$$f: \overset{x}{E} \longrightarrow \overset{y}{G}$$

injective and surjective = bijective

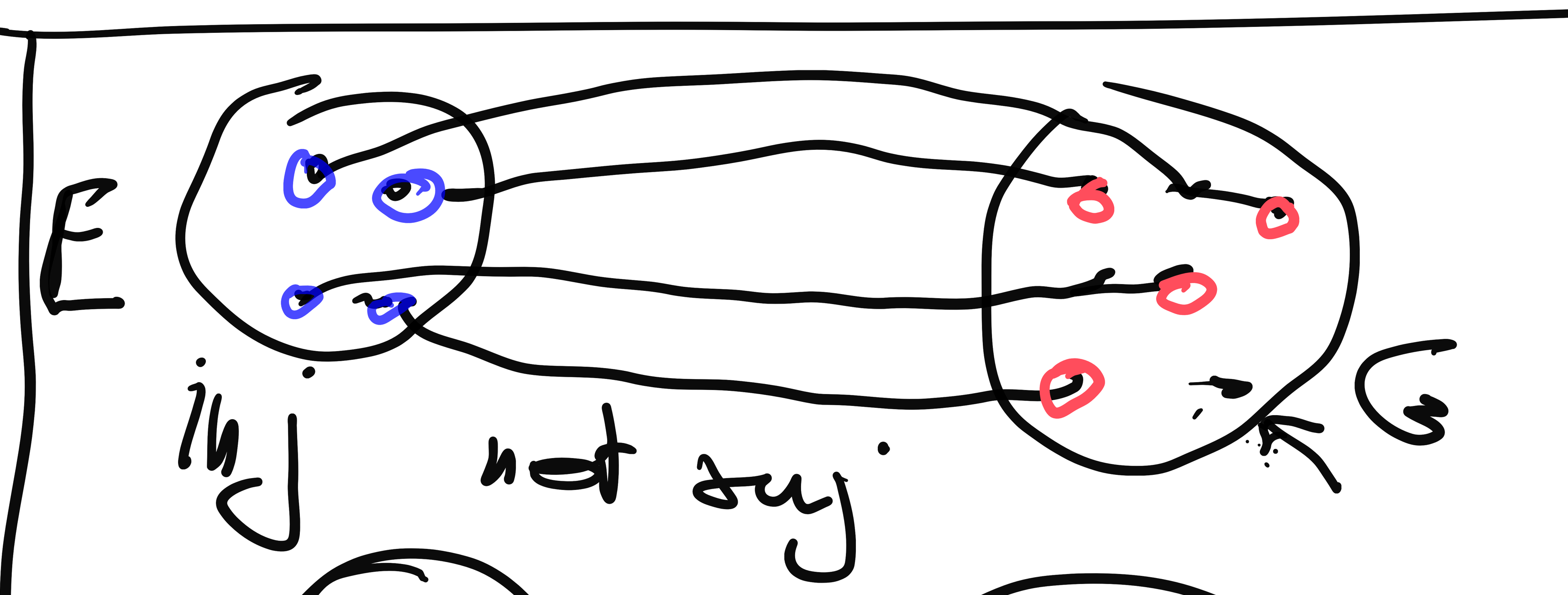
We call

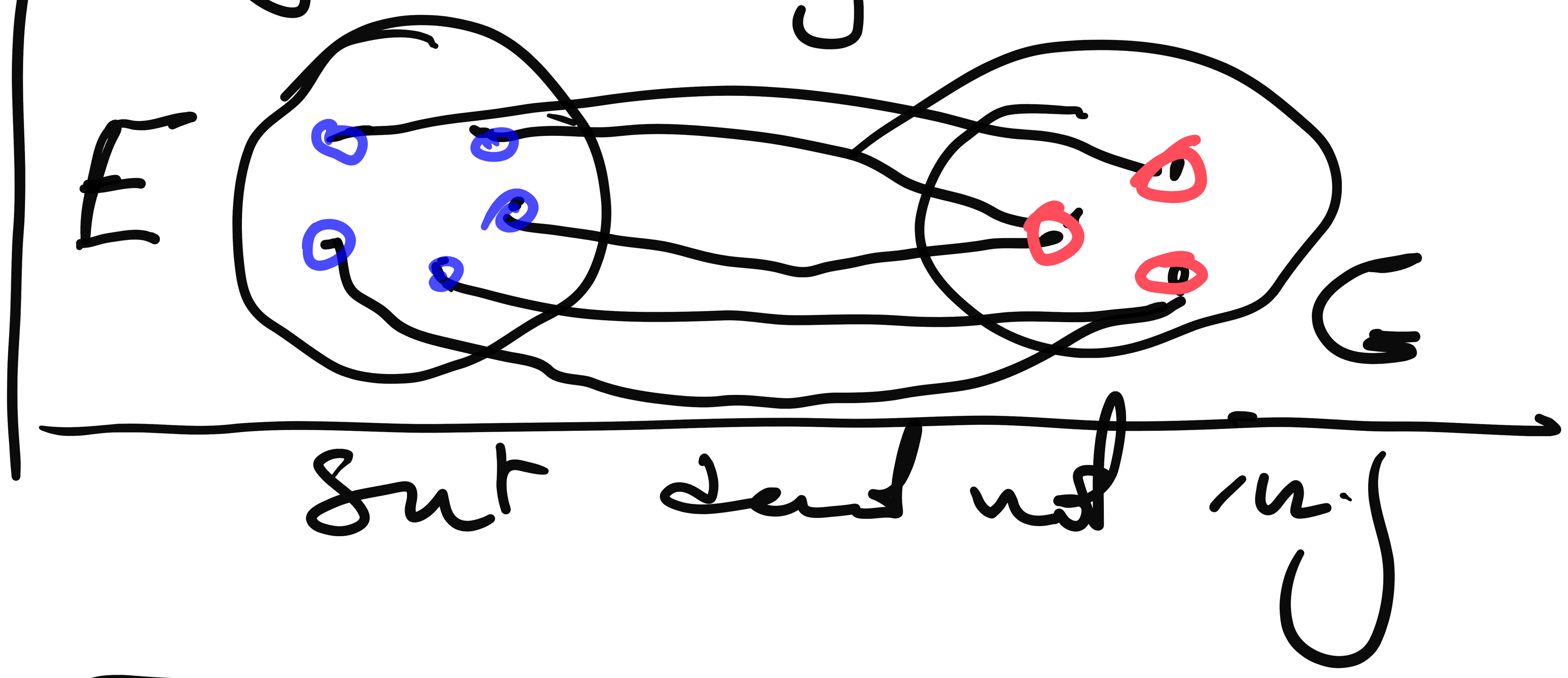
$h: G \longrightarrow F$

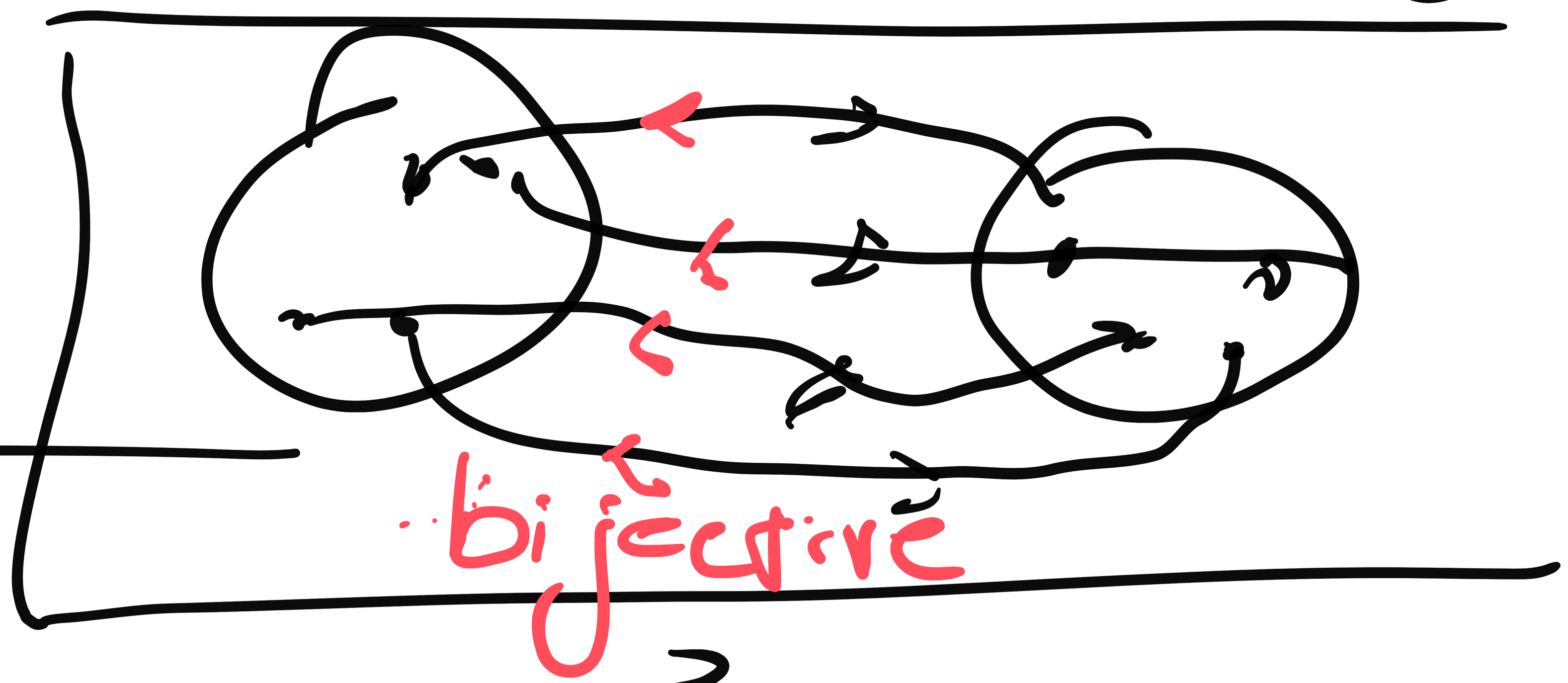
the inverse of f

if $h(y) =$ the unique element $x \in E$ s.that $f(x) = y$

$\forall y \in G$







Example:

$$f: x \longmapsto x^2$$
$$[0,+\infty[\longrightarrow [0,+\infty[$$

the inverse is

$$h: [0,+\infty[\longrightarrow [0,+\infty[$$
$$h(y) = \sqrt{y}$$
$$y \overset{?}{=} f(h(y)) = \left(\sqrt{y}\right)^2 = y$$

Notation for the $\boxed{\text{inverse}}$

$f : E \longrightarrow G$ bijective

we use f^{-1} to denote the inverse of f

Examples

$f(x) = x^2$ $\qquad f^{-1}(y) = \sqrt{y} = \boxed{y^{\frac{1}{2}}}$

$f(x) = x^7$ $\qquad f^{-1}(y) = \sqrt[7]{y} = \boxed{y^{\frac{1}{7}}}$

$a > 0 \qquad a \neq 1$

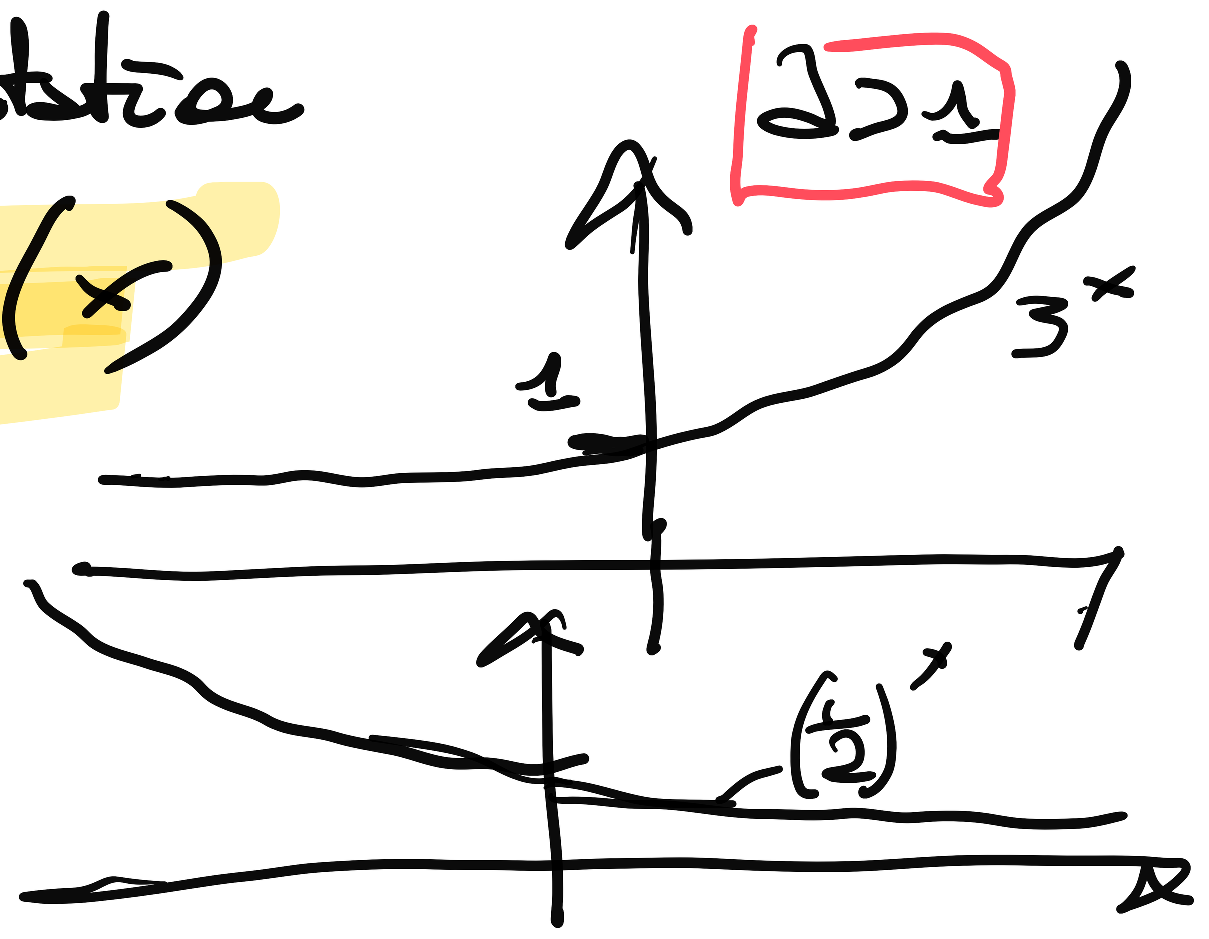
$\mathbb{R} \longrightarrow]0, +\infty[$ exponential with base a

$x \longmapsto a^x$

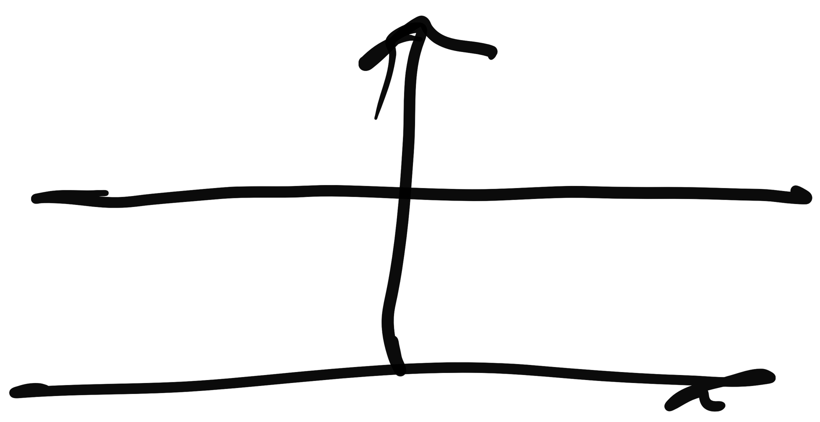
notation $\exp_a(x)$

$a = 3 \qquad x \longmapsto 3^x$

$a = \frac{1}{2} \qquad x \longmapsto \left(\frac{1}{2}\right)^x$



$a = 1 \qquad x \longmapsto (1)^x = 1$



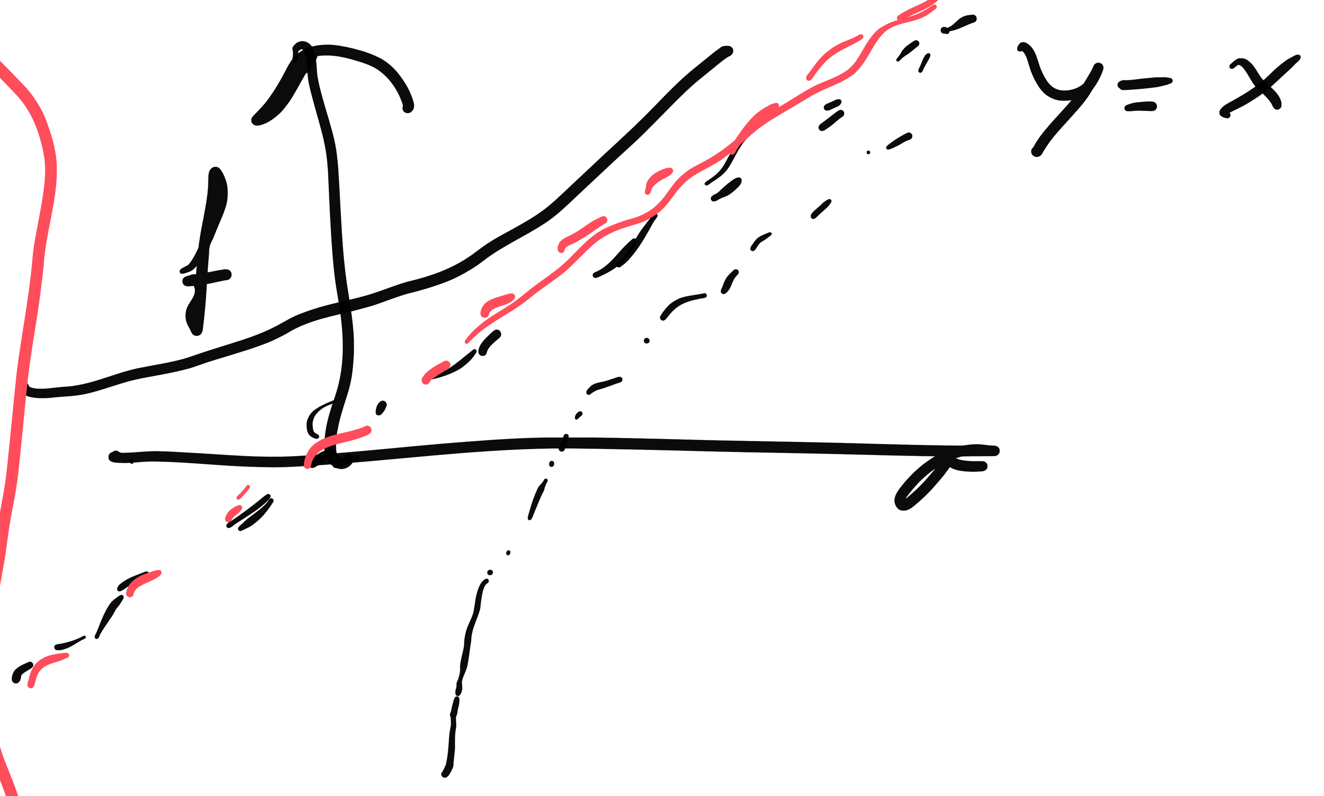
If $a \neq 1 \quad a > 0$

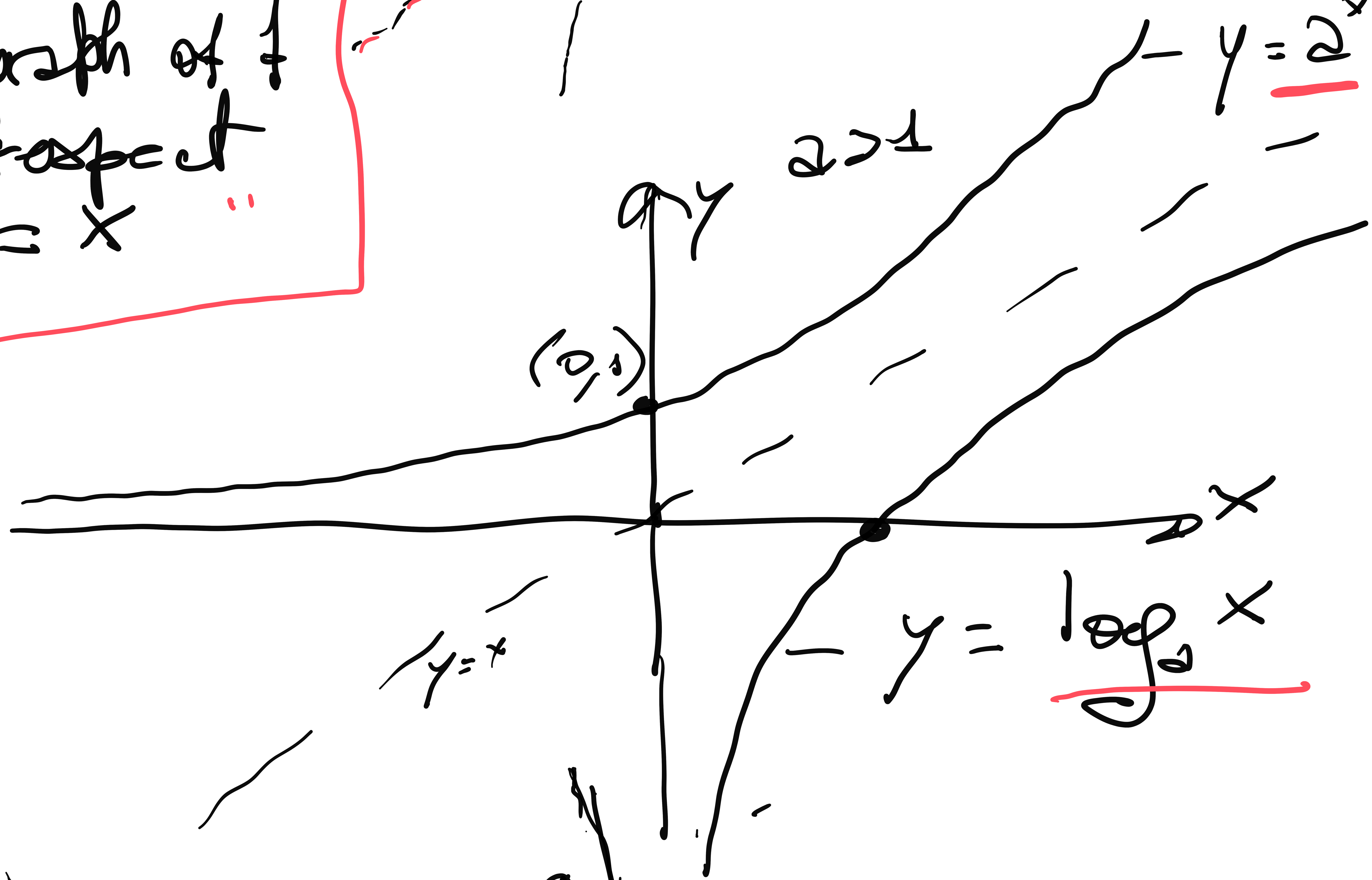
then $x \longmapsto a^x = \exp_a(x)$ is bijective, so it is invertible, the inverse

$x = \boxed{\log_a(y)}$ = the number x such that $a^x = y$

$a^{\log_a y} = y$

4 The graph of the inverse of f is symmetric to the graph of f with respect to $y = x$ "





$0 < a < 1$

$$\log_a :]0, +\infty[\longrightarrow \mathbb{R}$$

---

f is "invertible"

$\Updownarrow$

$$f(f^{-1}(y)) = y$$
$$f^{-1}(f(x)) = x$$

$$y \longmapsto f(f^{-1}(y)) = y$$

$$x \longmapsto f^{-1}(f(x)) = x$$

$z \mapsto z$ is called identity $id(z) = z$

$f(f^{-1})$ is the identity

$f^{-1}(f)$ is the identity

$$G \xrightarrow{f^{-1}} E \xrightarrow{f} G$$

$$y \longmapsto f^{-1}(y) \longmapsto f(f^{-1}(y)) = y$$

More in general COMPOSITION OF MAPS

$k : A_1 \to A_2$

$p : A_2 \to A_3$

$$\begin{tikzcd} A_1 \arrow[r, "k"] & A_2 \arrow[r, "p"] & A_3 \end{tikzcd}$$

$$\begin{tikzcd} a_1 \arrow[r, mapsto] \arrow[rr, bend right, mapsto] & k(a_1) \arrow[r, mapsto] & p(k(a_1)) \end{tikzcd}$$

$$\underbrace{p \circ k}(a_1) := p(k(a_1))$$

$$\underbrace{p \circ k} : A_1 \longrightarrow A_3$$

"p composed with k"

"p after k"

f is invertible if

→ $f^{-1} \circ f = \mathrm{id}$ (on the domain of f)

→ $f \circ f^{-1} = \mathrm{id}$ (on the codomain of f)

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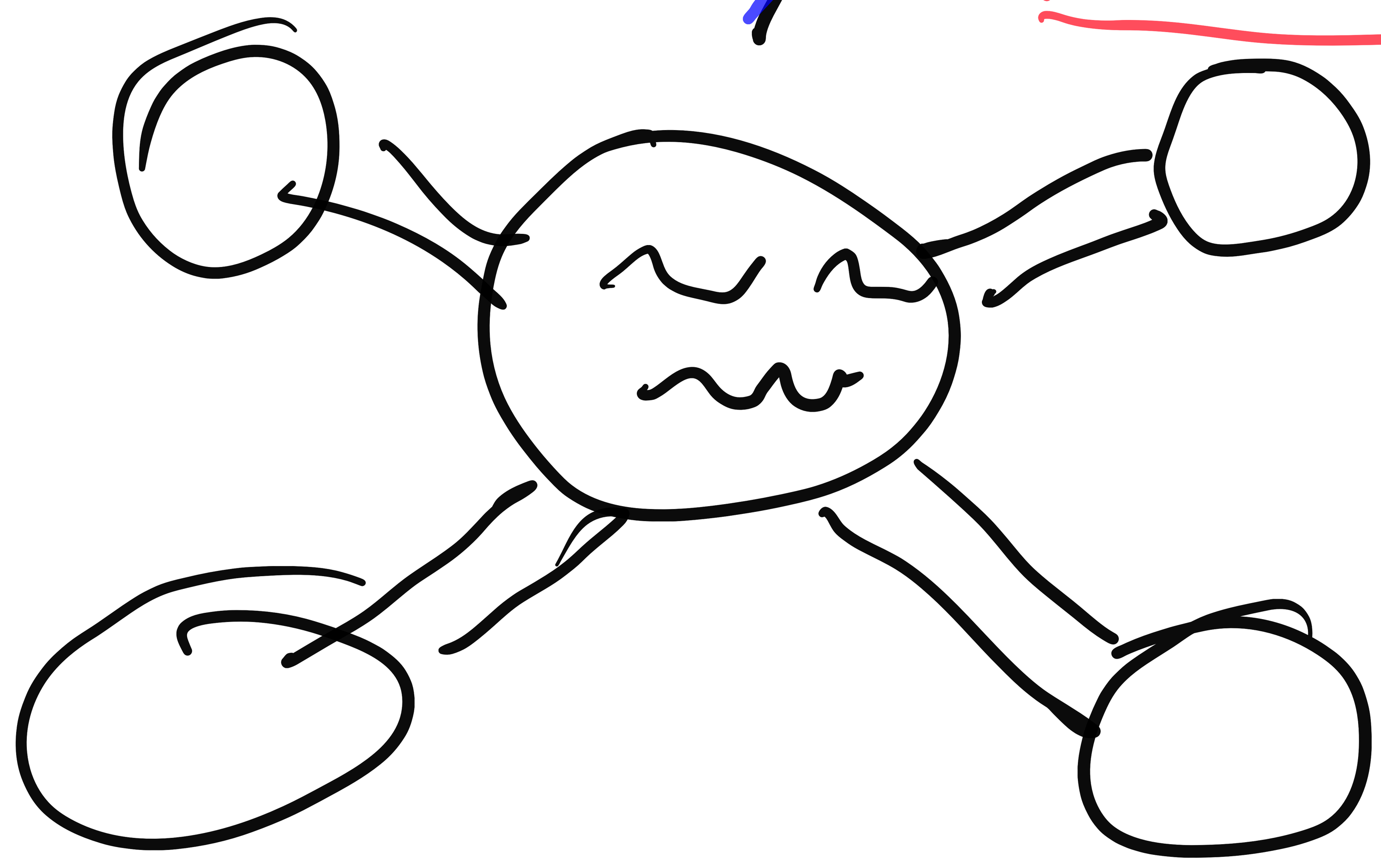
$f: E \to G$ bijective

$$\boxed{f^{-1}(x)} \neq \boxed{\left(f(x)\right)^{-1}} = \boxed{\frac{1}{f(x)}}$$

inverse of the number $f(x)$

inverse function of f.



Properties of $\log_a$

1) $a > 1$ $\log_a$ is strictly increasing

$0 < a < 1$ " " " decreasing

ii) $\boxed{\log_a(x \cdot y) = \log_a(x) + \log_a(y)}$

$$a^{\log_a(x \cdot y)} = a^{(\log_a(x) + \log_a y)}$$

$$x \cdot y = a^{\log_a(x)} \cdot a^{\log_a(y)}$$

$$x \cdot y \overset{\text{II}}{=} x \cdot y$$

also

$$\log_2\left(\frac{x}{y}\right) = \log_2(x) - \log_2(y)$$

IV) $\boxed{\log_a x = \log_a b \, \log_b x}$

$\forall\, x > 0 \quad a, b > 0 \quad a, b \neq 1$

$\Updownarrow$

$$a^{\log_a x} = a^{(\log_a b \cdot \log_b x)}$$

$\Updownarrow$

$$x = \left(a^{\log_a b}\right)^{\log_b x}$$

$\Updownarrow$

$$x = b^{\log_b x} = x$$

$$\log_2\left(\sqrt{x-1}\right) \geq \log_4 (x+4) \quad \text{Solve it in } \mathbb{R}$$

$$\log_2\left(\sqrt{x-1}\right) \geq \underline{(\log_4 2)}\,\underline{(\log_2 (x+4))}$$

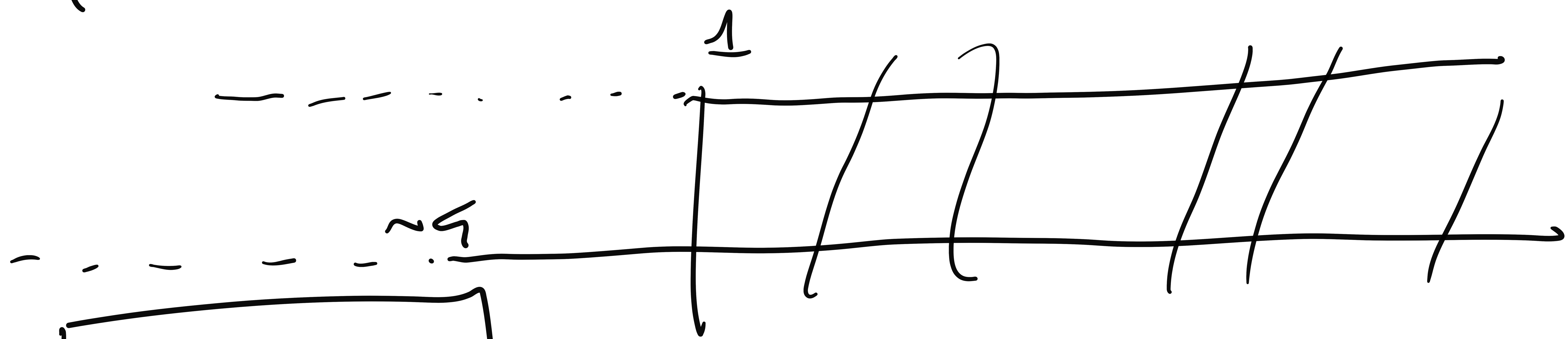
$$\log_4 2 = \frac{1}{2} \qquad \log_2\left(\sqrt{x-1}\right) \geq \frac{1}{2} \log_2 (x+4)$$

$$\left(\alpha \log_b x = \log_b x^{\alpha}\right)$$

$$\boxed{\log_e\left(\sqrt{x-1}\right) \geq \log_e\left(\left(x+4\right)^{\frac{1}{2}}\right)}$$

At the beginning we have to declare where the involved functions are defined

$$\begin{cases} x-1 \geq 0 \\ x-1 > 0 \\ x+4 > 0 \end{cases} \qquad \begin{cases} \boxed{x > 1} \\ x > -4 \end{cases}$$



$$\boxed{x > 1}$$

$$\Updownarrow$$

$$\sqrt{x-1} \geq \sqrt{(x+4)}$$

$$\Updownarrow$$

$$x-1 \geq x+4$$

$-1 \geq 4$ never

no solutions.

$$\log_2 \sqrt{x-1} + 2 \geq \log_4 (x+4)$$

solve this

$$E = \left\{ \underbrace{1 + \sqrt{\frac{n}{n+1}}} \quad n \in \mathbb{N} \right\}$$

Prove that

1) $\inf E = 1$ $\quad$ $\min E = 1$

2) $\sup E = 2$ $\quad$ $\max E$ does not exist

Proving 1) i) is $\underline{\forall n \in \mathbb{N}}$ $\quad 1 + \sqrt{\frac{n}{n+1}} \geq 1$

ii) $\forall \beta > 1 \quad \exists\, n$ s.t

$$1 \leq \underbrace{1 + \sqrt{\frac{n}{n+1}} \leq \beta}_{?}$$