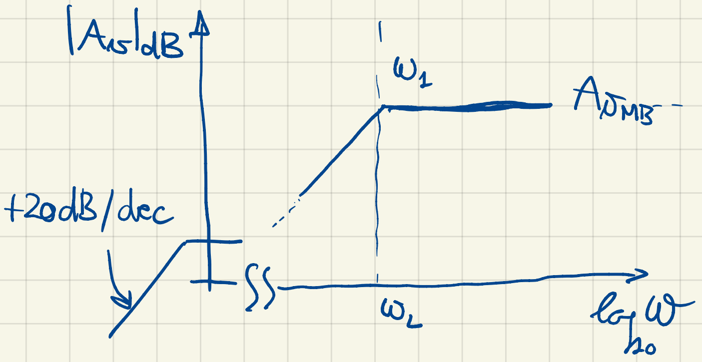


$$A_v(s) = \frac{s\tau_0 \,(1 + s\tau_z)}{1 + a_1 s + a_2 s^2} \cdot A_{vMB}$$

$\frac{1}{R_E C_E}$

#1 APPROACH ⟷ "BRUTE FORCE" APPROACH ☹ → TO DO AS EXERCISE

#2 APPROACH ⟵ "EDUCATED" APPROACH ☺ → USE THEOREMS

$$a_1 = R_S^0 C_S + R_E^0 C_E$$

$$a_2 = C_S C_E R_E^0 R_S^E .$$

$R_S^0 = R_s + R_1 /\!/ R_2 /\!/ R_{CER}^{IN}$ $\qquad R_{CER}^{IN} = r_\pi + (\beta_0 + 1) R_E \propto 10^4\ [\Omega]$

$R_E^0 = R_E /\!/ R_{CC}^{OUT}$ ⟵ $\qquad R_{CC}^{OUT} = \frac{R_1 /\!/ R_2 + r_\pi}{\beta_0 + 1} \propto 10^2\ [\Omega] \quad \left(\approx \frac{1}{g_m}\right)$

$R_S^E = R_s + R_1 /\!/ R_2 /\!/ R_{CE}^{IN}$ $\qquad R_{CE}^{IN} = r_\pi$

NOW a_1 AND a_2 ARE KNOWN ⟹ WE CAN FIND THE ROOTS OF $1 + a_1 s + a_2 s^2 = 0$

USING SCTC

$$\frac{a_1}{a_2} = \frac{1}{C_S R_S^{SC}} + \frac{1}{C_E R_E^{SC}} \stackrel{!}{=} 10^2\ \text{rad/s}$$

THIS TERM TENDS TO DOMINATE

$$R_S^{SC} = R_S^E \propto 10^3\ [\Omega]$$

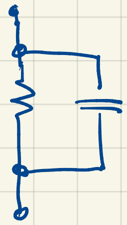
$$R_E^{SC} = R_E /\!/ \frac{R_s /\!/ R_1 /\!/ R_2 + r_\pi}{\beta_0 + 1} \propto 10^2\ [\Omega]$$

IF THERE IS A DOMINANT POLE $\omega_L \cong \frac{a_1}{a_2}$

DESIGN CRITERIA ARE $C_E \cong \frac{1}{R_E^{SC} \omega_L}$ $\qquad C_S \cong C_E$

WHAT ABOUT THE NUMERATOR? WE NEED TO FIND τ_0 AND τ_Z.

LET'S CONSIDER THE ADMITTANCE

$Y_E =$ 

$$Y_E = sC_E + \frac{1}{R_E} \overset{!}{=} \emptyset \rightarrow s = -\frac{1}{R_E C_E}$$

IF $Y_E = 0$ THEN $i_b = \emptyset$ $\left(i_b + \beta_0 i_b = \emptyset \right)$ AND THEN $v_o = \emptyset$!

WE HAVE A ZERO AT $\omega_z = \frac{1}{R_E C_E} \Rightarrow \underline{\underline{\tau_z = R_E C_E}}$

FINALLY, WE CAN FIND τ_0

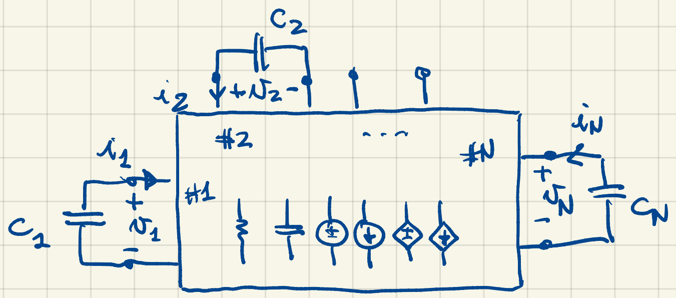
$$\lim_{\omega \to +\infty} A_v(j\omega) = A_{v_{MB}} \cdot \frac{\tau_0 \tau_z}{a_2} = A_{v_{MB}} \Rightarrow$$

$$\Rightarrow \tau_0 = \frac{a_2}{\tau_z} = \frac{C_S \cancel{C_E} R_E^0 R_S^E}{R_E \cancel{C_E}} = C_S \cdot \frac{R_E^0}{R_E} \cdot R_S^E$$

THE ANALYSIS IS NOW COMPLETE! TO BE COMPARED WITH BRUTE FORCE APPROACH!!

LESSON #3

BASIC NETWORK ANALYSIS



LINEAR CIRCUIT NETWORK

HYPOTHESIS:

1. CAPACITORS ARE THE ONLY REACTIVE COMPONENTS
2. CAPACITORS ARE ELECTRICALLY INDEPENDENT
3. THE RANK OF MATRIX $[R]$ IS FULL

N CAPACITORS $\Rightarrow$ $n = N = m$

LET'S CONSIDER THE NETWORK MATRIX $[R]$

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & & \\ \vdots & & & \\ R_{2N} & & & R_{NN} \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

$$R_{ij} \triangleq \left. \frac{v_i}{i_j} \right|_{i_k = \emptyset \ \forall k \neq j}$$

$$\boxed{R_{ii} = \frac{v_i}{i_i} \triangleq R_i^0}$$

(1) $\bar{v} = [R]\ \bar{i}$

AND THE PORT EQUATION

$$\begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = - \begin{bmatrix} C_1 & & & \emptyset \\ & C_2 & & \\ & & \ddots & \\ \emptyset & & & C_N \end{bmatrix} \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \\ \vdots \\ \dot{v}_N \end{bmatrix}$$

(2) $\bar{i} = -[C] \cdot \dot{\bar{v}}$

(1) AND (2) YIELD $\bar{v} = -[R][C]\dot{\bar{v}}$ (3)

REWRITING (3) MULTIPLYING TIMES $[C^{-1}][G]$ $G = R^{-1}$

$$\dot{\bar{v}} = \underbrace{-[C^{-1}G]}_{}\bar{v}$$

STATE MATRIX OF THE CIRCUIT

LET'S CALCULATE THE CHARACTERISTIC POLYNOMIAL OF THE NETWORK

$$\det\left[sI_N + C^{-1}G\right] = \emptyset \qquad \text{MULTIPLYING TIMES } \det[RC] \neq \emptyset$$

$$\Longleftrightarrow \quad \det\left[sRC + I_N\right] = \emptyset$$

LET'S SOLVE THE EQUATION (MANUALLY) FOR $N=3$

$$\det\begin{bmatrix} sR_{11}C_1+1 & sR_{12}C_1 & sR_{13}C_1 \\ sR_{21}C_2 & sR_{22}C_2+1 & sR_{23}C_2 \\ sR_{31}C_3 & sR_{32}C_3 & sR_{33}C_3+1 \end{bmatrix} =$$

$$= \left(sR_{11}C_1+1\right)\left[\left(sR_{22}C_2+1\right)\left(sR_{33}C_3+1\right) - s^2R_{32}R_{23}C_2C_3\right] +$$

$$- sR_{12}C_1\left[sR_{21}C_2\left(sR_{33}C_3+1\right) - s^2R_{23}R_{31}C_2C_3\right] +$$

$$+ sR_{13}C_1\left[s^2R_{21}R_{32}C_2C_3 - sR_{31}C_3\left(sR_{22}C_2+1\right)\right].$$

$$a_1 = R_{11}C_1 + R_{22}C_2 + R_{33}C_3 = \sum_{i=1}^{3} R_i^0 C_i$$ PROOF OF THEOREM #1

$$a_2 = R_{22}R_{33}C_2C_3 + R_{11}R_{22}C_1C_2 + R_{11}R_{33}C_1C_3 - R_{32}R_{23}C_2C_3 +$$

$$- R_{12}R_{21}C_1C_2 - R_{13}R_{31}C_1C_3 =$$

$$= \sum_{i=1}^{2}\sum_{j=i+1}^{3} C_iC_j\left(R_{ii}R_{jj} - R_{ij}R_{ji}\right) = \sum_{i=1}^{2}\sum_{j=i+1}^{3} C_iC_j R_i^0\left(R_j^0 - \frac{R_{ij}R_{ji}}{R_i^0}\right)$$

BUT WE CAN PROVE THAT

$$R_j^0 - \frac{R_{ij}R_{ji}}{R_i^0} = R_j^i$$

DEMONSTRATION: LET'S CALCULATE THE RELATION BETWEEN V_i, I_i AND V_j, I_j

$$\begin{cases} v_i = R_{ii}\, i_i + R_{ij}\, i_j \\ v_j = R_{jj}\, i_j + R_{ji}\, i_i \end{cases}$$

LET'S ASSUME THAT $v_i \equiv \emptyset$ (PORT i IS SHORTED!)

THEN

$$i_i = -\frac{R_{ij}\, i_j}{R_{ii}}$$

WHICH YIELDS

$$v_j = \left(R_{jj} - \frac{R_{ij} R_{ji}}{R_{ii}} \right) i_j$$

$$\left.\frac{v_j}{i_j}\right|_{v_i \equiv 0} \triangleq R_j^{i} = R_{jj} - \frac{R_{ij} R_{ji}}{R_{ii}} \qquad \text{c.v.d.}$$

IN THE SAME WAY, WE CAN ALSO PROVE THAT

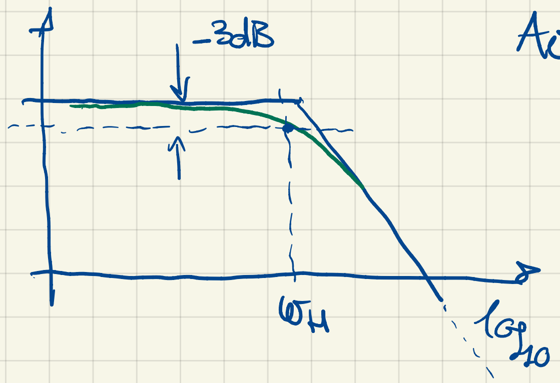
$$R_i^{j} = R_i^{0} - \frac{R_{ji} R_{ij}}{R_j^{0}}$$

FROM WHICH, IN CONCLUSION, FOLLOWS THAT

$$R_i^{j} R_j^{0} = R_j^{i} \cdot R_i^{0} \qquad \blacksquare$$

PROOF OF THEOREM #2

HIGH FREQUENCY RESPONSE



$$A_{V_{HF}}(s) = A_{V_{MB}} \frac{N(s)}{1 + a_1 s + a_2 s^2 + \dots + a_N s^N}$$

IN THIS CASE

$$N > m$$

ORDER OF THE NUMERATOR

$N - m \longrightarrow$ HIGH FREQUENCY ROLL-OFF

$$N - m = \begin{cases} 1 & -20 \text{ dB/dec} \\ 2 & -40 \text{ dB/dec} \\ \vdots & \end{cases}$$

REMEMBERING THAT

$$a_1 = \tau_1 + \tau_2 + \tau_3 + \dots$$

AND ASSUMING THAT THERE IS A DOMINANT POLE THEN

$$\omega_1 \ll \omega_2 < \omega_3 < \dots < \omega_N$$

THEN $a_1 \cong \tau_1 \Rightarrow$ WE ESTIMATE $\omega_H \cong \dfrac{1}{a_1}$

THE SO CALLED OCTC METHOD CONSISTS IN ASSUMING A DOMINANT POLE AND ESTIMATING

$$\omega_H \cong \frac{1}{\sum_{i=1}^{N} R_i^0 C_i}$$

IN THIS CASE $a_2 \cong \tau_1 \tau_2 \cong a_1 \tau_2 \Rightarrow \omega_2 \cong \dfrac{a_1}{a_2}$

AND THEREFORE

$$\frac{\omega_2}{\omega_1} = \frac{a_1^2}{a_2} \gg 1 \iff \text{WE HAVE A DOMINANT POLE}$$