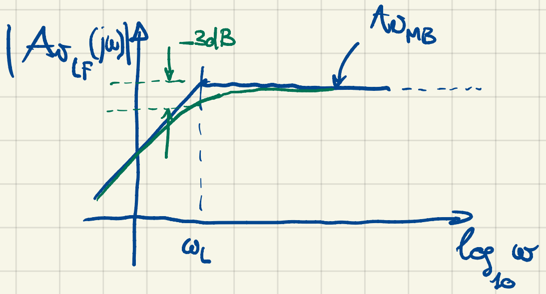
4/10/2022

◊ SHORT CIRCUIT TIME CONSTANT (SCTC) METHOD

PURPOSE: DERIVING AN ESTIMATION OF ω_L FROM THE CIRCUIT SCHEMATIC

$$A_{v_{LF}}(s) = A_{v_{MB}} \frac{N(s)}{1 + a_1 s + a_2 s^2 + \dots + a_n s^n}$$

LOW FREQUENCY RESPONSE



ORDER OF DENOMINATOR IS n

ORDER OF NUMERATOR IS m

$$m = n$$

$$\lim_{s \to +\infty} A_{v_{LF}}(s) = \text{CONST}$$

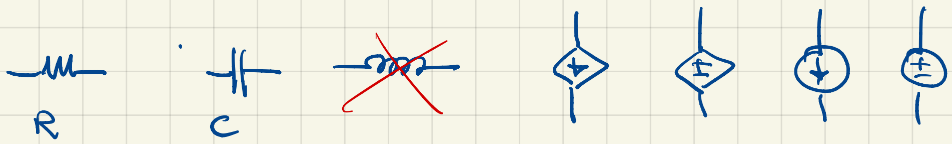
COEFFICIENTS a_i ARE REAL NUMBERS $\Rightarrow$ POLES ARE COMPLEX IN GENERAL, BUT ALWAYS IN COUPLES (CONJUGATE POLES)

IN AMPLIFIERS WHERE NO FEEDBACK IS APPLIED, POLES ARE OFTEN PURELY REAL AND, IN ANY CASE, NEGATIVE REAL PART. INDEED ANY ELECTRONIC AMPLIFIER IS NORMALLY OPEN LOOP STABLE

◊ Hypothesis: ALL POLES ARE REAL (AND NEGATIVE)

- AN AMPLIFIER CIRCUIT WITH CAPACITORS ONLY (NO INDUCTOR) AND NO FEEDBACK HAS REAL POLES
- THE CIRCUIT HAS BEEN LINEARIZED AROUND A SPECIFIC OPERATING POINT $\Longrightarrow$ FREQUENCY RESPONSE IS A SMALL SIGNAL ATTRIBUTE OF AN AMPLIFIER

LINEAR MODEL COMPONENTS



$$D(s) = 1 + a_1 s + a_2 s^2 + \dots + a_n s^n = \prod_{i=1}^{n} (1 + s\tau_i)$$

WITHOUT LOSS OF GENERALITY WE ASSUME THAT

$$\tau_1 < \tau_2 < \tau_3 < \dots < \tau_n$$

$$a_1 = \tau_1 + \tau_2 + \dots \tau_m$$

$$a_2 = \tau_1 \tau_2 + \tau_1 \tau_3 + \dots + \tau_{m-1}\tau_m$$

$$a_{m-1} = \tau_1 \tau_2 \dots \tau_{m-1} + \tau_1 \tau_2 \dots \tau_{m-2}\tau_m + \dots + \tau_2 \tau_3 \dots \tau_{m-1}\tau_m$$

$$a_m = \tau_1 \tau_2 \tau_3 \dots \tau_m$$

$$\frac{a_{m-1}}{a_m} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \dots + \frac{1}{\tau_m} = \omega_1 + \omega_2 + \dots \omega_m$$

HIGHEST FREQUENCY

a_1, a_2 AND $\frac{a_{m-1}}{a_m}$ CAN BE FOUND FROM THE CIRCUIT APPLYING THE FOLLOWING THEOREMS

FOR A LINEAR CIRCUIT WITH **N** INDEPENDENT CAPACITORS

THEOREM # 1

$$a_1 = \sum_{i=1}^{N} C_i \cdot R_i^0$$

R_i^0 : RESISTANCE SEEN FROM CAPACITOR C_i WHEN THE REMAINING ONES ARE **OPEN**

THEOREM # 2

$$a_2 = \sum_{i=1}^{N-1} \sum_{k=i+1}^{N} C_i C_k R_i^0 R_k^i = \sum_{i=1}^{N-1} \sum_{k=i+1}^{N} C_i C_k R_k^0 R_i^k$$

WHERE R_i^k IS THE RESISTANCE SEEN FROM CAPACITOR C_i WHEN THE REMAINING ONE ARE OPEN EXCEPT C_k THAT IS **SHORTED**

THEOREM # 3

$$\frac{a_{m-1}}{a_m} = \sum_{i=1}^{N} \frac{1}{C_i R_i^{SC}}$$

WHERE R_i^{SC} IS THE RESISTANCE SEEN FROM CAPACITOR C_i WHEN THE REMAINING ONES ARE SHORT-CIRCUITED

◊ SCTC METHOD

#1 LINEARISE THE AMPLIFIER CIRCUIT AROUND THE OP

#2 TURN ALL INDEPENDENT SOURCES OFF

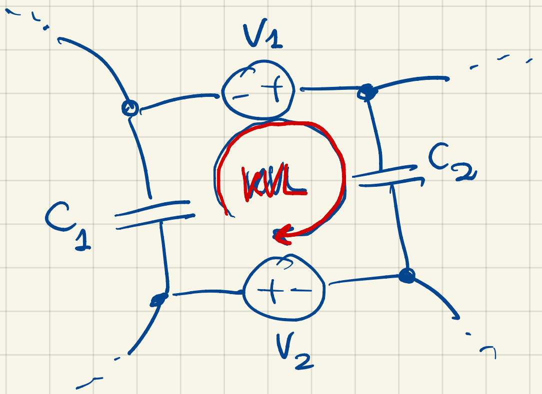
#3 ELIMINATE REDUNDANT CAPACITORS TO HAVE ONLY INDEPENDENT ONES IN THE CIRCUIT

#4 CALCULATE $\frac{a_{n-1}}{a_n}$ FROM THEOREM #3

#5 ASSUMING A DOMINANT POLE EXISTS, ESTIMATE ω_L AS

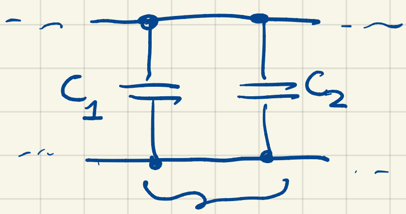
$$\omega_L \cong \frac{a_{n-1}}{a_n} = \sum_{i=1}^{N} \frac{1}{C_i R_i^{SC}}$$

INDEPENDENT CAPACITORS ?



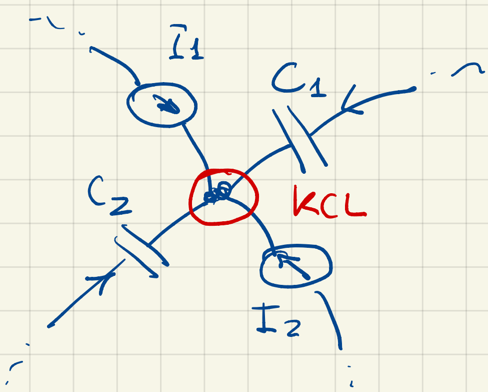
CAPACITORS IN A LOOP WITH VOLTAGE SOURCES ARE NOT INDEPENDENT!

AS A PARTICULAR CASE: $V_1 = V_2 = \emptyset$



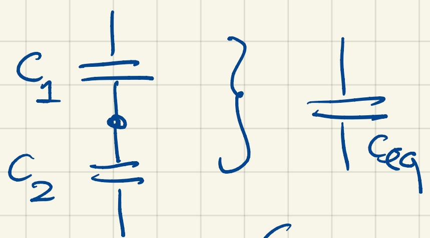
PARALLEL CONNECTED CAPS

$$C_{eq} = C_1 + C_2$$



CAPACITORS IN A NODE WITH CURRENT SOURCES ARE NOT INDEPENDENT

AS A PARTICULAR CASE, SETTING $I_1 = I_2 = 0$



SERIES CONNECTED CAPS

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

AFTER STRIPPING ALL DEPENDENT CAPACITORS OFF THE CIRCUIT

$N = m$

THE NUMBER OF CAPS IS EQUAL TO THE ORDER OF $D(s)$ i.e. TO THE NUMBER OF POLES.

SPECIAL CASE IS $m = N = \underline{\underline{2}}$. HERE WE CAN FIND:

$$a_1 = C_1 \cdot R_1^0 + C_2 R_2^0$$

$$a_2 = C_1 C_2 R_1^0 R_2^1 = C_1 C_2 R_2^0 R_1^2$$

THESE ARE THE EXACT COEFFICIENTS OF $D(s)$ (FOR A LINEAR CIRCUIT)

THEREFORE WE NOW KNOW $D(s) = 1 + a_1 s + a_2 s^2$ FROM WHICH WE CAN FIND P_1 AND P_2 (THE POLES OF THE NETWORK) $\Rightarrow$ WE CAN FIND ω_L EXACTLY

BUT WE CAN ALSO CALCULATE

$$\frac{a_1}{a_2} = \frac{\cancel{C_1 R_1^0}}{\cancel{C_1 R_1^0} C_2 R_2^1} + \frac{\cancel{C_2 R_2^0}}{\cancel{C_2 R_2^0} C_1 R_1^2} = \underbrace{\frac{1}{C_2 R_2^1} + \frac{1}{C_1 R_1^2}}_{\omega_1 + \omega_2}$$

$R_2^1 = R_2^{SC}$ AND $R_1^2 = R_1^{SC}$

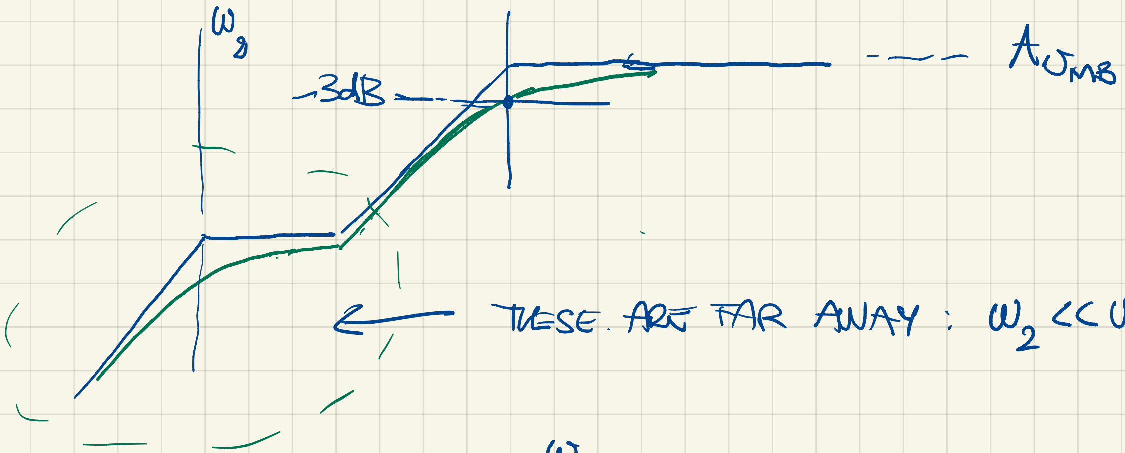
SO WE HAVE DEMONSTRATED THEOREM #3, AT LEAST FOR THIS CASE

IF WE FORGET ABOUT ALL THIS AND JUST TAKE

$$\omega_L \cong \frac{a_1}{a_2}$$ WE ARE APPROXIMATING THE VALUE OF ω_L

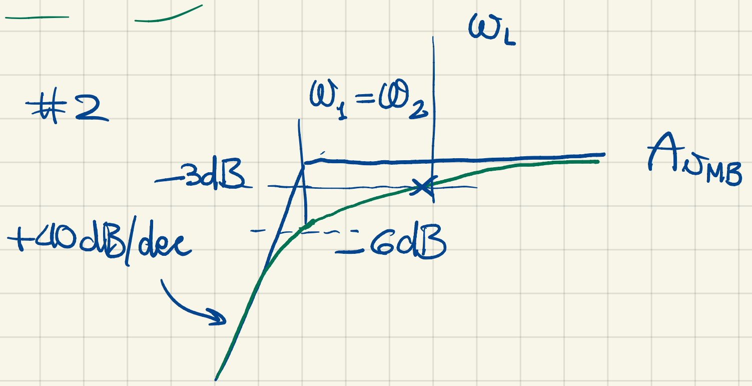
THE ESTIMATION IS ACCURATE AS LONG AS THERE IS A DOMINANT POLE, WHOSE FREQUENCY IS MUCH LARGER THAN THE REST.

CASE #1 $\omega_1 \cong \omega_L$ ACCURACY IS GOOD



THESE ARE FAR AWAY : $\omega_2 \ll \omega_1$

CASE #2

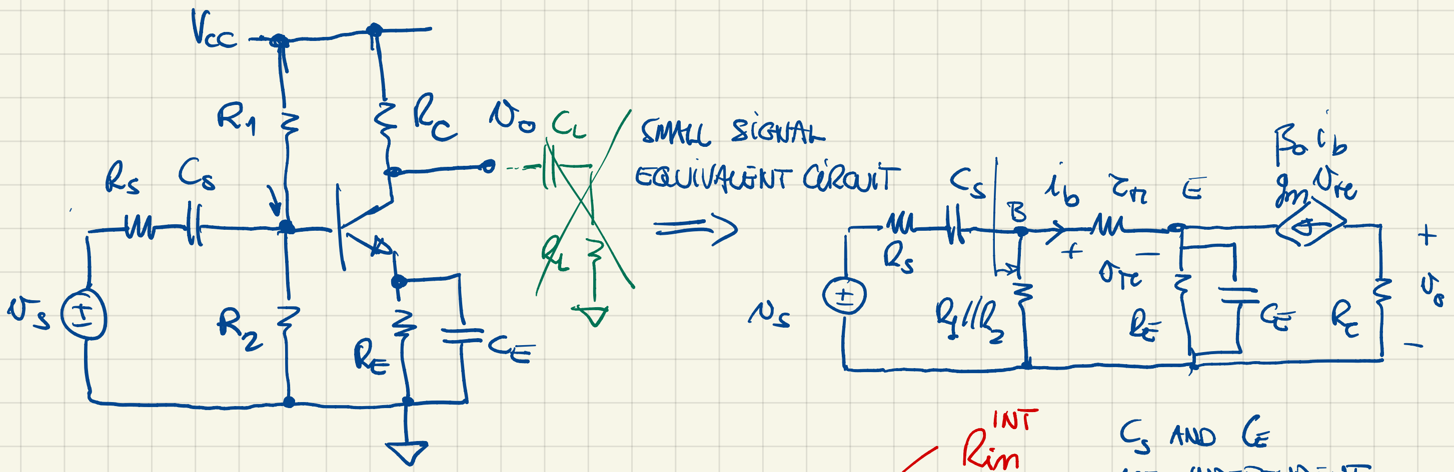


ESTIMATION OF ω_L

APPLYING SCTC WE WILL CONSIDER $\hat{\omega}_L = 2\omega_1$

QUESTION : HOW FAR IS ω_L FROM $\hat{\omega}_L$? FIND OUT AS AN EXERCISE !

EXAMPLE : LET'S CONSIDER A CE AMPLIFIER



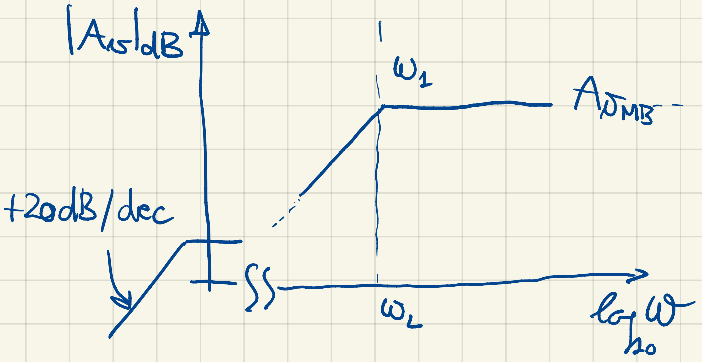
R_{in}^{INT}

C_S AND C_E ARE INDEPENDENT

$$A_{v_{MB}} = -g_m R_C \cdot \alpha_i = -\underbrace{\frac{R_1 // R_2 // r_\pi}{R_S + R_1 // R_2 // r_\pi}}_{<1} \cdot g_m R_C$$

INTRINSIC GAIN OF THE CE STAGE

INPUT ATTENUATION FACTOR



$$A_v(s) = \frac{s\tau_0 (1 + s\tau_z)}{1 + a_1 s + a_2 s^2} \cdot A_{vMB}$$

#1 APPROACH ⟷ "BRUTE FORCE" APPROACH ☹ → TO DO AS EXERCISE

#2 APPROACH ⟵ "EDUCATED" APPROACH ☺ → USE THEOREMS

$a_1 =$ USE THEOREM 1
$a_2 =$ USE THEOREM 2 $\Big\} \rightarrow \omega_L$

OR ELSE

$\frac{a_1}{a_2} = \ldots$ USE THEOREM 3 $\rightarrow \omega_L$