\{ x^4+ax^3+bx^2>0 \longrightarrow x^2(x^2+ax+b)>0$

$x^2>0\ \forall x\neq 0$

solutions of $x^2+ax+b>0$ must be $D[f(x)]$

$x<-\frac{1}{2} \vee x>2$

$\Updownarrow$

$\left(x+\frac{1}{2}\right)(x-2)>0$

$\Updownarrow$

$x^2+\left(\frac{1}{2}-2\right)x-1>0$

$x^2-\frac{3}{2}x-1>0$

$a=-\frac{3}{2}$, $b=-1$

IT IS THE ONLY PAIR (a,b)
THAT SOLVE THE PROBLEM

Thursday, October 13, 2022 1:29 PM

INDUCTION

1) n^2+n is even

- base case : $n=0$ $\quad 0^2+0=0$ is even
- induction step : if $P(n)$ is true than $P(n+1)$ is true

Hyp. n^2+n is even $\quad n^2+n=2m \quad m\in\mathbb{N}$

Prove that $(n+1)^2+(n+1)=2K \quad K\in\mathbb{N}$

$$n^2+2n+1+n+1=\underbrace{(n^2+n)}_{\text{is even}}+\underbrace{(2n+2)}_{\text{is even because is equal to } 2(n+1)=2K} \text{ is even}$$

2) $2^n+4^n\leq 5^n \quad n\geq 2$

- for $n=0$ $\quad 1+1 \not\leq 1$

$n=1$ $\quad 2+4\not\leq 5$

$n=2$ $\quad 4+16\leq 25$

- HP: $2^n+4^n\leq 5^n \rightsquigarrow 2^n+4^n-5^n\leq 0$

Prove $2^{n+1}+4^{n+1}\leq 5^{n+1}$

$$2\cdot 2^n+4\cdot 4^n\leq 4\underbrace{(2^n+4^n)}_{\leq 5^n}\leq 4\cdot 5^n\leq 5\cdot 5^n=5^{n+1}$$

3) $11 \mid 9^{n+1}+2^{6n+1} \quad n\geq 0$

divisible by 11

- $n=0$ $\quad 9^1+2^1=11=11m \quad m\in\mathbb{N}$
- HP $\quad 9^{n+1}+2^{6n+1}=11m \quad m\in\mathbb{N}$

$$9^{n+1}=11m-2^{6n+1}$$

$$9^{(n+1)+1}+2^{6(n+1)+1}=9^{n+2}+2^{6n+7}=9\cdot 9^{n+1}+2^6\cdot 2^{6n+1}$$

$$=9(11m-2^{6n+1})+2^6 2^{6n+1}$$

$$=\underbrace{99m}_{\text{divisible by 11}}+(2^6-9)2^{6n+1}$$

$64-9=55$ divible by 11

- $\displaystyle\sum_{i=0}^{n} i^3=\left(\sum_{i=0}^{n} i\right)^2 \quad n\geq 0$

$$\sum_{i=0} i^3 = \left(\sum_{i=0} i\right)$$

- $n=0$: $0 = 0$
- HP $\sum_{i=0}^{n} i^3 = \left(\sum_{i=0}^{n} i\right)^2$ $\rightarrow$ $\underbrace{0^3+1^3+\ldots n^3}+(n+1)^3 = \sum_{i=0}^{n} i^3 + (n+1)^3$

$$\rightarrow \sum_{i=0}^{n+1} i^3 = \underbrace{\sum_{i=0}^{n} i^3}_{HP} + (n+1)^3 = \left(\underbrace{\sum_{i=0}^{n} i}_{=\frac{n(n+1)}{2}}\right)^2 + (n+1)^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 =$$

$$= \frac{n^2(n+1)^2 + 4(n+1)^3}{4}$$

$$\left(\sum_{i=0}^{n+1} i\right)^2 = \left[\frac{(n+1)(n+2)}{2}\right]^2$$

$$= \frac{(n+1)^2}{4}\underbrace{\left(n^2+4(n+1)\right)}_{= n^2+4n+4 = (n+2)^2} = \frac{(n+1)^2(n+2)^2}{4}$$

COMPLEX

1) $\left|1+i - \frac{i}{1-2i}\right|$

$\rightarrow$ $a+ib$: $\frac{i}{1-2i}\left(\frac{1+2i}{1+2i}\right) = \frac{i(1+2i)}{1+4}$

$(1-2i)(1+2i) = 1^2 + (2i)\cancel{(-2i)} + \cancel{2i} - \cancel{2i}$ $= 1+4$; $= 4$

$$1+i - \frac{i(1+2i)}{5} = 1+i - \left(\frac{-2+i}{5}\right)$$

$$= 1+i+\frac{2}{5}-\frac{i}{5} = \frac{7}{5}+\frac{4}{5}i$$

$$\text{modulus}\left(\frac{7}{5}+\frac{4}{5}i\right) = \sqrt{\left(\frac{7}{5}\right)^2+\left(\frac{4}{5}\right)^2} = \frac{1}{5}\sqrt{49+16} = \frac{\sqrt{65}}{5}$$

2) $\sqrt{1+\sqrt{3}i}$ $\Rightarrow$ $z = 1+\sqrt{3}i$

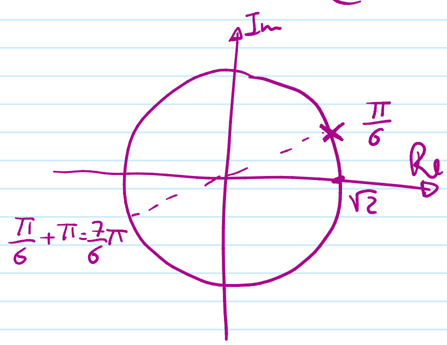
$\rightarrow$ $|z| = \sqrt{1+3} = 2$

$\angle z$: $\frac{Re(z)}{|z|} = \cos\theta$

$$\angle z : \begin{cases} \dfrac{Re(z)}{|z|} = \cos\theta \\ \dfrac{Im(z)}{|z|} = \sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} \cos\theta = \frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}}{2} \end{cases} \rightarrow \theta = \frac{\pi}{3} \rightarrow z = 2e^{i\left(\frac{\pi}{3} + 2\pi k\right)}$$

$$\sqrt{z} \longrightarrow \begin{cases} \rho_{12} = \sqrt{|z|} = \sqrt{2} \\ \alpha_{12} = \dfrac{\theta + 2k\pi}{2} = \dfrac{\frac{\pi}{3} + 2k\pi}{2} = \dfrac{\pi}{6} + k\pi \end{cases}$$



$$\alpha_1 = \sqrt{2}\, e^{i\frac{\pi}{6}} = \sqrt{2}\cos\frac{\pi}{6} + i\sqrt{2}\sin\frac{\pi}{6} = \frac{\sqrt{6}}{2} + i\frac{\sqrt{2}}{2}$$

$$\alpha_2 = \sqrt{2}\, e^{i\frac{7}{6}\pi} = -\frac{\sqrt{6}}{2} - i\frac{\sqrt{2}}{2}$$

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Saturday, 15 October 2022 16:17

Calculate the function domain:

2) $(1-x^2)^{\frac{x}{\ln|1-x^2|}}$

3) $\sqrt{kx^3+(k+1)x^2+x}$

4) $\log\left(\frac{1-3x}{x+2}\right)$ $\longrightarrow$ find k s.t. $\log\left(\frac{1-3x}{kx+2}\right)$ is odd/even function

5) find k | $f(x)=\frac{2}{4x^2+4x+1-2k}$ $D[f(x)]=\mathbb{R}$

Verify through induction;

4) $4 \mid 5^n-1$

5) $4 \mid 9^n+3$

7) $(1+a)^n \geq 1+na$

8) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

9) $\sum_{j=0}^{n} x^j = \frac{1-x^{n+1}}{1-x}$

10) $\sum_{k=0}^{n} \frac{1}{k^2} < 2$

11) $3+\sum_{i=0}^{n}(3+5i) = \frac{(n+1)(5n+6)}{2}$

Calculate the complex expressions:

Calculate the complex expressions :

2) $\left|\left(\frac{1+i}{1-i}-1\right)^2\right|$

3) convert $\frac{1}{3+3i}$ into (ρ, θ)

4) convert $\frac{4i}{\sqrt{3}+i}$ into (ρ, θ)